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Does Automation Technology increase Wage?

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Abstract

This paper examines the relationship between automation technology diffusion and the wage. In this model, producers either choose automation or non-automation technology, whichever is more profitable. Further, when the producers introduce automation technology, they must pay fixed costs, which differ between industries. The main results of this paper indicate that the improving the productivity of automation technology promotes automation diffusion, decreases labor share, and also decreases the wage when the level of automation technology diffusion is sufficiently high.

Keywords: automation, the wage, labor share decline, technology choice

JEL codes: E24, J23, O3

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1 Introduction

In recent years, many studies show that labor income share has been declining over the last several decades. Technological advancement is considered one of the main factors influencing this phenomenon (Karabarbounis and Neiman, 2014; Grossman et al., 2017; Alvarez-Cuadrado et al., 2018).[1] Acemoglu and Restrepo (2018b) and Autor et al. (2018) show that the diffusion of labor-saving automation technology, such as robots and machines, is one of the most important recent technological advances underlying this decline in labor income share. Labor income share decreases as labor is replaced by capital in the production process through the diffusion of automation. Bergholt et al. (2021) find that automation has become an increasingly important factor of the labor share decline since the early 2000s.

To analyze the benefit that individual workers gain from automation diffusion, this paper studies not only the effect on labor income share but also the effect on the wage. While many empirical studies examine the effect of automation technology on the wage, no consensus exists on whether the automation diffusion increases or decreases the wage. Graetz and Michaels (2018) analyze data on robot adoption and conclude that, on average, an industry's adoption of industrial robots positively affects worker wages. On the other hand, Acemoglu and Restrepo (2020a) focus on local labor markets data divided by commuting zones between 1990 and 2007 and demonstrate that the introduction of industrial robots negatively affects the wage. Further, Dauth et al. (2019) applies the approach used by Acemoglu and Restrepo (2020a) to data of Germany and Chiacchio et al. (2018) applies it to data of 6 countries in EU (Finland, France, Germany, Italy, Spain and Sweden). The research of the former shows that

[1] Several types of drivers of the decline in labor income share have been identified in the literature. Elsby et al. (2013) and Boehm et al. (2020) show that globalization, which promotes offshoring and reduces the ratio of labor-intensive industry in OECD countries, substantially contributes to a decline in labor income share. Autor et al. (2017, 2020), and Barkai (2020) emphasize the rising market concentration of sales and markups; as market sales are concentrated on a smaller fraction of productive firms, their markup rates increase, and the income share of profits compresses the income share of workers.

the average wage does not increase as a result of robot adoption, while the that of the latter shows the decrease in the average wage but the result is not robust.

A theoretical study on the relationship between automation diffusion and the wage is conducted by Acemoglu and Restrepo (2018b). Using the task-based model presented by Zeira (1998),[2] they analyze the impact of advancement in automation technology (i.e. when automation technology becomes available for more tasks) on the wage. The adoption of automation technology by enterprises displaces workers but simultaneously increases aggregate production and the labor demand. As a result of this combination of positive and negative effects on the labor demand, Acemoglu and Restrepo's static analysis shows that the wage might decline. However, the extended model taking capital accumulation into account concludes that the advancement in automation technology increases the wage because capital accumulation has an additionally positive effect.

This paper presents a general equilibrium model in which the wage could decrease even in the presence of capital accumulation. The model shows that an improvement in automation technology decreases the wage if the present level of automation diffusion is sufficiently high, while it increases the wage otherwise. This result suggests that the varying results of the aforementioned empirical researches may be consistently explained by focusing specifically on the level of automation diffusion.

The model is constructed using the task-based model presented by Zeira (1998) and the over-lapping generations model by Diamond (1965). Households live for two periods; they inelastically supply labor as workers in the first period and manage production units in industries as managers in the second period. They choose either automation technology or non-automation technology for the management of the production units to maximize profits. To introduce automation technology, they must pay fixed costs, which are different across industries. The fixed cost is considered as one of the main factors of technologically choice in previous theoretical studies (e.g.,

[2] Acemoglu and Autor (2011); Nakamura (2009); Nakamura and Nakamura (2008, 2019); Nakamura and Zeira (2018); Yuki (2016); Hémous and Olsen (2021); Aghion et al. (2019); Martinez et al. (2018) etc utilize the task-based model.

Jovanovic and Lach, 1989; Hall, 2004). Sirkin et al. (2015) specify the fixed costs, estimate their magnitude, and state that their burden impedes the adoption of robots.

In the model, managers choose the more profitable technology in each industry; automation technology is chosen in the industries that feature lower fixed costs. As automation technology improves, more industries with higher fixed costs can adopt automation technology.

The main result of this paper is that an improvement in automation productivity causes a decrease (an increase) in the wage when the level of automation diffusion is sufficiently high (low). Whether the wage decreases or not is associated with the level of difficulty that an industry encounters in introducing automation, that is, how high the fixed costs are in the industry. The advancement of diffusion displaces workers while it increases aggregate production. The former has a negative effect and the latter a positive effect on the wage. As automation technology diffuses, the fixed costs in industries in which automation technology is newly introduced become higher. To pay higher fixed costs in these industries, the returns from using automation technology must be larger. Because the returns are proportional to aggregate production, how much aggregate production increases is associated with how much fixed costs increase in newly automated industries. This model shows that unless the fixed costs in such industries increase rapidly, the positive effect of the increase in aggregate production on the wage is smaller than the negative effect of displacement when the level of automation diffusion is sufficiently high. In that case, in contrast to Acemoglu and Restrepo (2018b), the wage decreases when the level of automation diffusion is sufficiently high, even in the presence of capital accumulation.

In this model, the labor income share decreases with automation diffusion. While the income share of managers increases with the diffusion, the income share of workers decreases. Because the impact of the latter is greater than that of the former, the sum of their labor income shares decreases. This result is consistent with the trend over the last several decades.

This paper also considers two extensions. In the first extension, we introduce a technology frontier. Because of the technology frontier, managers who face high fixed costs cannot utilize automation technology; otherwise, they would use it. Acemoglu and Restrepo (2018b) adopt this assumption, and the extension of this model highlights the difference between their model and ours. In the model of Acemoglu and Restrepo (2018b), expanding the technology frontier increases the wage when capital accumulation exists. By contrast, our analysis shows that even in the presence of capital accumulation, the expansion of the technology frontier decreases the wage when the level of the frontier is sufficiently high. The second extension considers a policy that subsidizes the fixed costs. Because the technology choice depends on the return net of the fixed cost, such a policy ultimately affects the diffusion of automation technology. The benefits of subsidy policy vary across households. Subsidy rates are shown to exist that which improve the welfare of all households. The analysis shows that there exists subsidy rates that improve the welfare of all households.

The rest of the paper is organized as follows. Section 2 constructs the model. Section 3 solves the equilibrium and describes the conditions of the existence of a unique steady state. Section 4 examines the comparative statics and presents an extension of the model considering the creation of new intermediate goods. Section 5 discusses the differences between this paper and Acemoglu and Restrepo (2018b), and modifies the model by introducing an automation technology frontier to clearly demonstrate the differences. Section 6 introduces a subsidy policy into the model. Section 7 concludes this paper

.

2 The Model

The consumer side of the model is based upon the two-period OLG model presented by Diamond (1965). Each household supplies labor as a worker and chooses a technology during the first period of her life and then manages a production unit producing intermediate goods in an industry during the second period. There are many industries over which households are distributed during the first period. The producer side consists of managers and final goods producers. The final goods producers use intermediate goods supplied by the managers for production, and the final goods are utilized for consumption and savings. The factor of production for operating a unit in an industry producing intermediate goods is either labor or capital; managers choose the more profitable input. When capital is chosen, managers can make use of automation technology,[3] which enables them to produce without using labor, but instead they must pay fixed costs that are heterogeneous between industries.

2.1 Households

There is a unit measure of households in each industry $j \in [0,1]$. Households live for two periods. During the first (young) period, they inelastically supply labor as workers and allocate their wages to consumption and savings. During the second period (old), they manage production units in industries of intermediate goods to receive profits and they consume the returns from their savings and profits. Let us consider a household born in period t and working in industry j. Her utility maximization problem is expressed as follows:

$$U_t(j) = \max_{c_t^y, c_{t+1}^o} \log(c_t^y(j)) + \beta \log(c_{t+1}^o(j)), \quad \beta \in (0,1) \tag{1}$$

$$s.t. \; c_t^y(j) + s_t(j) = w_t, \tag{2}$$

$$c_{t+1}^o = R_{t+1} s_t(j) + \pi_{t+1}(j),$$

where $U_t(j)$, $c_t^y(j)$, $c_{t+1}^o(j)$, $s_t(j)$, and $\pi_{t+1}(j)$ represent lifetime utility, consumption during the first period, consumption during the second period, savings, and the profits that she receives during the second period, respectively; R_{t+1} is the gross interest rate, and β is the discount factor. She is assigned to industry $j \in [0,1]$ before she decides consumption and savings during the first period. Thus, the decision accounts for the profits she generates during the second period.

From her optimal conditions, savings $s_t(j)$ are expressed by the following equation:

$$s_t(j) = \frac{\beta}{1+\beta} w_t - \frac{\pi_{t+1}(j)}{(1+\beta)R_{t+1}}. \tag{3}$$

[3] This structure of the production side is similar to that of Zeira (1998), who is the pioneer in the literature that examines mechanization by using the task-based model.

2.2 Producers

There are two types of producers: final goods producers and managers producing intermediate goods. Final goods producers combine a unit measure of intermediate goods. Their technology is

$$\log Y_t = \int_0^1 \log y_t(j) dj, \tag{4}$$

where Y is the amount of final goods and $y(j)$ is the amount of the intermediate good $j \in [0, 1]$. [4] From the profit maximization problem of final goods producers, the demand for each intermediate good $y(j)$ is

$$y_t(j) = \frac{Y_t}{p_t(j)}, \tag{5}$$

where $p_t(j)$ is the price of intermediate good j.

A manager operating a production unit in industry j produces intermediate good j. In each industry, a unit measure of managers exists.[5] To produce intermediate goods, managers must choose between non-automation technology and automation technology. When managers choose non-automation technology, they employ only labor as their factor of production. On the other hand, when they choose automation technology, they use only capital as their factor of production. To introduce automation technology, they must pay fixed costs before commencing operations. The choice of technology is made at the end of the first period.

Acemoglu and Restrepo (2018b) assume that labor productivity varies between tasks. Capital has a comparative advantage when used in tasks with low labor productivity and thus automation technology is adopted in such tasks. In our model, the

[4] In the model of Acemoglu and Restrepo (2018b), the range of intermediate goods is from $N-1$ and N, where $N \geq 1$ and N increases over time. They interpret a change in N as the creation of new tasks. We set $N = 1$ here, but Section 4.3 develops a model in which the range is from $N-1$ to N and analyzes the effect of an increase in N on the wage.

[5] This assumption ensures that the intermediate goods market is competitive, in spite of households' immovability between industries.

fixed costs of introducing automation technology are the source of different comparative advantages across industries. Sirkin et al. (2015) specify the components of the cost to introduce a spot welding, an example of automation technology, and estimate their magnitudes. They state that the price of the robotic hardware and software to introduce a spot welding robot is only one-quarter of the total cost.[6] Some of other components of the cost are fixed costs, such as the cost of programming and integrating a robot into a factory and the cost of safety structures that protect workers and robots themselves. These fixed costs account for two-thirds of the total. The argument that fixed costs inhibit the adoption of new technologies has been the subject of many theoretical papers (e.g., Jovanovic and Lach, 1989; Hall, 2004).

Because managers gain profits from producing intermediate goods, they choose the more profitable technology as follows:

$$\pi_{t+1}(j) = \max\{\pi^l_{t+1}(j), \pi^k_{t+1}(j)\}, \tag{6}$$

where $\pi^l_{t+1}(j)$ and $\pi^k_{t+1}(j)$ are the profits from producing intermediate goods by using non-automation technology and automation technology in industry j, respectively.

When a manager utilizes non-automation technology, she can produce intermediate goods without incurring any fixed costs. The production function is given by as following:[7]

$$y^l_t(j) = l_t(j), \tag{7}$$

where $l_t(j)$ is the amount of labor inputs. The profit maximization problem then becomes

$$\pi^l_t(j) = \max_{l_t(j)} \; p^l_t(j) y^l_t(j) - w_t l_t(j), \tag{8}$$

[6] Spot welding is a type of electric resistance welding used to weld various sheet metal products, and it achieves a labor-saving in a welding line (Takayama and Takahashi, 2014). It is applied to production processes, such as the production of automobile and aerospace vehicles (Li and Duarte, 2018).

[7] For simplicity, the production function for non-automation technology is linear. The case in which the function is non-linear is discussed in section 4.2.

where w_t represents the wage. Since the production function is linear, the price of the intermediate goods produced by using non-automation technology is as follows:

$$p^l_t(j) = w_t. \tag{9}$$

Thus, the profit from non-automation technology is equal to zero.

The production function for automation technology is[8]

$$y^k_t(j) = \gamma k_t(j)^\alpha, \quad \alpha \in (0,1), \ \gamma > 0, \tag{10}$$

where $k_t(j)$ is the capital used in industry j and γ is the productivity of automation technology. The profit maximization problem for managers choosing automation technology is

$$\pi^k_t(j) = \max_{k_t(j)} \ p^k_t(j) y^k_t(j) - R_t k_t(j) - C(j), \tag{11}$$

$$where \ \ C(j) \equiv C \cdot j, \quad C > 0. \tag{12}$$

In the above equation, $C(j)$ represents the fixed cost function required to introduce automation technology into industry j and is paid for by final goods.[9] Since fixed costs are increasing in j, the comparative advantage of labor to capital is also increasing in j.[10] The first order condition is the following equation:

$$R_t = p^k_t(j)\gamma\alpha k_t(j)^{\alpha-1}. \tag{13}$$

By substituting this equation into (11), profit becomes as follows:

$$\pi^k_t(j) = \frac{1-\alpha}{\alpha}(p^k_t(j)\gamma\alpha)^{\frac{1}{1-\alpha}} R_t^{\overline{\frac{\alpha}{\alpha-1}}} - C(j). \tag{14}$$

3 Equilibrium

At equilibrium, the optimal conditions for consumers and producers and market clearing conditions for each market are satisfied. By using (5), (10) and (13), from

[8] Rosenfeld et al. (2004) study the productivity of foraging robots. They show that the marginal productivity is decreasing in the number of robots.

[9] Chen and Koebel (2017) estimates fixed costs by industry and show that they are different.

[10] In the model of Acemoglu and Restrepo (2018b), the productivity of labor depends on the industry index. Our setting differs, but it is the same in terms of the comparative advantage.

the intermediate goods market clearing condition, the price of intermediate good j produced by using automation technology $p_t^k(j)$ is given by the following:

$$p_t^k(j) = \frac{Y_t^{1-\alpha}}{\gamma}(\frac{R_t}{\alpha})^{\alpha}. \tag{15}$$

Substituting this equation into (14), $\pi_t^k(j)$ can be rewritten as

$$\pi_t^k(j) = (1-\alpha)Y_t - C(j). \tag{16}$$

The fixed costs are increasing in the industry index. This paper focuses on the case in which a threshold $j^* \in (0,1)$ exists and managers in industry $j \leq j^*$ choose automation technology.

