

**Financial friction sources in emerging economies:
Structural estimation of sovereign default models**

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Structural estimation of sovereign default models*

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Abstract

There are two literature strands that explain stylized facts in emerging economies: the stochastic productivity trend or financial frictions. However, financial frictions are driven by both trend and stationary productivity shocks, thus distinguishing their impact from the direct role of output fluctuations is essential. We estimate sovereign default models, full-nonlinear dynamic stochastic general equilibrium (DSGE) with micro-founded financial imperfections, applying a particle filter, and evaluate the source of financial frictions. The main finding is that stationary shocks rather than trend shocks account for financial frictions and the resulting countercyclicality, except for the post-1977 period in Mexico; however, the exception disappears for 1902–2005 as long-run data. The sources of financial frictions are determined by the persistence and volatility of shocks, asymmetric domestic cost of sovereign default, and mismatch between sovereign default and business cycles.

Keywords: Sovereign default, Business cycles, Financial imperfections, Particle filter, Sequential Monte Carlo, Full nonlinear DSGE

JEL classification: E32, E62, F41, F44

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1. Introduction

The empirical regularities of business cycles in emerging economies are excessive volatility in consumption, the countercyclical current account balance, countercyclical interest rates, and frequent default at equilibrium (e.g., Neumeyer and Perri, 2005; Uribe and Yue, 2006; Aguiar and Gopinath, 2006, 2007; García-Cicco et al., 2010; Uribe and Schmitt-Grohé, 2017). There are two literature strands that explain these stylized facts. The first claims the most important source of these characteristics is a permanent productivity shock. The second emphasizes financial frictions over nonstationary productivity shocks.

For instance, Aguiar and Gopinath (2007) introduce nonstationary productivity shocks into an open-economy real business cycle (RBC) model, successfully replicating the characteristics of emerging economies. By contrast, García-Cicco et al. (2010) and Chang and Fernández (2013) emphasize financial frictions over nonstationary productivity shocks. They add financial frictions such as stochastic high debt-elastic interest rate premia to an RBC model with trend shocks, and report that the role of a stochastic trend is drastically decreased. Specifically, the persistence and volatility of trend shocks are assigned small values. Additionally, a financial frictions model performs better than frictionless RBC models with stochastic trends, as a frictionless RBC model tends to generate nearly random-walk trade balances and fails to replicate excess volatility in consumption.

The above two seminal papers suggest that financial frictions considerably account for the volatile Solow residuals¹ of frictionless RBC. However, financial frictions are not driven only by a stationary transitory productivity shock, but also a trend shock (Aguiar and Gopinath, 2006; Aguiar

¹ As other sources of nonstationarity, Boz et al. (2011) suggest informational frictions, and Naoussi and Tripier (2013) the level of income, quality of institutions, and size of credit markets. Álvarez-Parra et al. (2013) report only a minor effect for trend output shocks in economies with durable nondurable and goods. These elements are important, but outside the scope of this paper.

et al., 2016). As such, a stochastic trend may play an indirect but important role in causing financial frictions. Although Chang and Fernández (2013) have already shown trend shocks do not have a significant effect on the bond spread in linearized equations, empirical studies reveal the responses of credit risks to economic shocks have nonlinear properties: moderate debtors suffer little impacts from shocks, whereas countries close to borderline solvency would face a steeper increase in a bond spread (Daliyami et al., 2008; Jeanneret and Souissi, 2016; Galstyan and Velic, 2017), suggesting a stochastic trend may affect financial frictions in a nonlinear economy.

Sovereign default models are suitable for addressing this issue, since their credit risks are caused nonlinearly through asset incompleteness. Indeed, García-Cicco et al. (2010) and Chang and Fernández (2013) point out that a promising area for future research is estimating and evaluating a dynamic stochastic general equilibrium (DSGE) model of an emerging economy with micro-founded financial imperfections. The literature on sovereign default models has repeatedly succeeded in replicating the stylized facts of business cycles in emerging economies² but, to the best of our knowledge, there is no estimation attempt focusing on a random-walk stochastic trend.³ Therefore, we estimate a structural sovereign default model by applying a Bayesian state space framework, and test which trend shocks or transitory shocks largely drive the countercyclical current account, countercyclical interest rates, and frequent default in emerging markets.

Our main result indicates that stationary shocks rather than trend shocks principally account for financial frictions, which is consistent with the findings of García-Cicco et al. (2010) and Chang and Fernández (2013). Additionally, the only exception is Mexico during 1977–2013, which is in line

² See, for example, Arellano (2008), Cuadra and Sapriza (2008), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Yue (2010), Boz (2011), Mendoza and Yue (2012), and Durdu et al. (2013).

³ To the best of our knowledge, except for this paper, only Gumus et al. (2017) estimate the Arellano (2008) model applying a maximum simulated likelihood estimation. However, our work is focusing on the effect of a stochastic trend, whereas they compare the performance of predicting the timing of default events of Arellano (2008) with that of a logit-model using filtered series. Moreover, our estimation assumes random-walk productivity shocks using a Bayesian nonlinear state-space model, which we describe in detail in Section 4.

with Aguiar and Gopinath's (2007) results. However, long-run data for 1902–2005 shows the role of trend shocks on financial frictions is drastically reduced, which is consistent with García-Cicco et al.'s (2010) findings: an empirical analysis of the post-1980 period may be problematic, since 1980–2005 contains only between one and a half and two cycles. On the other hand, due to high persistence, trend shocks play an important role in volatile trends (as per Aguiar and Gopinath (2007)), although their impact on financial frictions is limited.

The seemingly contradictory results stem from four elements. First, stationary shocks are even more persistent than trend shocks, which implies that stationary shocks drive financial frictions more than trend shocks. The intuition behind this is derived in Section 4. A highly persistent positive shock implies income is higher today but even higher in the next period, thus strengthening the ability to access international financial markets to bring forward expected income and leading to significant debt accumulation with low interest rates. The more persistent a shock becomes, the more intense the effect is.

Second, as the volatility of shocks increases, an expected value of utility conditional on the positive realization of output increases, leading to higher debt accumulation and lower credit spreads⁴