Since the profit from non-automation technology is equal to zero, the return from automation technology for industry j_t^* is zero. Thus, by using (16) and (12),

$$j_t^* = \frac{(1-\alpha)Y_t}{C}. \tag{17}$$

From (13) and (15),

$$k_t(j) = \frac{\alpha Y_t}{R_t}. \tag{18}$$

The capital market clearing condition is

$$K_t = \int_0^{j_t^*} k_t(j)dj, \tag{19}$$

where K_t represents aggregate capital. Thus, the interest rate is equal to

$$R_t = \frac{j_t^* \alpha Y_t}{K_t}. \tag{20}$$

From (5), (7), and (9),

$$l_t(j) = \frac{Y_t}{w_t}. \tag{21}$$

Since the labor supply is unity, the labor market clearing condition is

$$1 = \int_{j^*}^{1} l_t(j)dj. \tag{22}$$

Thus, the equation for the wage equals the following:

$$w_t = (1 - j_t^*)Y_t. \tag{23}$$

Combining (18) and (21) with (4), (7) and (10) yield the amount of final goods Y_t as follows:

$$Y_t = \left[\gamma \left(\frac{K_t}{j_t^*}\right)^{\alpha}\right]^{j_t^*} \left(\frac{1}{1-j_t^*}\right)^{1-j_t^*}. \tag{24}$$

By using (17), this equation can be rewritten as

$$\frac{Cj_t^*}{1-\alpha} = \left[\gamma \left(\frac{K_t}{j_t^*}\right)^{\alpha}\right]^{j_t^*} \left(\frac{1}{1-j_t^*}\right)^{1-j_t^*}. \tag{25}$$

Thus, K_t equals

$$K_t = \left(\frac{Cj_t^*}{1-\alpha}\right)^{\frac{1}{\alpha j_t^*}} (1-j_t^*)^{\frac{1-j_t^*}{\alpha j_t^*}} j_t^* \gamma^{-\frac{1}{\alpha}} \equiv \Phi(j_t^*). \tag{26}$$

From the final goods market clearing condition, savings are used to produce capital for the next period. Thus, the equation for capital accumulation is

$$K_{t+1} = \int_0^1 s_t(j)dj. \tag{27}$$

By combining (3), (16), (17), (20), and (23), this equation can be rewritten as follows:

$$K_{t+1} = \frac{2\alpha\beta C}{(1-\alpha)\{2\alpha(1+\beta)+(1-\alpha)\}}(1-j_t^*)(j_t^*) \equiv \Psi(j_t^*). \tag{28}$$

Since $K_{t+1} = \Phi(j_{t+1}^*)$ from (26), the equation for capital accumulation is then converted to the dynamic equation for j^* as follows:

$$\Phi(j_{t+1}^*) = \Psi(j_t^*). \tag{29}$$

From this equation, the conditions of the existence and the stability of a unique steady state with $j^* \in (0,1)$ are obtained as the following propositions.[11]

Proposition 1. *When the parameter of the fixed cost function C is sufficiently low, a unique steady state exists in $j^* \in (0,1)$.*

[11] $j^* = 1$ and $j^* = 0$ do not hold in a steady state. When $j_t^* = 1$, since workers receive no income, no capital is left and no manager can choose automation technology in the next period (i.e., $j_{t+1}^* = 0$). When $j_t^* = 0$, workers make savings $K_{t+1} > 0$. When fixed cost function is (12), $\pi^k(0) > \pi^l(0) = 0$, and thus $j_{t+1}^* > 0$.

Proposition 2. *Suppose that a unique steady state in $j^* \in (0,1)$ exists.*[12] *When automation productivity γ is low enough or C is high enough that the level of automation diffusion j^* is less than $1/2$, this steady state is globally stable. If $j^* > 1/2$ and C is sufficiently low or high, a threshold of $j^* \in (1/2, 1)$ exists below which the steady state is locally stable.*

In Appendix A, we provide more formal conditions and proofs for Proposition 1 and 2 and illustrate the dynamics graphically. The analysis in this paper focuses only on the cases in which there exists a unique steady state that is globally and locally stable.

4 Analysis

4.1 Comparative Statics

In this section, we focus on the unique steady state and analyze the long-run effects on the wage and income shares of an improvement in automation technology productivity, γ, and of the cost-saving technological changes which reduces the parameter of the fixed cost function, C. The former effect is summarized in the following proposition. Proofs of the propositions presented in this section are provided in Appendix.B.

Proposition 3. *Suppose that Proposition 2 holds and thus a unique and locally stable steady state exists. If the productivity of automation technology γ is high (low) enough or if the fixed cost parameter C is low (high) enough that the level of automation diffusion j^* is greater (less) than $1/2$, an improvement in automation productivity γ decreases (increases) the wage in the long-run.*

This proposition shows that the wage decreases (increases) due to the improvement in the automation productivity when the threshold of a unique steady state j^* is

[12] Strictly speaking, this proposition requires the additional assumption that C is small enough that $\Phi(j^*)$ is an increasing function in the steady state as shown in Proposition A.2.

greater (less) than 1/2. From (17) and (23), the wage in the steady state is

$$w = (1 - j^*)Y, \tag{30}$$
$$= \frac{C}{1-\alpha} j^*(1 - j^*). \tag{31}$$

Lemma B.1 presented in Appendix.B shows that j^* increases as γ rises or C falls. (31) demonstrates that the level of j^* determines the sign of the marginal effect of γ on the wage. This proposition shows that when γ is high (low) enough or C is low (high) enough that $j^* > 1/2$ ($j^* < 1/2$), an increase in γ decreases (increases) the wage.

From (30), the wage is the product of the mass of industries employing labor $1 - j^*$ multiplied by the amount of final goods Y. When j^* increases, labor is substituted for capital in newly automated industries. Thus, this channel decreases labor demand and the wage. On the other hand, an increase in j^* causes an increase in Y from (17), thus pulls up the wage from (30). For automation technology to be introduced in industries with higher fixed costs, Y needs to be large. When $j^* > 1/2$ ($j^* < 1/2$), the negative (positive) effect is dominant, thus the wage decreases (increases) with an increase in γ. This result might appear to depend on the assumption that the fixed cost function is linear. We discuss how robust the result is when the fixed cost function is more general.

The following proposition shows the effect of a cost-saving technological change on the wage.

Proposition 4. *Suppose that Proposition 1 holds and thus a unique steady state exists. If γ is high enough or C is low enough that $j^* > 1/2$, the wage decreases with a decrease in C. If $j^* < 1/2$, there exists a threshold of C, C_{w2}, and the wage decreases with a decrease in C for any j^* when $C < C_{w2}$. When $C \geq C_{w2}$, the wage increases with a decrease in C if the level of j^* is intermediate in $(0, 1/2)$, while the wage decreases if j^* is sufficierntly high or sufficiently low.*

This proposition shows the effects of a cost-saving technological change that reduces the fixed cost of automation technology on the wage. When γ is high enough or C is low enough that $j^* > 1/2$, a decrease in C reduces the wage. When $j^* < 1/2$, it also reduces the wage if $C < C_{w2}$. Whereas the indirect effect of a decrease in C through an increase in j^* raises the wage when $j^* < 1/2$, the direct effect reduces the wage (see (31)). When the level of C is sufficiently low, the negative direct effect is larger than the positive indirect effect, and thus the wage decreases with a decrease in C. By contrast, when $j^* < 1/2$ and $C \geq C_{w2}$, there exists the region in which the wage increase by the technological change. The wage increases when j^* is intermediate in $(0, 1/2)$, whereas the wage decreases when j^* is large enough or small enough in $(0, 1/2)$.

From Proposition 3 and 4, when the technology level is high, that is γ is high enough or C is low enough that $j^* > 1/2$, the wage decreases with both an improvement of automation productivity and a cost-saving technological progress. On the other hand, when the technology level is low enough that $j^* < 1/2$, the wage increases with γ from Proposition 3. By contrast, as for the cost-saving technological progress, except for the case in which $C > C_{w2}$ and j^* is intermediate in $(0, 1/2)$, the wage decreases with a decrease in C from Proposition 4. These propositions suggest that when the technology level reaches a high enough state of advancement and thus, the diffusion level is sufficiently high, both types of technological progress lower the wage.

While many empirical researches examine the effect of automation diffusion on the wage, these studies obtain different results. Some researches show that the diffusion of automation increases the wage (e.g., Autor and Salomons, 2017; Graetz and Michaels, 2018). Others, such as Acemoglu and Restrepo (2020a), Dauth et al. (2019) and Chiacchio et al. (2018), show that the effect of automation on the wage is negative or ambiguous. Proposition 3 and 4 show that whether the technological progress promoting automation diffusion increases or decreases the wage is determined by the level of automation diffusion in the economy. Existing researches do not empirically examine the influence of the diffusion level upon the effect of an introduction of new automation technology on the wage. The results in this paper suggest a new empirical question regarding the relationship between automation diffusion and the wage.

The following proposition analyzes the effect of an increase in γ or a decrease in C

on the labor income share. In this model, workers and managers exist. The definition of the labor income share in SNA includes the income share of managers, so in the next proposition, the labor income share is the sum of the income shares of both workers and managers.[13]

Proposition 5. *When Proposition 2 holds and thus a unique and locally stable steady state exists, the income share of managers increases (decreases), and that of workers decreases (increases) with γ (C). The total income share of managers and workers decreases (increases) with γ (C).*

The income share of managers increases (decreases), and that of workers decreases (increases) with γ (C). Their total income share decreases (increases) with γ (C). This result is consistent with many researches, such as Elsby et al. (2013) and Karabarbounis and Neiman (2014) that show a decline in the labor income share in many countries since the 1980s.[14]

As Piketty and Saez (2003) and Jones and Kim (2018) suggest, the gap between of income share of workers and entrepreneurs has been widening in the US since the 1970s.

(Figure 1 around here)

Figure 1 illustrates graphically the effect of a productivity improvement of automation technology on the extent of automation technology diffusion, the amount of final goods production, wage, the income share of workers, the income share of managers, and the total labor income share at the steady state. The parameters are set to be $\alpha = 0.3,\ \beta = 0.82,$ and $C = 4.4$. The time preference β is a standard value for OLG model. The parameter value of the production function of automation technology α does not change qualitative results. The value of C and the range of γ are chosen to ensure the unique stable steady state. As shown in Proposition 3, when j^* is over 1/2

[13] Let $\mathcal{C}$ be aggregate fixed cost, and Π be aggregate profit. The income share of managers and workers are $\Pi/(Y - \mathcal{C})$ and $w/(Y - \mathcal{C})$, respectively.

[14] The definition of labor income share in Karabarbounis and Neiman (2014) does not include the element equivalent to profits in our model.

(γ is approximately 5), aggregate capital and the wage decrease with γ. As shown in Proposition 5, the income share of workers decreases, the income share of managers increases, and the total income share decreases with γ.

4.2 Why Can Wage Decrease?

In the model of Acemoglu and Restrepo (2018b) and many other task-based models analyzing mechanization, the wage increases because of technological change when the technology choice depends upon producers' decision.[15][16] By contrast, our model shows that the wage decreases with an increase in the automation productivity γ when the level of automation diffusion j^* is sufficiently large. This section explains the mechanisms underlying the decrease in the wage presented in Proposition 3 and 4.

We assume that the fixed cost function, which was linear in the previous section as (12), is a continuous, differentiable, and increasing function $C(j)$, $C' > 0$. The equation determining the technology choice threshold j^* is transformed from (17) into the following:

$$C(j^*) = (1 - \alpha)Y. \tag{32}$$

In industry j^*, the price of intermediate goods is equal to the average cost because the profit from automation technology is zero. The price depends on the demand for the intermediate goods, which is proportional to Y from (5). The average cost includes

[15] Acemoglu and Restrepo (2018b) suggest the possibility of a decrease in the wage when some producers cannot utilize automation technology because of a technology frontier. The model considering the technology frontier will be discussed in Section 5. Berg et al. (2018) also obtain a result similar to that of Acemoglu and Restrepo (2018b) using a different model.

[16] Acemoglu and Restrepo (2020b) analyze the effect of automation diffusion on labor markets, including the wage by focusing on a skill difference. Then, they suggest theoretically and empirically that the automation diffusion enhances the gap between the wages of low- and high-skilled labor and the decline in the wage of low-skilled labor. In their model, low-skilled labor performs tasks that automation technology is able to replace. Still, high-skilled labor is not replaced because they are engaged in tasks that automation technology cannot perform. However, as noted by Autor (2015), the progress of automation tends to encroach upward in abstract tasks in that high-skilled labor is apt to engage. Thus, the approach of Acemoglu and Restrepo (2020b) has a limitation. This paper proposes a mechanism for the decrease in the wage that does not depend on skill differences.

the fixed cost. Thus, as shown in (32), Y is related to the fixed cost in industry j^*. Since the fixed cost function is increasing in j^*, an increase in Y enhances automation diffusion j^*. Intuitively, this is because, as the output of final goods increases, the difference between the returns from automation and non-automation technology increases, enabling more industries to pay these fixed costs and thus introduce automation.

The wage in the steady state is

$$\begin{aligned} w &= (1 - j^*)Y, \\ \Leftrightarrow \log w &= \log(1 - j^*) + \log Y. \end{aligned} \tag{33}$$

Thus, the effect of an increase in j^* on the logarithm of the wage is

$$\frac{d \log w}{dj^*} = -\frac{1}{1 - j^*} + \frac{d \log Y}{dj^*}. \tag{34}$$

Following Acemoglu and Restrepo (2018b), we call the first term the *displacement effect*, and the second term the *productivity effect*. The diffusion of automation technology captured by an increase in j^* displaces labor because the industries in which automation technology is newly introduced cease employing labor. The displacement effect has a negative effect on the labor demand, and then on the wage. On the other hand, automation diffusion has a positive effect on the wage through the increase in the output of final goods. When the negative effect dominates the positive effect, the wage decreases.

The scale of the productivity effect depends on the rate of change in the fixed cost function from (32):

$$\frac{d \log Y}{dj^*} = \frac{d \log C(j^*)}{dj^*}. \tag{35}$$

From (34) and (35):

$$\frac{d \log w}{dj^*} = -\frac{1}{1 - j^*} + \frac{d \log C(j^*)}{dj^*}. \tag{36}$$

The rate of change of the fixed cost function determines whether the wage increases or decreases with the automation diffusion. When the rate of change is small enough, the wage decreases since the negative displacement effect dominates the positive productivity effect. In the model presented in Section 2, $d \log w / dj^* < 0$ when $j^* > 1/2$

because of the linear fixed cost function. The condition of linearity can be relaxed. For example, when the fixed cost function is an exponential function $C(j) = \exp(a \cdot j)$, where $a > 1$, $d \log w/dj^* < 0$ when $j^* > 1 - 1/a$. Even when $\lim_{j \to 1} C(j) = \infty$, the wage can decrease when j^* is high enough. Let us consider the case of the following function.

$$C(j) = \frac{\exp(a_1 j + a_3)}{(1-j)^{a_2}}, \ where\ 0 < 1 - a_2 < a_1,\ a_2 > 0.$$

This function satisfies that $\lim_{j \to 1} C(j) = \infty$ and $\lim_{j \to 1} C'(j)/C(j) = \infty$. In this case, $d \log w/dj^* < 0$ when $j^* > (a_1 + a_2 - 1)/a_1$. Therefore, the results of Proposition 3 and 4 hold even if the fixed cost function increases with j much more rapidly than the linear function. For simplicity, we assume the linear function is specified as (12) except for this subsection.

4.3 Effects of New Industries

Acemoglu and Restrepo (2018b) point out that the progress of automation technology creates new tasks and show theoretically that this task creation, which increases the tasks performed by non-automation technology, stimulates labor demand and increases the wage. This subsection examines the effect of the creation of new industries on wages. In Acemoglu and Restrepo (2018b), the displacement of labor by capital due to the diffusion of automation technology is viewed as the factor that reduces the wage, and the task creation is viewed as the countervailing factor that increases the wage. By contrast, in this paper, task creation works as a factor reducing the wage.

In this subsection, to introduce new industries, the range of intermediate good industries is modified to $j \in [N-1, N]$. The technology of final goods is rewritten as

$$\log Y_t = \int_{N-1}^{N} \log y_t(j) dj,$$

where N indicates the number of industries. An increase in N is interpreted as the creation of new industries. This setup is the same as in Acemoglu and Restrepo

(2018b). (23), (26) and (28) are rewritten as:

$$
\begin{aligned}
w &= (N - j_t^*)Y_t, \\
\Phi^N(j_{t+1}^*) &= \left(\frac{Cj_{t+1}^*}{1-\alpha}\right)^{\frac{1}{\alpha(j_{t+1}^* - (N-1))}} (1 - j_{t+1}^*)^{\frac{N - j_{t+1}^*}{\alpha(j_{t+1}^* - (N-1))}} (j_{t+1}^* - (N-1))\gamma^{-\frac{1}{\alpha}} \\
\Psi^N(j_t^*, j_{t+1}^*) &= \left[1 + \frac{1-\alpha}{2\alpha(1+\beta)} \frac{j_{t+1}^* - N + 1}{j_{t+1}^*}\right]^{-1} \frac{\beta C}{(1+\beta)(1-\alpha)} (N - j_t^*) j_t^*
\end{aligned}
$$

From these equations, let us numerically demonstrate the effects of new industries on the steady state. The values of α, β, and C are the same as before, and the value of γ is 4.65. The range of N is set to $(1, 1.15)$ and the parameter values ensure the existence of a stable equilibrium. As N increases, the fixed cost for automation technology increases.[17]

(Figure 2 around here)

Figure 2 demonstrates how each variable in the steady state depends on N. This figure shows that an increase in N leads to a decrease in j^*, Y, and w. As in the previous subsections, the effect on the wage is separated into two channels; the effect through $1 - j^*$ and the effect through Y. The effect of the former on the wage is positive because an increase in N and the resulting decrease in j^* stimulate labor demand. However, the effect of the latter on the wage is negative because the decrease in j^* leads to a decrease in Y, and is stronger than the effect of the former. Thus, in this model, the creation of new industries typically leads to a decrease in wages.[18]

[17] When N is sufficiently large, fixed costs are too high to introduce the automation technology even for managers in industry $N - 1$, and thus no one uses this technology. In this case, the equilibrium conditions do not hold.

[18] When γ is large, the decrease in Y is relatively gentle, and the negative effect is weak. Thus, the total effect of N on the wage becomes positive when N is small. However, an increase in N magnifies the decrease in Y and the corresponding negative effect. Therefore, even in this case, the wage decreases with N when N is not small.

5 Automation Technology Frontier

In the model in the previous section, automation technology is potentially applied to all industries. However, some industries may not adopt automation because the technology to automate them does not exist. Thus, this section considers the case in which automated industries are limited to industry $j \leq \bar{j} < j^*$, where $\bar{j}$ is the upper bound for automated industries. Managers in $j \in (\bar{j}, j^*]$ who want to introduce automation cannot do so because of this technological limitation. Therefore, this section examines the effects on the wage of the technological changes, in particular, an improvement in automation productivity γ and an incremental change in automation frontier $\bar{j}$. The assumption regarding $\bar{j}$ is similar to that in the model of Acemoglu and Restrepo (2018b). The differences between their model and the present one are highlighted in this analysis.

In this section, the range of intermediate goods industries is the same as in Section 2, $j \in [0, 1]$. Managers in $j \in [0, \bar{j}]$ choose automation technology. Thus, the gross interest rate (20), the wage (23), and the final goods (24) are rewritten as follows:

$$R_t = \frac{\alpha Y_t}{K_t} \cdot \bar{j}, \tag{37}$$

$$w_t = (1 - \bar{j}) Y_t, \tag{38}$$

$$Y_t = \left[\gamma \left(\frac{K_t}{\bar{j}} \right)^{\alpha} \right]^{\bar{j}} \left(\frac{1}{1 - \bar{j}} \right)^{1 - \bar{j}}. \tag{39}$$

From (37), (38), savings (3), and the profits from automation technology (16), the capital accumulation equation (27) becomes

$$K_{t+1} = \frac{\beta}{1+\beta} \bar{j}(1 - \bar{j}) Y_t - \frac{K_{t+1}}{\alpha(1+\beta)} \left[(1 - \alpha) - \frac{C\bar{j}}{2Y_{t+1}} \right]. \tag{40}$$

At first, the following proposition examines the effect of γ on the wage.

Proposition 6. *When a technological frontier $\bar{j} < j^*$ exists, the effect of an increase in γ on the wage is positive in the long run.*

The proof of this proposition is provided in Appendix B. This proposition shows that an improvement in automation technology γ increases the wage. Since an increase in γ does not influence the number of industries using automation technology, the displacement effect does not emerge, and the productivity effect increases the wage.

Let us consider the effect of an incremental change in the automation frontier on the wage. This effect is represented as the following equation from (38):

$$\frac{dw}{d\bar{j}} = -Y + (1-\bar{j})\frac{dY}{d\bar{j}}. \tag{41}$$

As in the previous section, we call the first term the displacement effect and the second term the productivity effect.

To analyze the productivity effect, we differentiate (39) with respect to $\bar{j}$:

$$\begin{aligned}\frac{dY}{d\bar{j}} &= \frac{\partial Y}{\partial K}\frac{dK}{d\bar{j}} + \frac{\partial Y}{\partial \bar{j}},\\ &= \alpha\bar{j}\frac{Y}{K}\frac{dK}{d\bar{j}} + Y\left[(1-\alpha) + \log\gamma\left(\frac{K}{\bar{j}}\right)^{\alpha}(1-\bar{j})\right].\end{aligned} \tag{42}$$

The first term of (42) reflects the indirect effect through capital accumulation, and the second term reflects the direct effect. From (40), the effect of an increase in $\bar{j}$ on capital accumulation is

$$\begin{aligned}\frac{dK}{d\bar{j}} = &\left(1 + \frac{1-\alpha}{\alpha(1+\beta)} - \frac{C\bar{j}}{2(1+\beta)\alpha Y}\right)^{-1}\\ &\times\left[\frac{\beta}{1+\beta}(1-2\bar{j})Y + \frac{CK}{2(1+\beta)\alpha Y} + \left(\frac{\beta}{1+\beta}\bar{j}(1-\bar{j}) - \frac{1}{Y}\frac{CK\bar{j}}{2(1+\beta)\alpha Y}\right)\frac{dY}{d\bar{j}}\right].\end{aligned} \tag{43}$$

Equations (42) and (43) determine $dK/d\bar{j}$ and $dY/d\bar{j}$. Because the signs of them are not analytically clear, we conduct a numerical analysis.

(Figure 3 around here)

Figure 3 illustrates how the wage depends on γ and $\bar{j}$.[19] The x-axis is γ, the y-axis is $\bar{j}$, and the z-axis is the wage. The parameters are $\alpha = 0.3,\ \beta = 0.82,$ and $C = 4.4$. The figure shows that when the level of $\bar{j}$ is sufficiently high (low), the effect of an

[19] This figure focuses on the ranges of γ and $\bar{j}$ assuring the existence of a unique steady state.

increase in $\bar{j}$ is negative (positive) when γ is sufficiently low or high. In the model of Acemoglu and Restrepo (2018b), the wage always increases with $\bar{j}$. By contrast, the wage decreases in some ranges of γ and $\bar{j}$ in this model.

(Figure 4 around here)

To understand why the wage decreases with $\bar{j}$ when $\bar{j}$ is sufficiently large, we focus on the productivity effect. As previously described, the first term of (42) corresponds to the indirect effect through capital accumulation, and the second term reflects the direct effect. Figure 4 shows how the amount of aggregate capital K depends on γ and $\bar{j}$. Since K is increasing in $\bar{j}$ from this numerical analysis, the indirect effect is positive, which is identical to the result in Acemoglu and Restrepo (2018a).

By using (37) and (38), the direct effect is rewritten as follows:

$$\frac{\partial Y}{\partial \bar{j}} = \left[\left(\log \frac{1}{w} - \log \frac{\gamma}{R^{\alpha}} \right) + \alpha \log \alpha - (1-\alpha) \log Y + 1 - \alpha \right] Y. \tag{44}$$

The part depending on effective factor prices is the effect pointed out by Acemoglu and Restrepo (2018a) and many other models based on their work. The remaining part of (44) emerges because the aggregate final goods production function (39) is decreasing return to scale (when $\alpha = 1$, the additional part disappears), and thus this part decreases with Y.

In Acemoglu and Restrepo (2018b), the negative displacement effect is always dominated by the positive productivity effect because of the indirect effect through capital accumulation. In this model, the wage could decrease with $\bar{j}$, as shown in Figure 3, which means that the sum of the displacement effect and the productivity effect could be negative. Because the displacement effect is the same as Acemoglu and Restrepo (2018b), the productivity effect is smaller than their model and even may be negative. One of the reasons for this difference is the decreasing return to scale of the final good production function.

6 Subsidy Policy and Taxation

Sirkin et al. (2015) suggest that a high initial cost prevents the introduction of automation technology. This section introduce a subsidy to fixed costs financed by tax on young households that encourage the diffusion of automation technology, and examines the effects of such a policy. Since this model utilizes OLG setting, over-accumulation of capital might occur. A subsidy on the introduction of automation technology might possibly increase aggregate output and ultimately lead to an improvement in overall economic welfare. Let us consider a subsidy that compensates a certain percentage of fixed costs and that is financed by a lump-sum tax on young households.

This policy changes the budget constraint for young households such that

$$c_t(j) = w_t - s_t(j) - \tau_t, \tag{45}$$

where τ represents the lump-sum tax. The profit from automation technology is

$$\pi_t^k(j) = \max_{k_t(j)} \; p_t(j)y_t^k(j) - R_t k_t(j) - (1-\sigma)C(j), \tag{46}$$

where $\sigma \in [0,1]$ is the subsidy rate. A policymaker's budget constraint is

$$\tau_t = \int_0^{j_t^*} \sigma C(j) dj. \tag{47}$$

This section numerically examines the effects of this subsidy. The parameters are the same as in the previous numerical example except for γ ($\gamma = 4.65$). The values of γ and C ensure that a unique steady state exists.

(Figure 5 around here)

Figure 5 illustrates the effects of the subsidy on the wage, the final good production Y, the extent of automation technology diffusion j^*, aggregate capital K, interest rate R, and economic welfare at the steady state. Welfare is measured by the aggregation of each household's lifetime utility. Figure 5 shows that this subsidy encourages the adoption of automation technology because it reduces the associated fixed costs. Thus, the automation technology diffusion level increases when the subsidy rate increases. Automation diffusion leads to a decline in $1 - j^*$, but an increase in Y. Therefore, as in the previous section, this subsidy has both positive and negative effects on the wage. Figure 5 shows that the negative effect dominates the positive effect, and thus the subsidy decreases the wage.[20] Aggregate capital K declines because the increase in tax and the decrease in the wage depress savings. R increases with σ because the decrease in the supply of capital caused by the decrease in savings and the increase in the demand for capital resulting from the increases in j^* and Y have negative effects on R.

Figure 5 also shows that as the subsidy rate σ increases, welfare increases at first; however, it subsequently decreases. The welfare of each household is determined by lifetime income and R. The lifetime income is

$$w_t - \tau_t + \frac{\pi^i_{t+1}(j)}{R_{t+1}}, \quad for\ i = l, k,\ j \in [0,1], \tag{48}$$

$$where\ \ \pi^\ell_{t+1}(j) = 0, \ \ \pi^k_{t+1}(j) = (1-\alpha)Y_{t+1} - (1-\sigma)C(j). \tag{49}$$

Increases in R and τ and a decrease in w depress the lifetime income, and an increase in the profits net of the payment of fixed costs driven by increases in Y and σ increases the lifetime income. The increase (decrease) in the lifetime income pulls up (pushes down) the welfare. R has not only an indirect effect through lifetime income, but also a direct effect on the welfare because R increases the value of savings. When the positive effects are larger (smaller) than the negative effects, welfare increases (decreases). Figure 5 shows that average welfare increases when σ is low and decreases when σ is high. The reason for the increase in welfare for a low σ may be that the policy solves over-accumulation of capital by imposing tax, which decreases K and increases R.

In this model, there are different types of households. Some households may not receive benefits from the subsidy policy, particularly those households that do not choose automation technology in response to the subsidy policy because of the high fixed costs (thereafter called *Non-automation type*). These households receive no direct gain from the policy. In addition, households that face zero fixed costs (thereafter called *Zero fixed cost type*) also receive no direct gains.

[20] If we set a different parameter set, for example, a larger γ, the positive effect becomes stronger, and a region in which the wage increases with σ emerges.

Let us consider the dynamics of their welfare to examine whether there exist subsidy rates that benefit every household of every generation. At first, the economy is in a steady state, and the subsidy policy is unexpectedly enforced in Period 3.

(Figure 6 around here)

(Figure 7 around here)

Figure 6 illustrates the dynamics of the welfare of households who are old in period t. The upper left panel illustrates the dynamics of the average welfare of households. The upper right panel illustrates the welfare of the Non-automation type, and the lower left panel illustrates the welfare of the Zero fixed cost type. In each panel, the blue line (round), green line (square), and red line (diamond) indicate the dynamics when the subsidy rate, σ, is 0.01, 0.04, and 0.06, respectively. The black (dotted) line in each panel indicates the welfare level in the steady state without the subsidy policy. Figure 7 demonstrates the dynamics of the lifetime income of each type of household examined in Figure 6 and of the interest rate. The blue (round), green (square), and red (diamond) lines indicate the dynamics when σ is 0.01, 0.04, and 0.06, respectively. The black (dotted) lines indicate the levels without the policy.

The upper left panel of Figure 6 shows that the subsidy policy improves average welfare compared with the steady state economy without the subsidy policy at any time. However, not all households may benefit from the policy. Figure 6 focuses on two types of households — the Non-automation type and the Zero fixed cost type — that receive no direct gain from the policy.[21]

The upper right panel of Figure 6 illustrates that the welfare of the Non-automation type households increases in Period 4, and that their welfare in subsequent periods is greater than their welfare without the policy.[22] They pay the tax during the young

[21] Households with positive fixed costs that introduce automation technology obtain greater benefits than the two types of households.

[22] Welfare of old households of the Non-automation type and the Zero fixed cost type in Period 3, in which the policy is implemented, are the same as in previous periods. Figure 7 shows that R and their lifetime income in Period 3 are the same as in previous periods. The subsidy policy does not influence R in Period 3 since the supply of capital depends on savings in Period

period and receive no subsidy. From Figure 7, their lifetime income after the policy is lower in all cases than in the steady state without the policy. However, since R after the policy is greater than the level without the policy in Figure 7, their income during the old period, funded by savings, is higher. The positive effect is dominant, and their welfare improves.

The lower left panel of Figure 6 shows that the welfare of the Zero fixed cost type households after Period 5 is greater than their welfare without the policy, while their welfare in Period 4 is lower when σ is 0.01 and greater when σ is 0.04 and 0.06 than the welfare without the policy.

Since households that introduce automation technology by paying positive fixed costs receive the subsidy directly, they gain greater benefits from the policy than the Non-automation type and the Zero fixed cost type households. Thus, when σ are 0.04 and 0.06, all households of all generations benefit from the policy.[23] However, when σ is sufficiently high, an increase in σ decreases the average welfare; consequently, many households do not benefit from a marginal increase in the subsidy rate.

7 Conclusion

This paper investigates the effects of fixed costs on automation technology diffusion and examines the relationship between the diffusion and the wage. In this model, households choose either automation or non-automation technology for production; when the households choose the automation technology, they pay fixed costs. Thus, their choice determines the extent of automation technology diffusion and fixed costs play an essential role in their decision.

2, and the demand for capital depends on the technology choice made at the end of Period 2. Lifetime income of these households do not change because their profits in Period 3 are not influenced by the policy, i.e., $\pi = 0$ for the Non-automation type and $\pi = (1-\alpha)Y$ for the Zero fixed cost type, where Y is determined by K and j^* Thus, while the subsidy policy increases the lifetime income of old households with positive fixed costs in Period 3, who introduce automation technology, it does not affect the lifetime income of the two types examined in Figure 7 in Period 3.

[23] When σ is from about 0.032 to 0.068, the welfare of every household of every generation increases by the policy. When σ is over 0.068, some households suffer welfare loss.

The analysis in this paper yields two notable results. First, the wage can decrease with an improvement in automation productivity or decreases in fixed costs, both of which accelerate automation diffusion. The diffusion has both positive and negative effects on the wage. The positive (negative) effect dominates in the long-run when the extent of automation technology diffusion is low (high) enough. In Acemoglu and Restrepo (2018b), the wage increases in the long-run because of the positive effect through capital accumulation. By contrast, in this study, the wage decreases when the level of diffusion is sufficiently high, even in the presence of capital accumulation.

Second, the labor income share, which is the sum of the income share of workers and managers, decreases with automation diffusion. This result is consistent with the recent empirical studies that point out a declining labor income share over the last several decades (Karabarbounis and Neiman, 2014; Grossman et al., 2017; Alvarez-Cuadrado et al., 2018).

Subsequently, this paper examines two extensions of this model. First, we introduce an automation technology frontier. This setting is the same as that in Acemoglu and Restrepo (2018b). Because of the technology frontier, automation is not available to some industries. In contrast to Acemoglu and Restrepo (2018b), in which an expansion of the frontier always increases the wage in the long-run, in this study, it can decrease the wage in the long-run. Second, this paper explores the long- and short-run effects of a subsidy policy for fixed costs on the economy. The analysis demonstrates that such a subsidy policy has both positive and negative effects on the long-run wage and welfare. This paper shows that there exist subsidy rates that improve the welfare of all households of all generations.

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Appendix A. Existence and Stability of Unique Steady State

In the main part of this paper, Proposition 1 shows the existence of a unique steady state, and Proposition 2 examines its stability. In this section, we show the condition of the existence and the stability in a more formal way. Proposition A.1 corresponds to Proposition 1, and Proposition A.3 and A.4 corresponds to Proposition 2.

A.1 Existence of Unique Steady State

In the main part of the paper, the two types of capital dynamics are described as:

$$K_t = \Phi(j_t^*) \tag{A1}$$

$$K_{t+1} = \Psi(j_t^*) \tag{A2}$$

$$where \quad \Phi(j_t^*) = \left(\frac{Cj_t^*}{1-\alpha}\right)^{\frac{1}{\alpha j_t^*}} (1-j_t^*)^{\frac{1-j_t^*}{\alpha j_t^*}} j_t^* \gamma^{-\frac{1}{\alpha}} \tag{A3}$$

$$\Psi(j^*) = \frac{2\alpha\beta C}{(1-\alpha)\{2\alpha(1+\beta)+(1-\alpha)\}}(1-j_t^*)(j_t^*). \tag{A4}$$

Let us express these dynamics in logarithmic form.

$$\phi(j_{t+1}^*) \equiv \log \Phi(j_{t+1}^*) = \log(\frac{Cj_{t+1}^*}{1-\alpha})^{\frac{1}{\alpha j_{t+1}^*}} (1-j_{t+1}^*)^{\frac{1-j_{t+1}^*}{\alpha j_{t+1}^*}} j_{t+1}^* \gamma^{-\frac{1}{\alpha}}, \tag{A5}$$

$$\psi(j_t^*) \equiv \log \Psi(j_t^*) = \log AC(1-j_t^*)(j_t^*), \tag{A6}$$

$$where \quad A \equiv \frac{2\alpha\beta}{(1-\alpha)\{2\alpha(1+\beta)+(1-\alpha)\}}, \tag{A7}$$

$$\lim_{j\to 0} \phi(j_{t+1}^*) = -\infty, \quad \lim_{j\to 1} \phi(j_{t+1}^*) = \frac{1}{\alpha}\log\left(\frac{C}{\gamma(1-\alpha)}\right), \tag{A8}$$

$$\lim_{j\to 0} \psi(j_t^*) = -\infty, \quad \lim_{j\to 1} \psi(j_t^*) = -\infty. \tag{A9}$$

Taking the limits of $\phi(j_{t+1}^*)$ and $\psi(j_t)$ to zero and one; For simplicity, we ignore the time scripts. The gap between ϕ and ψ is

$$\phi(j^*) - \psi(j^*) =$$
$$\frac{1}{j^*\alpha}\left(\log(\frac{C}{1-\alpha}) - j^*\log(\gamma(AC)^\alpha) + \log j^* + (1-j^*(1+\alpha))\log(1-j^*)\right). \tag{A10}$$

Thus, $\lim_{j^*\to 0} \phi(j^*) - \psi(j^*) = -\infty$ and $\lim_{j^*\to 1} \phi(j^*) - \psi(j^*) = \infty$

Therefore, if the slope of ϕ is higher than ψ in $[0, 1]$, the fixed point is unique. The gap of the slope between ϕ and ψ is

$$\frac{\partial \phi(j^*)}{\partial j^*} - \frac{\partial \psi(j^*)}{\partial j^*} = \frac{1}{\alpha(j^*)^2(1-j^*)} \left\{ (1-j^*) \left(\log(\frac{1-\alpha}{Cj^*(1-j^*)}) + 1 - j^* \right) + (j^*)^2 \alpha \right\} \tag{A11}$$

$$\equiv \frac{1}{\alpha(j^*)^2(1-j^*)} Q(j^*)$$

The limits of $Q(j^*)$ are:

$$\lim_{j^* \to 0} Q(j^*) = \infty, \tag{A12}$$

$$\lim_{j^* \to 1} Q(j^*) = \alpha. \tag{A13}$$

We take the differential with respect to j^*,

$$\frac{\partial Q(j^*)}{\partial j} = Q'(j^*) = \log(\frac{Cj^*(1-j^*)}{1-\alpha}) + 2(1+\alpha)j^* - \frac{1}{j^*}. \tag{A14}$$

And we differentiate $Q(j^*)$ with respect to j^* one more time,

$$\frac{\partial^2 Q(j^*)}{\partial (j^*)^2} = \frac{-2(1+\alpha)(j^*)^3 + 2\alpha(j^*)^2 + 1}{(j^*)^2(1-j^*)}. \tag{A15}$$

(Figure A.1 around here)

Let $Q_n(j^*)$ be the numerator of the above equation. Since the denominator is positive, the sign of $Q_n(j^*)$ determines the sign of $Q''(j^*)$. The limits of $Q_n(j^*)$ are $\lim_{j^* \to 0} Q_n(j^*) = 1$ and $\lim_{j^* \to 1} Q_n(j^*) = -1$. Because $Q_n'(j^*) = [-6(1+\alpha)j^* + 4\alpha]j^*$, the smaller extreme point of $Q_n(j^*)$ is zero and its other extreme point is in $[0, 1]$. Thus, $Q_n(j^*) = 0$ has one solution in $[0, 1]$ and $Q_n(j^*) > 0$ for smaller j^* and $Q_n(j^*) < 0$ for larger j^* (see Figure A.1 (a)). Therefore, $Q'(j^*)$ is an inverse U-shaped function (see Figure A.1 (b1) and (b2)). When the maximized value of $Q'(j^*)$ in $[0, 1]$ is negative or when the value of $Q(j^*)$ evaluated at the smaller solution for $Q'(j^*) = 0$ is positive, the slope of ϕ is higher than ψ and a unique steady state exist (see figures except (a) in Figure A.1).

The above discussion describes the sketch of the following proposition. It shows the condition of the existence of a unique steady state in a more formal way than Proposition 1.

Proposition A.1.

(1) If the following condition holds, the maximized value of $Q'(j^*)$ in $[0,1]$ is negative, $Q(j^*) > 0$ and a unique steady state exists:

$$C < C_1, \tag{A16}$$

$$\begin{aligned} \textit{where} \quad C_1 &= \frac{(1-\alpha)e^{[-2(1+\alpha)j^{s1}+\frac{1}{j^{s1}}]}}{j^{s1}[1-j^{s1}]}, \\ j^{s1} &= \frac{1}{1+\alpha}\left(\sqrt[3]{\nu + \frac{1+\alpha}{2}\sqrt{\nu}} + \sqrt[3]{\nu - \frac{1+\alpha}{2}\sqrt{\nu}} + \frac{\alpha}{3}\right), \\ \nu &= \frac{(1+\alpha)^2}{4} + \frac{\alpha^3}{27}. \end{aligned}$$

(2) If the following condition holds, the maximized value of $Q'(j^*)$ in $[0,1]$ is positive and the value of $Q(j^*)$ evaluated at the smaller solution for $Q'(j^*) = 0$ is positive, $Q(j^*) > 0$ and a unique steady state exists:

$$C_1 < C < C_2, \tag{A17}$$

$$\textit{where} \quad C_2 = \frac{(1-\alpha)e^{[-2(1+\alpha)j^{s2}+\frac{1}{j^{s2}}]}}{j^{s2}[1-j^{s2}]}, \ \textit{and} \tag{A18}$$

j^{s2} is the unique solution of the following equation in $[0,1]$:

$$-(1+\alpha)(j^{s2})^3 + 2\alpha(j^{s2})^2 + 2(j^{s2}) - 1 = 0. \tag{A19}$$

Proof of Proposition A.1.

(1) This case is illustrated in the upper row of Figure A.1. As shown in the paragraph just below (A15), $Q_n(j^*) = 0$ has one solution j^{s1} in $[0,1]$. By applying Cardano formula, we obtain the value of j^{s1}. (A16) is derived from $Q'(j^{s1}) < 0$.

(2) This case is illustrated in the lower row of Figure A.1. When $Q'(j^{s1}) > 0$ and thus $C > C_1$, $Q'(j^*) = 0$ has two solutions (Figure A.1 (b2)). Let j^m be the smaller

solution. When $Q(j^m) > 0$, $Q(j^*) > 0$ for any $j^* \in [0, 1]$ and a unique steady state exists in $[0, 1]$. Since $Q'(j^m) = 0$, from (A14),

$$\log C = \log \frac{1-\alpha}{j^m(1-j^m)} - 2(1+\alpha)j^m + \frac{1}{j^m}. \tag{A20}$$

By using (A20), the condition for the uniqueness of the solution is:

$$\begin{aligned} Q(j^m) = (1-j^m)\left(\log(\frac{1-\alpha}{Cj^m(1-j^m)}) + 1 - j^m\right) + (j^m)^2\alpha > 0 \\ \Leftrightarrow -(1+\alpha)(j^m)^3 + 2\alpha(j^m)^2 + 2j^m - 1 > 0. \end{aligned} \tag{A21}$$

Let $Q_L(j)$ be the left-hand side of (A21). The values of $Q_L(j)$ at endpoints are $Q_L(0) = -1$ and $Q_L(1) = \alpha$. Because $Q_L''(j) = -6(1+\alpha)j + 4\alpha$, the inflection point $j = 4\alpha/6(1+\alpha)$ is in $(0, 1)$. Thus, since $Q_L(j)$ is a cubic function, $Q_L(j) = 0$ has one solution in $(0, 1)$. Let the solution j^{s2}, and thus $Q(j^m) = 0$ when $j^m = j^{s2}$. Therefore, $Q(j^m) > Q(j^{s2}) = 0$ is the condition of the existence of a unique steady state from (A21).

Then, we show the existence of C_2, which is the level of the fixed cost satisfying $Q(j^m) = 0$. When $C > C_1$, the graphs are illustrated as the lower row in Figure A.1. From (A11), $Q(j^*)$ shifts down as C increases since $dQ(j^*)/dC < 0$, and $\lim_{C\to\infty} Q(j^*) = -\infty$. Thus, there exists C_2 such that $Q(j^m) = 0$. Therefore, when $C_1 < C < C_2$, as illustrated by Figure A.1 (c2), $Q(j^m) > 0$ and there exists a unique steady state.[24] (A18) is derived from (A20) when $Q(j^m) = 0$. □

A.2 Stability of Unique Steady State

In this section, we confirm the stability of the dynamic system of this model. The steady state level of j_t^* is denoted by j^* below. We divide two cases in terms of j^*: $j^* \le 1/2$ and $j^* > 1/2$. For the former case, the global stability holds as shown below by Proposition A.3. For the latter case, the local stability holds when j^* is small

[24] We also examine the condition of existence of a unique steady state when $C > C_2$, and shows that a unique equilibrium or multiple equilibria occur depending on the parameters. This paper focuses on the case of a unique equilibrium for simplicity.

enough. Proposition A.4 shows conditions under which there exists a threshold level of j^* such that the dynamic system is locally stable (unstable) when j^* is smaller (larger) than the threshold. In this section, we assume that a unique steady state exists as shown by Proposition A.1 and that ϕ is a monotonically increasing function. At first, we derive the condition of the monotonicity, then show the stability of the dynamic system.

A.2.1. The Monotonicity of ϕ

To examine the stability of the dynamic system, ϕ and ψ, we derive the condition to assure that ϕ is a monotonically increasing function. From (A5), if differentiate $\phi(j^*_{t+1})$ with respect to j^*_{t+1},

$$\frac{\partial \phi(j^*_{t+1})}{\partial j^*_{t+1}} = \frac{1}{\alpha (j^*_{t+1})^2}\left[\log\left(\frac{1-\alpha}{Cj^*_{t+1}(1-j^*_{t+1})}\right) + 1 - (1-\alpha)j^*_{t+1}\right]. \tag{A22}$$

We define the bracket of this equation as $\tilde{\phi}(j^*_{t+1})$. We differentiate $\tilde{\phi}(j^*_{t+1})$ with respect to j^*_{t+1},

$$\frac{\partial \tilde{\phi}(j^*_{t+1})}{\partial j^*_{t+1}} = \frac{1}{j^*_{t+1}(1-j^*_{t+1})}\left[(1-\alpha)(j^*_{t+1})^2 + (1+\alpha)j^*_{t+1} - 1\right]. \tag{A23}$$

Let j^s be the value in $[0,1]$ when the bracket is equal to zero. Then,

$$j^s = \frac{-(1+\alpha)+\sqrt{(1+\alpha)^2+4(1-\alpha)}}{2(1-\alpha)}. \tag{A24}$$

For $j^*_{t+1} < j^s$, $\partial\tilde{\phi}(j^*_{t+1})/\partial j^*_{t+1} < 0$ and $\tilde{\phi}(j^*_{t+1})$ is decreasing function, and for $j^*_{t+1} > j^s$, $\tilde{\phi}(j^*_{t+1})$ is an increasing function. Thus, $\tilde{\phi}(j^*_{t+1})$ is minimized at $j^*_{t+1} = j^s$. Therefore, when $\tilde{\phi}(j^s) > 0$, $\partial\phi(j^*_{t+1})/\partial j^*_{t+1} > 0$ and $\phi(j^*_{t+1})$ increases monotonically. From (A22), $\tilde{\phi}(j^s) > 0$ corresponds to the following condition;

$$C < \frac{(1-\alpha)e^{1-(1-\alpha)j^s}}{j^s(1-j^s)} \tag{A25}$$

From the above discussion, the following proposition is obtained:

Proposition A.2.
If $C < \frac{(1-\alpha)e^{1-(1-\alpha)j^s}}{j^s(1-j^s)}$*, where* $j^s = \frac{-(1+\alpha)+\sqrt{(1+\alpha)^2+4(1-\alpha)}}{2(1-\alpha)}$*,* ϕ *increases monotonically in* $[0,1]$.

A.2.2. The Global Stability When $j^* \leq 1/2$

We show the global stability when $j^* \leq 1/2$. Suppose that Proposition A.1 and A.2 hold. From (A1) and (A5) and the monotonicity of ϕ,

$$j^*_{t+1} = \phi^{-1}[\log K_{t+1}]. \tag{A26}$$

The dynamic system satisfies $\phi(j^*_{t+1}) = \psi(j^*_t)$ from (29). Thus, by substituting (A2) into (A26),

$$j^*_{t+1} = \phi^{-1}[\psi(j^*_t)] \equiv h(j^*_t). \tag{A27}$$

We examine the function $h(j^*_t)$. The derivative of $h(j^*_t)$ is

$$\frac{dh(j^*_t)}{dj^*_t} = \frac{d\psi(j^*_t)/dj^*_t}{d\phi(j^*_{t+1})/dj^*_{t+1}} \tag{A28}$$

Whether $h(j^*_t)$ is an increasing function or a decreasing function depends on the derivatives of $\phi(j^*_{t+1})$ and $\psi(j^*_t)$. Because Proposition A.2 holds, $\phi(j^*_{t+1})$ is an increasing function. $\psi(j^*)$ is increasing for $j^*_t \leq 1/2$ and decreasing for $j^*_t > 1/2$. Hence, $h(j^*)$ is increasing for $j^*_t \leq 1/2$ and decreasing for $j^*_t > 1/2$ from (A28).

Moreover, the following two condition hold:

$$\lim_{j^*_t \to 0} h(j^*_t) = 0 \textit{ and } \lim_{j^*_t \to 0} h'(j^*_t) = \infty. \tag{A29}$$

The latter holds because $\lim_{j^*_t \to 0} d\psi(j^*_t)/dj^*_t = \infty$ from (A6) and ϕ is an increasing function. The reason of the former is that $\lim_{j^*_t \to 0} \psi(j^*_t) = -\infty$ from (A9), and $\lim_{\psi(j^*_t) \to -\infty} \phi^{-1}[\psi(j^*_t)] = 0$ because $\lim_{j^*_{t+1} \to 0} \phi(j^*_{t+1}) = -\infty$ from (A8) and ϕ is an increasing function.

Figure A.2 illustrate $h(j^*_t)$ and patterns of convergence. The upper two panels in Figure A.2 illustrate that $j^*_{t+1} > j^*_t$ ($j^*_{t+1} < j^*_t$) when $j^*_t < j^* \leq 1/2$ ($j^* < j^*_t \leq 1/2$), and j^*_t converges to j^*. The lower two panels illustrate that when $j^*_t > 1/2$ and $j^*_{t+1} \geq j^*$, $j^* < j^*_{t+2} < j^*_{t+1} < 1/2$, and when $j^*_t > 1/2$ and $j^*_{t+1} < j^*$, $j^*_{t+1} < j^*_{t+2} < j^*$. Therefore, when $j^* \leq 1/2$, this dynamic system is globally stable.

From the above discussion, the following proposition is obtained:

Proposition A.3. *Suppose that Proposition A.1 and A.2 hold. Then, if $j^* \leq 1/2$, the steady state is globally stable.*

(Figure A.2 around here)

A.2.3. The Local Stability When $j^* > 1/2$

Then, we consider the case of $j^* > 1/2$. Suppsoe that Proposition A.1 and A.2 hold. From (A5), (A6) and (A28),

$$\frac{dj^*_{t+1}}{dj^*_t} = \frac{-\left[\frac{1}{1-j^*_t} - \frac{1}{j^*_t}\right]}{\frac{1}{\alpha(j^*_{t+1})^2}\left\{\left[1 - \log\frac{Cj^*_{t+1}}{1-\alpha}\right] - \log(1-j^*_{t+1}) - (1-\alpha)j^*_{t+1}\right\}}. \tag{A30}$$

To assure local stability, the following condition is necessary:

$$\left|\frac{dj^*_{t+1}}{dj^*_t}\right|_{j^*_t = j^*_{t+1} = j^*} < 1 \Leftrightarrow -1 < -\frac{-d\psi(j^*)/dj^*}{d\phi(j^*)/dj^*} < 1. \tag{A31}$$

When Proposition A.2 holds, $d\phi/dj^* > 0$. Thus, (A31) is equivalent to:

$$\frac{d\phi(j^*)}{dj^*} + \frac{d\psi(j^*)}{dj^*} > 0 \quad and \quad \frac{d\phi(j^*)}{dj^*} - \frac{d\psi(j^*)}{dj^*} > 0 \tag{A32}$$

Evaluating the conditions above at a steady state, $d\phi(j^*)/dj^* - d\psi(j^*)/dj^*$ is equivalent to (A11). Thus, the latter condition is satisfied when $Q(j) > 0$ for any $j \in [0, 1]$. Because Proposition A.1 holds, $Q > 0$, and thus the latter condition is satisfied.

The former condition of (A32) is:

$$\frac{d\phi(j^*)}{dj^*} + \frac{d\psi(j^*_t)}{dj^*} = \frac{1}{\alpha(j^*)^2(1-j^*)}Z(j^*) > 0, \tag{A33}$$

$$where \quad Z(j^*) \equiv (1-j^*)\left(\log\left(\frac{1-\alpha}{Cj^*(1-j^*)}\right) + 1 - j^*\right) + 2\alpha j^*(1-j^*) - \alpha(j^*)^2. \tag{A34}$$

When $j^* = 1/2$, $Q(j^*) = Z(j^*)$ from(A11). This relationship implies that

$$\left[\frac{d\phi(j^*)}{dj^*} + \frac{d\psi(j^*)}{dj^*}\right]\Bigg|_{j^*=1/2} = \left[\frac{d\phi(j^*)}{dj^*} - \frac{d\psi(j^*)}{dj^*}\right]\Bigg|_{j^*=1/2}. \tag{A35}$$

Because $Q(j^*) > 0$ for any j^* when Proposition A.1 hold, $\frac{d\phi(j^*)}{dj^*} - \frac{d\psi(j^*)}{dj^*} > 0$ from (A11). Hence, $\frac{d\phi(j^*)}{dj^*} + \frac{d\psi(j^*)}{dj^*} > 0$ around $j^* = 1/2$, and thus the dynamic system is locally stable. Because $\lim_{j^* \to 1} Z(j^*) = -\alpha$, the steady state is unstable around $j^* = 1$. This means that both stable areas and unstable areas exist in $j^* \in (1/2, 1]$.

We derive the sufficient conditions that there exists a unique threshold, which divides the stable area with the unstable area. Before presenting a proposition, we show the following lemma.

Lemma A.1. *Suppose that Proposition A.1 and A.2 hold. Then, in* $j^* \in (1/2, 1]$, $Z''(j^*) = 0$ *has only one solution* j^{z1}, *and* Z'' *is positive for* $j^* < j^{z1}$ *and negative for* $j^* > j^{z1}$.

Proof of Lemma A.1. From (A34), the second derivative of Z is:

$$\frac{\partial^2 Z(j^*)}{\partial (j^*)^2} = \frac{2(3\alpha - 1)(j^*)^3 - 6\alpha(j^*)^2 + 1}{(j^*)^2(1 - j^*)}. \tag{A36}$$

The numerator determines the sign of Z''. When $j^* = 0$ and $j^* = 2\alpha/(3\alpha - 1)$, it achieves extreme values. Taking the limits to $1/2$ and 1, the numerator of Z'' are:

$$\lim_{j^* \to 1/2} [2(3\alpha - 1)(j^*)^3 - 6\alpha(j^*)^2 + 1] = \frac{3}{4}(1 - \alpha) > 0$$
$$\lim_{j^* \to 1} [2(3\alpha - 1)(j^*)^3 - 6\alpha(j^*)^2 + 1] = -1.$$

Taking it into account that the numerator is the cubic function and the above two values, the shape of Z'' is one of the two forms depicted in Figure A.3. In both cases, $Z''(j^*) = 0$ has only one solution j^{z1} in $j^* \in (1/2, 1]$, and Z'' is positive for $j^* < j^{z1}$ and negative for $j^* > j^{z1}$. □

(Figure A.3 around here)

Next, we consider when there is a unique threshold dividing the area featuring stable steady state with the area featuring unstable steady state. As we described, this is satisfied when $Z(j^*) = 0$ has a unique solution. To examine the shape of $Z(j^*)$, we focus on the relationship between $Z(j^*)$ and $Z'(j^*)$, which is illustrated in Figure A.4.

Lemma A.1 implies that $Z'(j^*)$ has a single peak. $Z'(j^*)$ equals

$$Z'(j^*) = \frac{1}{j^*} \left[j^* \log \left\{ \frac{Cj^*(1 - j^*)}{1 - \alpha} \right\} + 2(1 - 3\alpha)(j^*)^2 + 2\alpha j^* - 1 \right]. \tag{A37}$$

Taking the limits to 1/2 and 1, the values of $Z'(j^*)$ are:

$$\lim_{j^* \to 1/2} Z'(j^*) = \log\left\{\frac{C}{4(1-\alpha)}\right\} - (1+\alpha), \tag{A38}$$

$$\lim_{j^* \to 1} Z'(j^*) = -\infty. \tag{A39}$$

If (A38) is non-negative, that is $C \geq 4(1-\alpha)e^{1+\alpha} \equiv C^{z1}$, since $Z'(j^*)$ has a single peak, $Z'(j^*) = 0$ has a unique solution in $j^* \in [1/2, 1]$, and thus $Z(j^*)$ has a single peak in $[1/2, 1]$. $Z(j^*)$ is positive around $j^* = 1/2$ since $Z(1/2) = Q(1/2)$, and negative around $j^* = 1$ since $\lim_{j^* \to 1} Z(j^*) = -\alpha$. Thus, $Z(j^*) = 0$ has also a unique solution (see Figure A.4 (a)).

If (A38) is negative, $Z'(j^{z1}) \leq 0$ or $Z'(j^{z1}) > 0$. Since $Z'(j^{z1})$ is extreme value of $Z'(j^*)$, $Z'(j^*) \leq 0$ in $j^* \in [1/2, 1]$ when $Z'(j^{z1}) \leq 0$. Thus, $Z(j^*)$ is weakly decreasing in j^*, and $Z(j^*) = 0$ has a unique solution (see Figure A.4 (b1)). When $Z'(j^{z1}) > 0$, $Z'(j^*) = 0$ has two solutions. Let the solutions be j^{z2} and j^{z3}, and assume that $j^{z2} < j^{z3}$. The extreme values, $Z(j^{z2})$ and $Z(j^{z3})$, determine the shape of $Z(j^*)$. When $Z(j^{z2}) > 0$, $Z(j^*) = 0$ has one solution between j^{z3} and 1 (see Figure A.4 (b21)). When $Z(j^{z2}) < 0$, if $Z(j^{z3}) < 0$, $Z(j^*) = 0$ has one solution between 1/2 and j^{z2} (see Figure A.4 (b23)).

Following proposition is obtained from the above discussion:

Proposition A.4. *Suppose that Proposition A.1 and Proposition A.2 hold, and that* $j^* > 1/2$. *Unless the following condition holds, there is a unique threshold of* j^*, *and the dynamic system is locally stable (unstable) for smaller (larger)* j^*:

$$C^{z2} \leq C \leq C^{z3}, \tag{A40}$$

$$\textit{where} \quad C^{z2} = \frac{(1-\alpha)e^{-\frac{\alpha(j^{z2})^2}{1-j^{z2}} + (2\alpha-1)j^{z2} + 1}}{j^{z2}(1-j^{z2})}, \tag{A41}$$

$$C^{z3} = \frac{(1-\alpha)e^{-\frac{\alpha(j^{z3})^2}{1-j^{z3}} + (2\alpha-1)j^{z3} + 1}}{j^{z3}(1-j^{z3})} \tag{A42}$$

and j^{z2} *and* j^{z3} *are solutions of* $Z'(j) = 0$ *and* $j^{z2} < j^{z3}$.

Proof of Proposition A.4. Above this proposition, we explained all cases in which there exists a unique threshold of j^* such that the dynamic system is locally stable for j^* smaller than the threshold. When $Z(j^*) = 0$ has multiple solutions, there does not exist such unique threshold. Since $dZ(j^*)/dC < 0$ from (A34), an increase in C shifts down $Z(j^*)$. Let C^{z2} (C^{z3}) be C satisfying $Z(j^{z2}) = 0$ ($Z(j^{z3}) = 0$), where $j^{z2} < j^{z3}$. When $C^{z2} \leq C \leq C^{z3}$, $Z(j^*) = 0$ has multiple solutions (see Figure A.4 (b22)). Because $Z(j^*)$ has a unique solution when $C < C^{z2}$ and $C > C^{z3}$ (see Figure A.4 (b21) and (b23)), there exists the unique threshold of j^* unless $C^{z2} \leq C \leq C^{z3}$. (A41) and (A42) are derived from (A34) evaluated at j^{z2} and j^{z3}. $\square$

(Figure A.4 around here)

Following figures show graphically the cases of dynamics. In each figure, the solid line and the dot line represents $\phi(j^*_{t+1})$ and $\psi(j^*_t)$, respectively. Since $j^*_t = j^*_{t+1}$ in the steady states, the intersections correspond to the steady states. Figure A.5 demonstrates the case of the unique steady state with monotonic convergence, and Figure A.6 does the case of the unique steady state with cyclical convergence.

(Figure A.5 around here)

(Figure A.6 around here)

Appendix B. Proofs of Propositions

B.1. The Proofs of Proposition 3 and 4

At first, we propose a lemma that shows the relationship between γ and j^*.

Lemma B.1.

Suppose that Proposition A.1 holds. Then:

$$\frac{dj^*}{d\gamma} > 0$$
$$\frac{dj^*}{dC} < 0$$

Proof of Lemma B.1. When Proposition A.1 holds, $\phi(j^*) - \psi(j^*)$ in the unique steady state is increasing in j^* since $Q(j^*) > 0,\ j^* \in [0, 1]$. From (A11), in the steady state,

$$\frac{d\Omega(j^*)}{dj^*} = \frac{1}{\alpha(j^*)^2(1-j^*)}Q(j^*) > 0, \tag{B1}$$

$$where\ \ \Omega(j^*) \equiv \phi(j^*) - \psi(j^*). \tag{B2}$$

Since $\phi(j^*)-\psi(j^*) = 0$ in the steady state, by applying the Implicit Function Theorem to (A10),

$$\frac{dj^*}{d\gamma} = \frac{1}{\gamma\alpha}\left(\frac{d\Omega(j^*)}{dj^*}\right)^{-1} > 0. \tag{B3}$$

$$\frac{dj^*}{dC} = -\frac{1-\alpha j^*}{C\alpha j^*}\left(\frac{d\Omega(j^*)}{dj^*}\right)^{-1} < 0. \tag{B4}$$

□

By using this lemma, Propositions 3 is proved.

Proof of Proposition 3. From (17) and (23) in the main script, the wage in a steady state are characterized by j^*,

$$w = j^*(1-j^*)\frac{C}{1-\alpha}. \tag{B5}$$

Thus, the first order differentiation with respect to γ is

$$\frac{dw}{d\gamma} = \frac{dw}{dj^*}\frac{dj^*}{d\gamma} = \frac{C(1-2j^*)}{1-\alpha}\frac{dj^*}{d\gamma}. \tag{B6}$$

Since $dj^*/d\gamma > 0$ from Lemma B.1, (B6) implies that if the threshold at equilibrium j^* is lower than 1/2, the productivity improvement of automation technology pushes the wage up. On the other hand, if $j^* > 1/2$, it pulls the wage down. □

Proof of Proposition 4. Suppose that the condition of Proposition A.1 holds. From

(B1), (B4) and (B5),

$$\begin{aligned} \frac{dw}{dC} &= \frac{\partial w}{\partial C} + \frac{\partial w}{\partial j^*}\frac{dj^*}{dC} \\ &= \frac{j^*(1-j^*)}{1-\alpha}\left[1 + \frac{(1-\alpha j^*)(2j^*-1)}{Q(j^*)}\right] \\ &= \frac{j^*(1-j^*)}{(1-\alpha)Q(j^*)}\left[Q(j^*) + (1-\alpha j^*)(2j^*-1)\right] \\ &= \frac{j^*(1-j^*)}{(1-\alpha)Q(j^*)}W_c(j^*), \qquad \text{(B7)} \\ \textit{where} \quad W_c(j^*) &\equiv Q(j^*) + (1-\alpha j^*)(2j^*-1). \qquad \text{(B8)} \end{aligned}$$

From Proposition A.1, $Q(j^*) > 0$ for $j^* \in (0,1)$. If $j^* \geq 1/2$, since $(1-\alpha j^*)(2j^*-1) \geq 0$, $W_c > 0$ and thus $dw/dC > 0$. If $j^* < 1/2$, there exists the region of j^* in which $dw/dC < 0$ depending on C. We show below that there exists a threshold C_{w2}, and $dw/dC > 0$ for all j^* when $C < C_{w2}$.

The second derivative of $W_c(j^*)$ is:

$$W_c''(j^*) = Q''(j^*) - 4\alpha. \qquad \text{(B9)}$$

Because $Q''(j^*)$ is a decreasing function for $j^* \in (0,1/2)$ and $Q''(1/2) = 2\alpha + 6 > 4\alpha$ from (A15), $W_c''(j^*) > 0$ for $j^* \in (0,1/2)$. Thus, $W_c'(j^*) = Q'(j^*) - 4\alpha j^* + \alpha + 2$ is an increasing function for $j^* \in (0,1/2)$. Because $\lim_{j^*\to 0} W_c'(j^*) = -\infty$, $W_c(j^*)$ for $j^* \in (0,1/2)$ is a decreasing function when $W_c'(1/2) \leq 0$, and is an U-shaped function when $W_c'(1/2) > 0$. Figure B.1 illustrates the two cases: the upper row shows the former case and the lower row shows the latter case.

For the former case, since Proposition A.1 assures $Q(1/2) > 0$, $W_c(1/2) > 0$ from (B8). Therefore, if the following condition holds, $W_c(j^*) > 0$ and thus $dw/dC > 0$ for $j^* \in (0,1/2)$:

$$W_c'(1/2) \leq 0 \Leftrightarrow C \leq C_{w1} \equiv \exp\{\log[4(1-\alpha)] - 1\}. \qquad \text{(B10)}$$

When $W_c(j^*)$ is an U-shaped function, if the value of $W_c(j^*)$ at the extreme point j^c is positive, that is $W_c(j^c) > 0$, $dw/dC > 0$ for $j^* \in (0,1/2)$. Since an increase in C lowers Q from (A11), it also lowers W_c from (B8). Let C_{w2} be the level of C

and j^{cc} be the level of j^c such that $W_c(j^c) = 0$ Then, when following condition holds, $dw/dC > 0$ for any j^*:

$$C < C_{w2} \equiv \frac{(1-\alpha)\exp\left[\frac{(1-\alpha)(j^{cc})^2+\alpha j^{cc}}{1-j^{cc}}\right]}{j^{cc}(1-j^{cc})}, \tag{B11}$$

$$where \quad -(1-\alpha)(j^{cc})^3 - 2\alpha(j^{cc})^2 + (3+\alpha)j^{cc} - 1 = 0. \tag{B12}$$

Conversely, when $C \geq C_{w2}$, there exists the region of j^* in which $dw/dC \leq 0$. The rightmost two figures at the lower row in Figure B.1 illustrate the cases of $C < C_{w2}$ and $C \geq C_{w2}$. The figure of the case of $C \geq C_{w2}$ shows that $W_c(j^*) > 0$ and thus $dw/dC > 0$ when j^* is sufficiently small and sufficiently large, and $W_c(j^*) < 0$ and thus $dw/dC < 0$ when j^* is intermediate. □

We numerically examine the relationship between C_{w1}, C_{w2} and C_2 because the relationship affects the sign of the marginal impact on the wage, but the relationship cannot be derived analytically. From (A18), (A19), (B10), (B11) and (B12), C_{w1}, C_{w2}, and C_2 depend only on α. Proposition A.1 shows that $C < C_2$ is the condition of the existence of a unique steady state. Thus, if $C_{w2} < C_2$ for any α, the region in which $dw/dC \leq 0$ could exist when $C \geq C_{w2}$. Figure B.2 illustrates the relationship between C_{w1} (red circle), C_{w2} (black diamond), C_2 (blue square) and α. The figure shows $C_{w1} < C_{w2} < C_2$ for all α. Because $C_{w2} < C_2$, when $C_{w2} \leq C < C_2$, there exists the region in which $dw/dC \leq 0$. From the graph of $W_c(j^*)$ in the bottom right figure of Figure B.1, $W_c > 0 \Leftrightarrow dw/dC > 0$ when $j^* \in (0, 1/2)$ is low enough and high enough, and $W_c \leq 0 \Leftrightarrow dw/dC \leq 0$ when j^* is intermediate.

B.2. The Proof of Proposition 5

From (17) and (11) in the main script, aggregate profit of managers at the steady state is:

$$\Pi \equiv \int_0^{j^*} \pi^k(j)dj + \int_{j^*}^1 \pi^l(j)dj = \frac{1}{2}C(j^*)^2 + 0 = \frac{1}{2}C(j^*)^2. \tag{B13}$$

The aggregate fixed cost is:

$$\mathcal{C} \equiv \int_0^{j^*} C \cdot j dj = \frac{1}{2} C (j^*)^2. \tag{B14}$$

Thus, the income share of managers is

$$\frac{\Pi}{Y - \mathcal{C}} = \frac{(1-\alpha) j^*}{2 - (1-\alpha) j^*}. \tag{B15}$$

The marginal effect of of an improvement of automation technology on the labor share of income is:

$$\frac{d\Pi/(Y - \mathcal{C})}{d\gamma} = \frac{2(1-\alpha)}{[2 - (1-\alpha) j^*]^2} \cdot \frac{dj^*}{d\gamma} > 0 \tag{B16}$$

When Proposition A.1 holds, Lemma B.1 assures the inequality above.

On the other hand, the income share of labor is given by (B5):

$$\frac{w}{Y - \mathcal{C}} = \frac{2(1 - j^*)}{2 - (1-\alpha) j^*} \tag{B17}$$

Thus, since the marginal effect is

$$\frac{dw/(Y - \mathcal{C})}{d\gamma} = -\frac{2(1+\alpha)}{[2 - (1-\alpha) j^*]^2} \frac{dj^*}{d\gamma} < 0, \tag{B18}$$

The wage decreases as automation technology improves. As for total income share:

$$\frac{d}{d\gamma} \left(\frac{\Pi}{Y - \mathcal{C}} + \frac{w}{Y - \mathcal{C}} \right) = \frac{-4\alpha}{[2 - (1-\alpha) j^*]^2} \frac{dj^*}{d\gamma} < 0. \tag{B19}$$

Thus, the total labor income share decreases with γ. The effect of a change in C on labor income share can be proved in the same manner.

B.3. The Proof of Proposition 6

In the steady state, (40) is:

$$H \equiv \frac{\beta}{1+\beta} \bar{j}(1 - \bar{j}) Y - \frac{K}{\alpha(1+\beta)} \left[1 - \alpha - \frac{C\bar{j}}{2Y} \right] - K = 0. \tag{B20}$$

By totally differentiating the above equation, the effect of an increase in γ on K is:

$$\frac{dK}{d\gamma} = -\frac{\partial H}{\partial Y}\left(\frac{\partial H}{\partial K}\right)^{-1}\frac{dY}{d\gamma}, \tag{B21}$$

$$where \quad \frac{\partial H}{\partial Y} = \frac{\beta}{1+\beta}\bar{j}(1-\bar{j}) - \frac{1}{Y}\frac{K \cdot C\bar{j}}{2(1+\beta)\alpha Y}, \tag{B22}$$

$$\frac{\partial H}{\partial K} = -\frac{1-\alpha}{\alpha(1+\beta)} + \frac{C\bar{j}}{2(1+\beta)\alpha Y} - 1. \tag{B23}$$

By using (B20), (B23) is:

$$\frac{\partial H}{\partial K} = -\frac{\beta}{1+\beta}\bar{j}(1-\bar{j})\frac{Y}{K}. \tag{B24}$$

From (39), the effect of an increase in γ on Y is:

$$\frac{dY}{d\gamma} = \bar{j}\frac{Y}{\gamma} + \alpha\bar{j}\frac{Y}{K}\frac{dK}{d\gamma}. \tag{B25}$$

By substituting (B21) into (B25):

$$\frac{dY}{d\gamma} = \bar{j}\frac{Y}{\gamma}\left[1 + \alpha\bar{j}\frac{Y}{K}\left(\frac{\partial H}{\partial K}\right)^{-1}\frac{\partial H}{\partial Y}\right]^{-1}. \tag{B26}$$

From (B22) and (B24),

$$\left(\frac{\partial H}{\partial K}\right)^{-1}\frac{\partial H}{\partial Y} = -\frac{K}{Y}\left[1 - \frac{2\alpha\beta(1-\bar{j})Y^2}{CK}\right]. \tag{B27}$$

Substituting (B27) into (B26):

$$\frac{dY}{d\gamma} = \bar{j}\frac{Y}{\gamma}\left[(1-\alpha\bar{j}) + \alpha\bar{j}\frac{CK}{2\alpha\beta(1-\bar{j})Y^2}\right]^{-1} > 0. \tag{B28}$$

Thus, effect of an increase in γ on the wage as the following equation is positive:

$$\frac{dw}{d\gamma} = (1-\bar{j})\frac{dY}{d\gamma} > 0. \tag{B29}$$

□

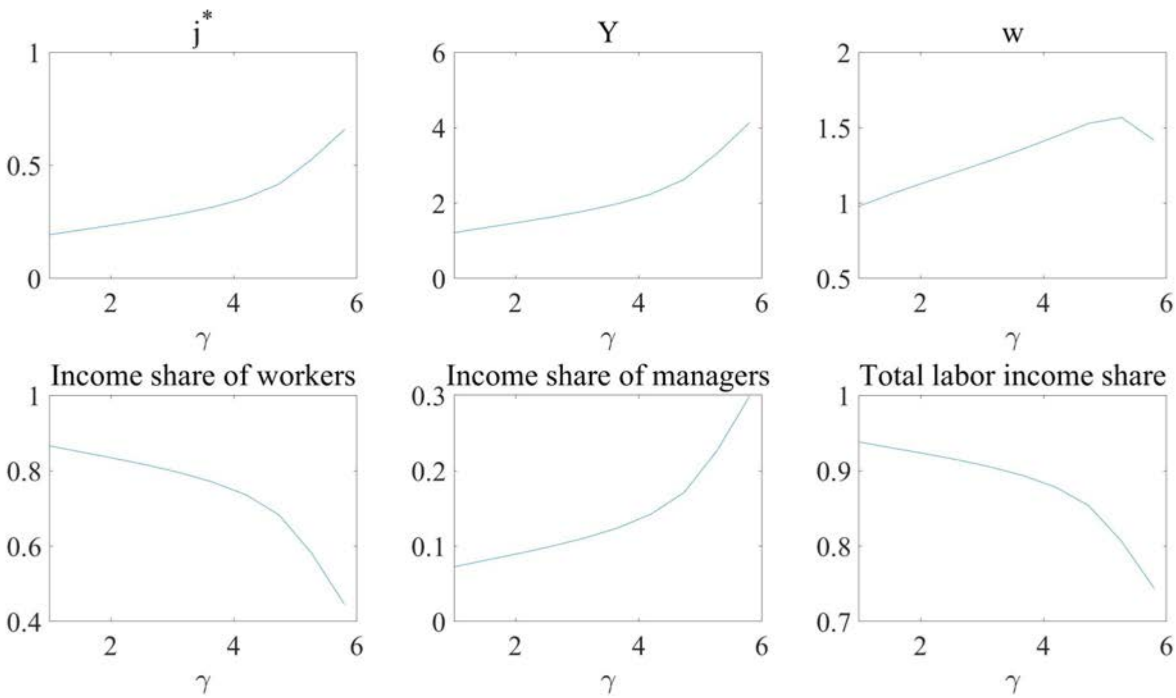

Figure 1 Effects of an improvement of automation productivity in the steady state.

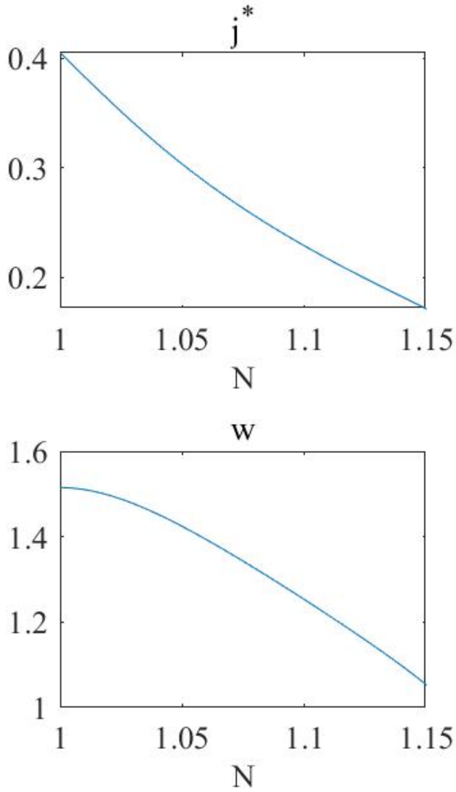

Figure 2 Effects of a creation of new industries in the steady state.

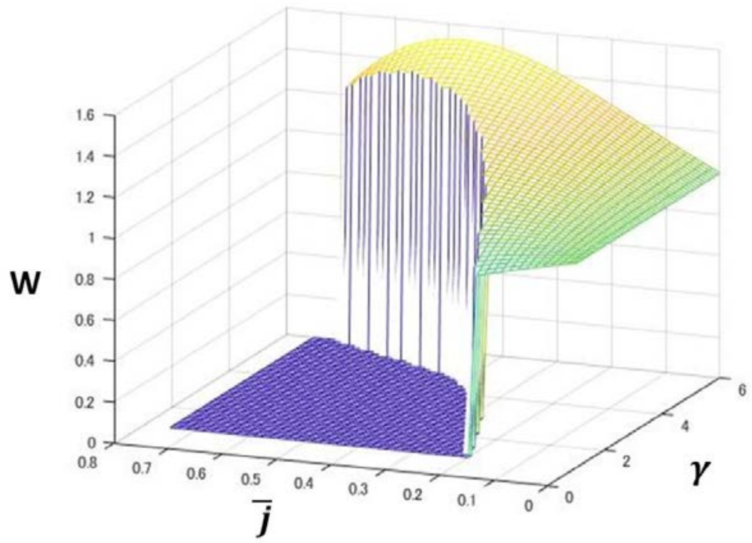

Figure 3 The wage when there is a technological limitation $\overline{j}$. This figure illustrates the case of $\overline{j} < j^*$.

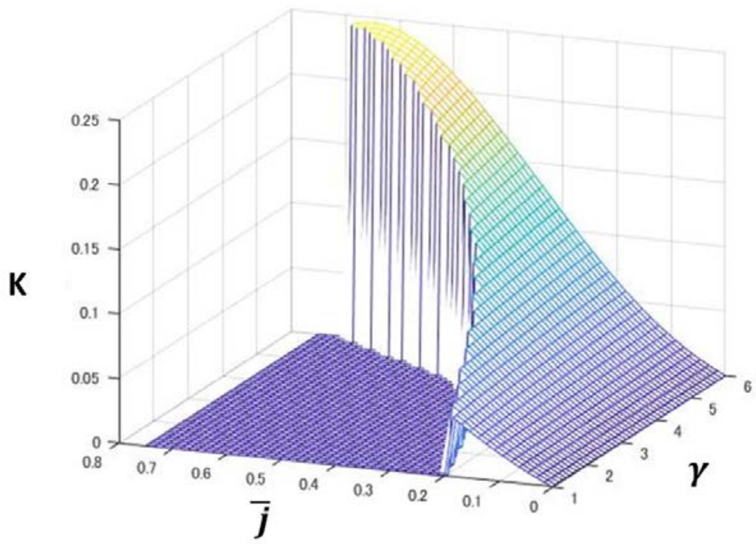

Figure 4 Aggregate capital when there is a technological limitation $\overline{j}$. This figure illustrates the case of $\overline{j} < j^*$.

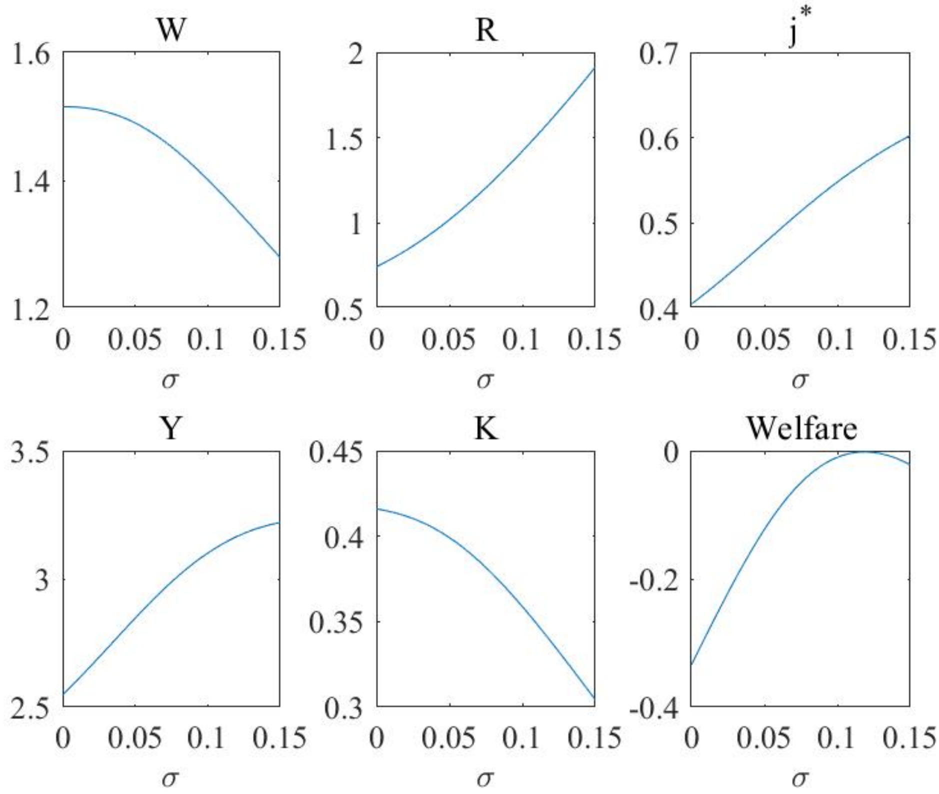

Figure 5 Effects of a subsidy policy in the steady state.

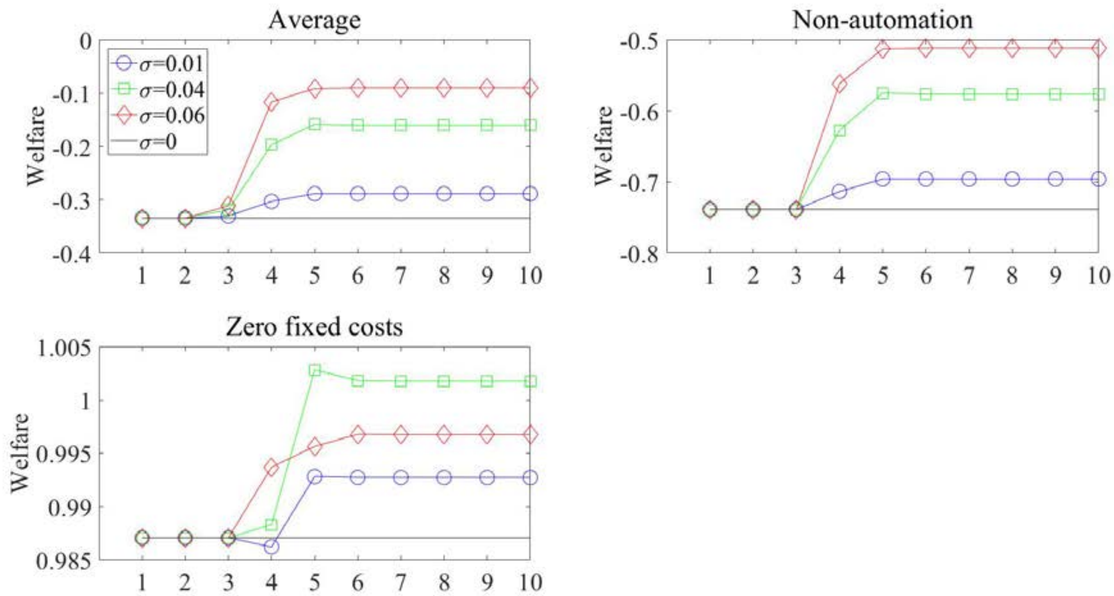

Figure 6 Dynamics of welfare: aggregate welfare of all households, welfare of households who choose non-automation technology and welfare of households who face zero fixed costs.

Note: In each panel, the blue line (round), green line (square), and red line (diamond) indicate the dynamics in cases in which the subsidy rate, σ, is 0.01, 0.04, and 0.06, respectively. The black (dotted) line in each panel indicates the level of welfare in the previous steady state.

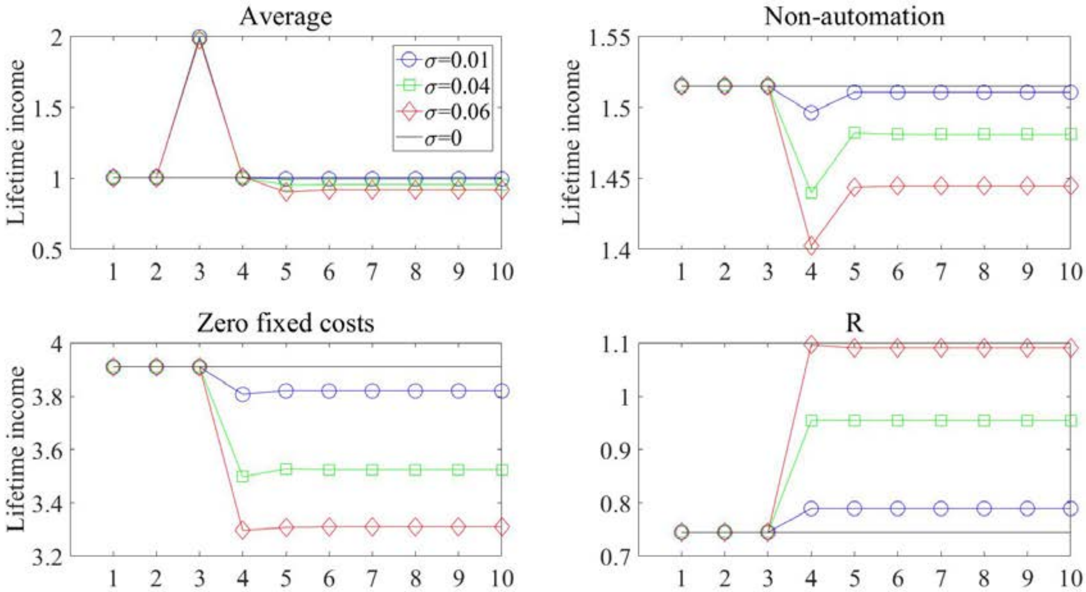

Figure 7 Dynamics of the interest rate and lifetime income of each type of household presented in Figure 6.

Note: In each panel, the blue line (round), green line (square), and red line (diamond) indicate the dynamics in cases in which the subsidy rate, σ, is 0.01, 0.04, and 0.06, respectively. The black (dotted) line in each panel indicates the level of welfare in the previous steady state.

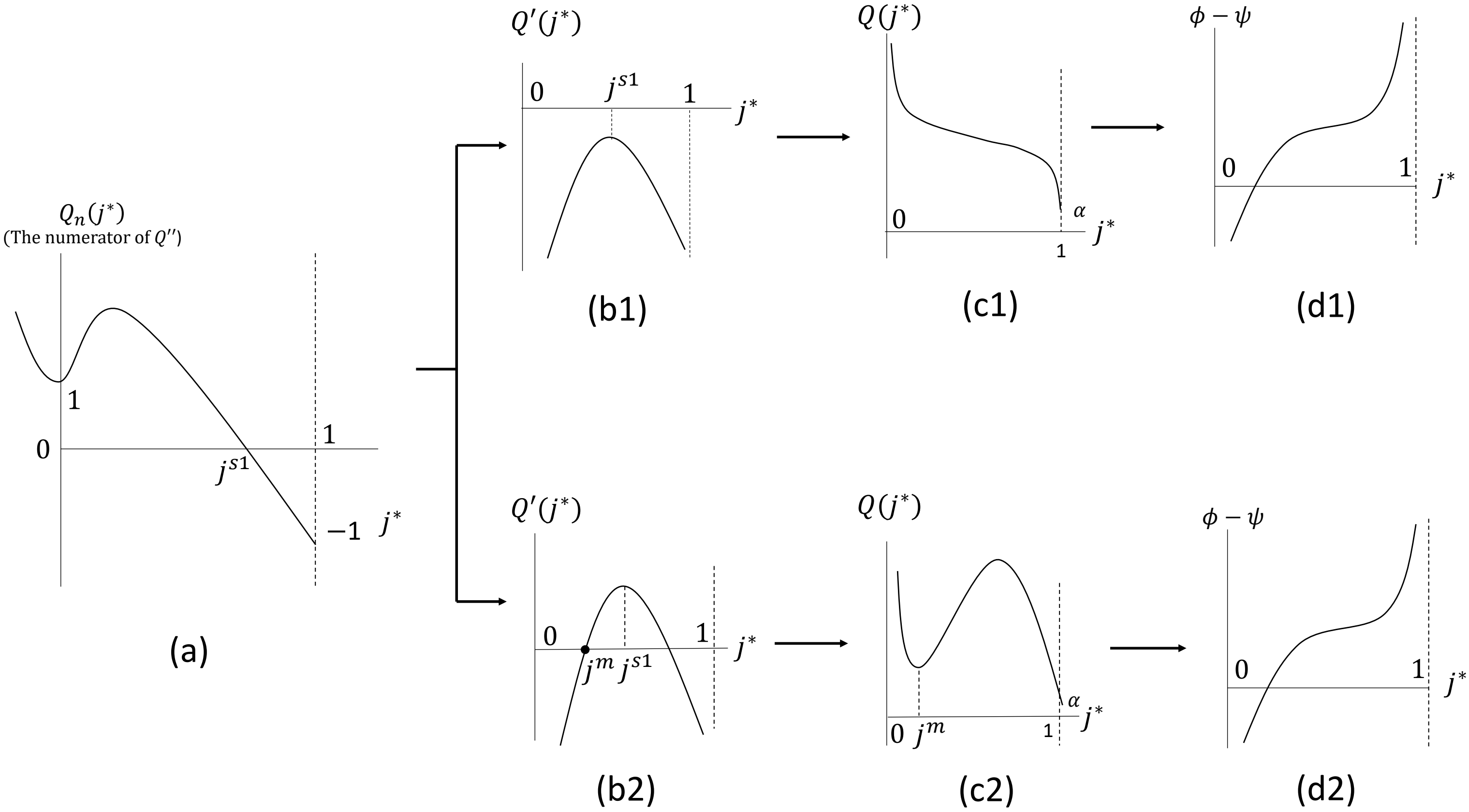

Figure A.1 Cases of Proposition A.1.

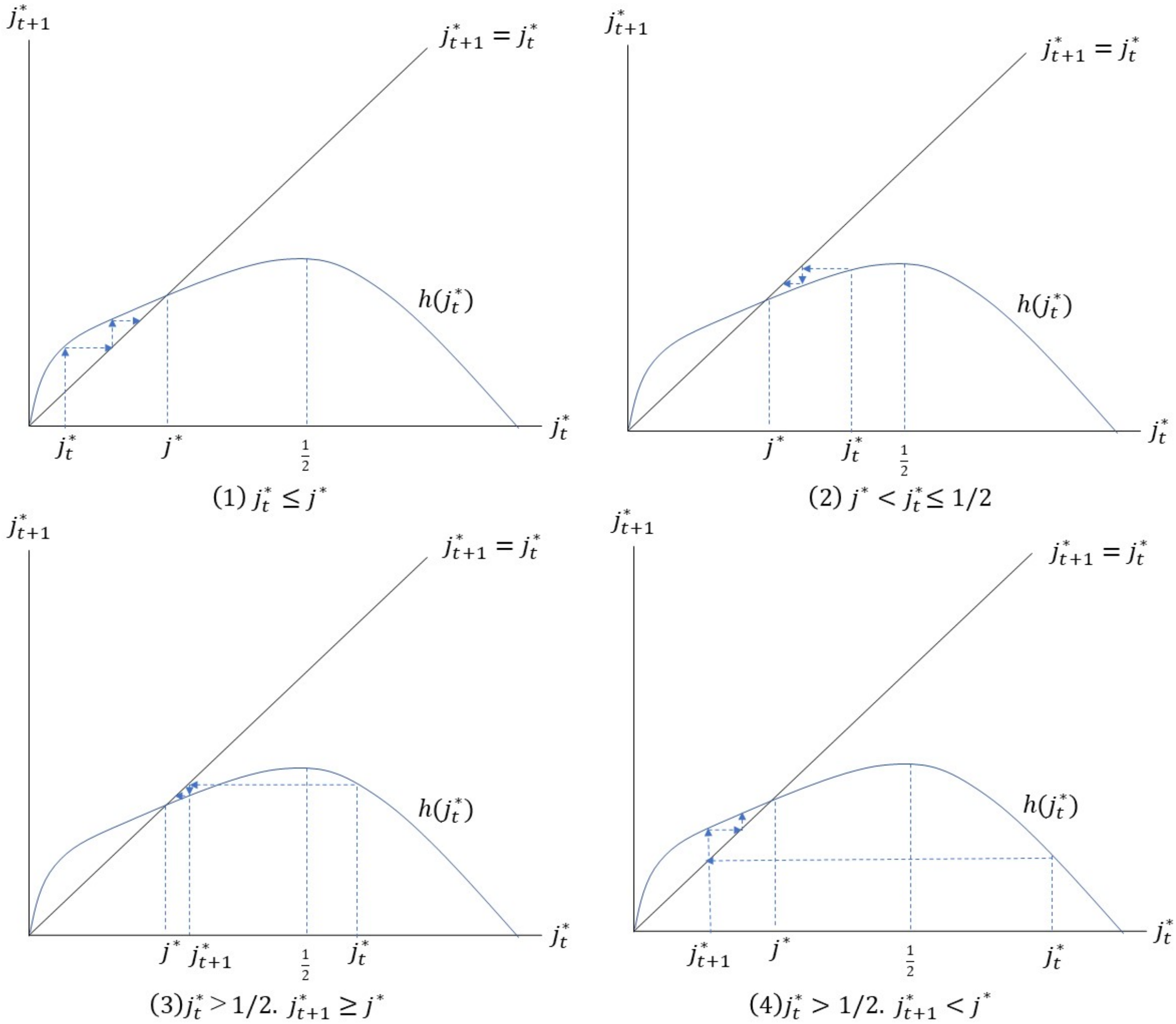

Figure A.2 Patterns of convergence when $j^* \leq 1/2$.

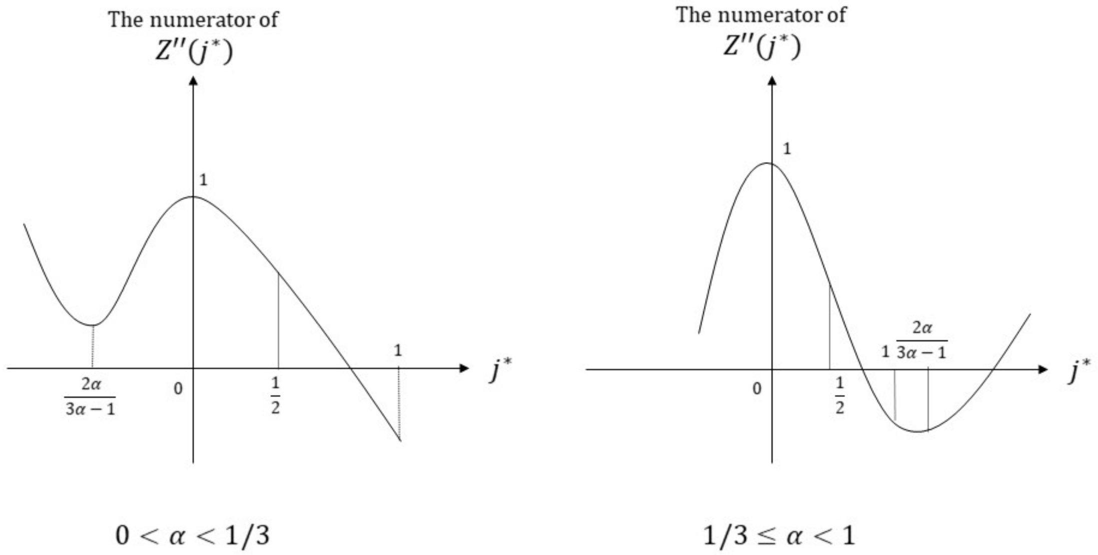

Figure A.3 Shape of $Z''(j^*)$ in Lemma A.1.

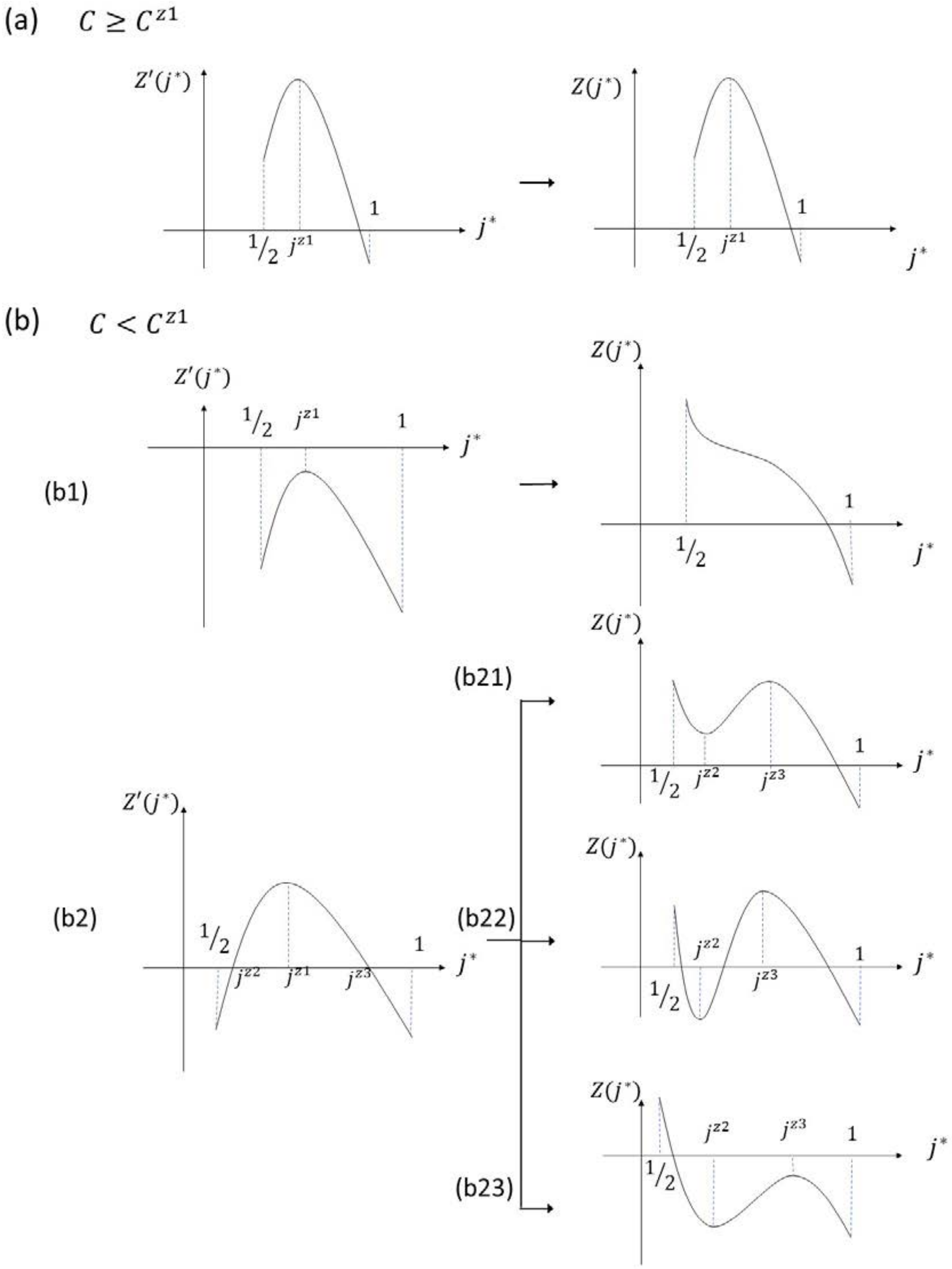

Figure A.4 Shape of $Z(j^*)$ in Proposition A.4.

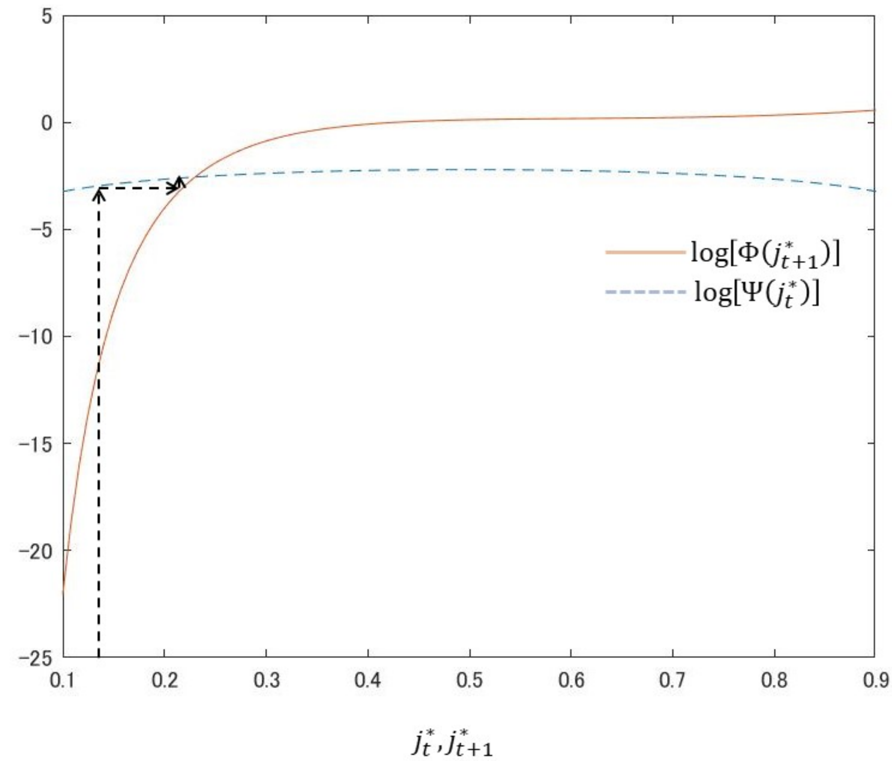

Figure A.5 Case of the unique steady state with monotonic convergence.

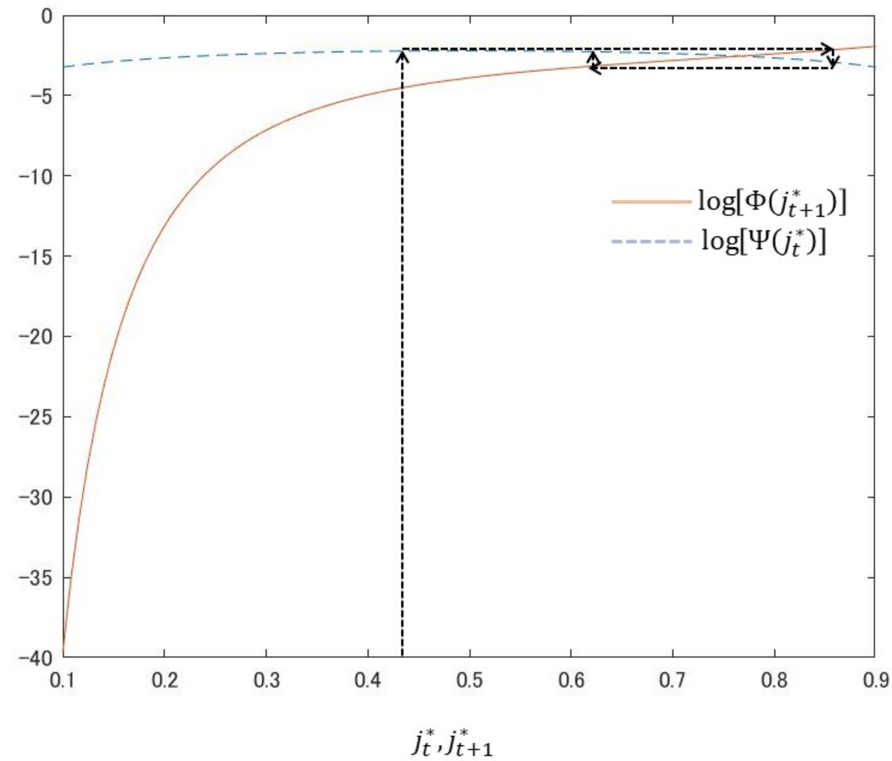

Figure A.6 Case of the unique steady state with cyclical convergence.

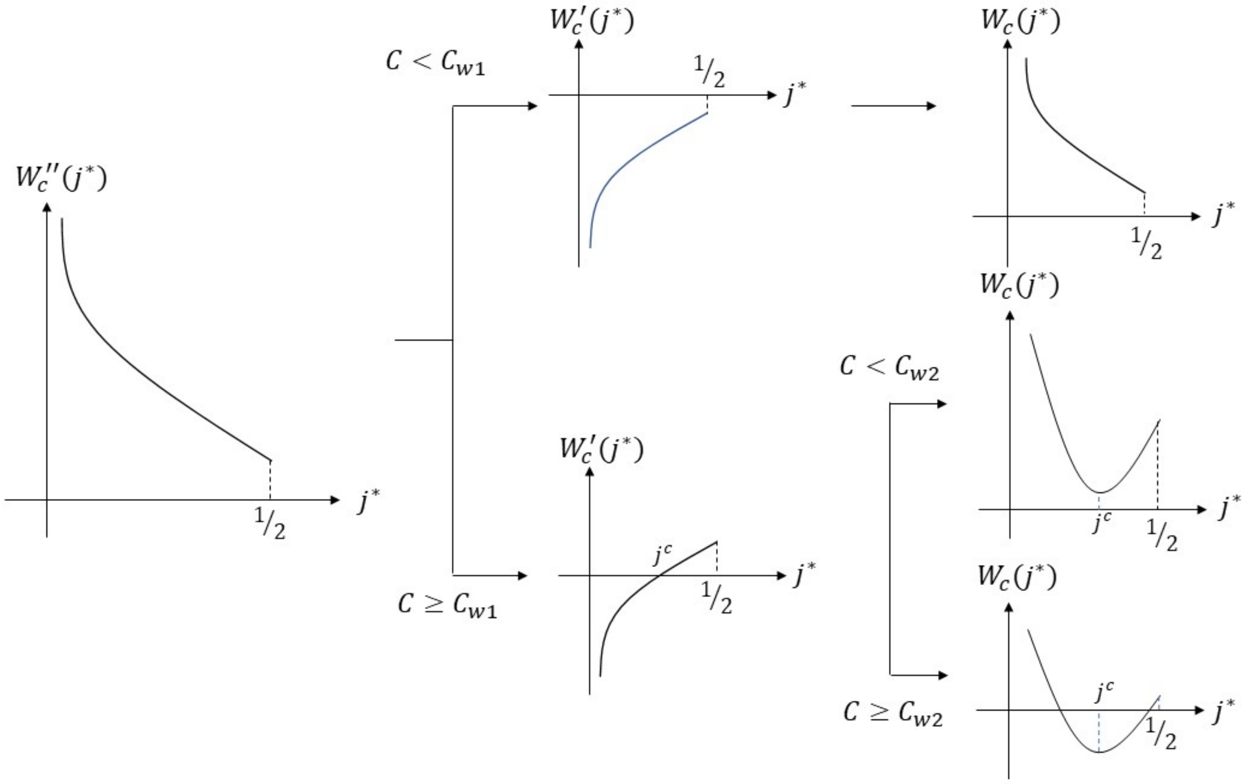

Figure B.1 The patterns of the shape of $W_c(j^*)$.

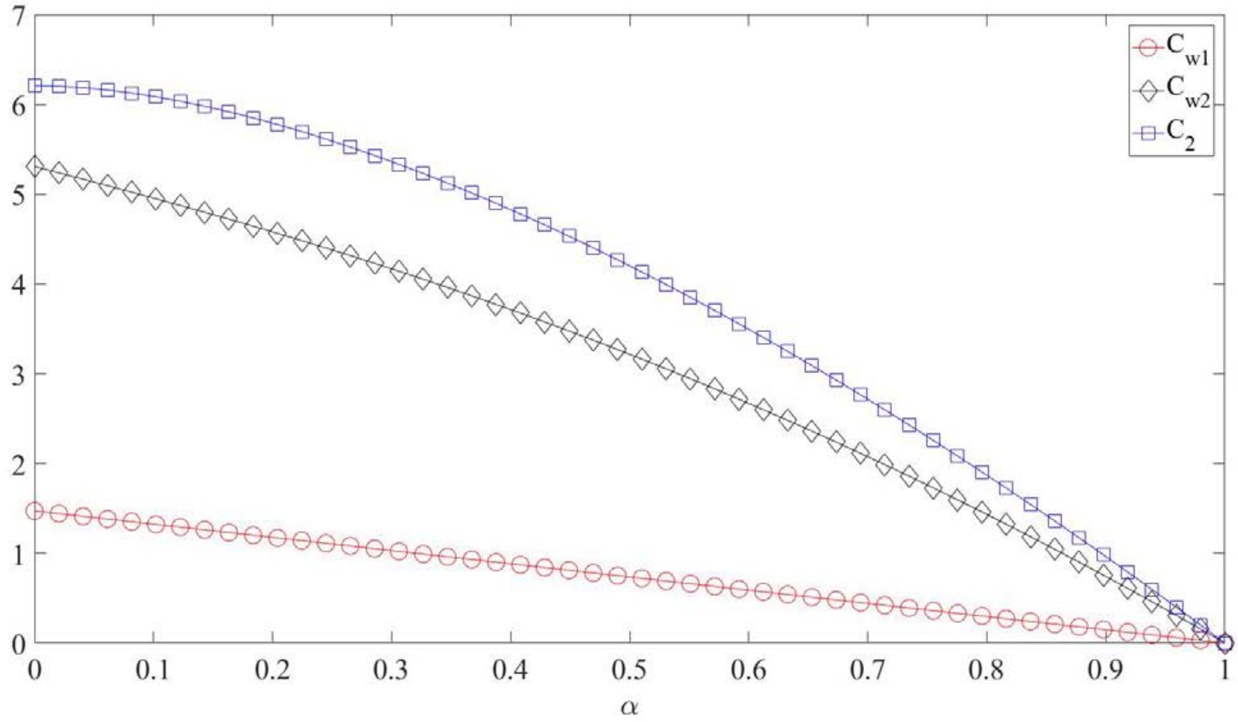

Figure B.2 Thresholds of fixed cost C: C_{w1} (red circle) , C_{w2} (black diamond), and C_2 (blue square) with respect to α.

Note: When $C < C_{w1}$, $W_c'(1/2) < 0$. When $C < C_{w2}$, there exists an area of j^* in which $dw/dC < 0$. When $C < C_2$, a unique steady state exists.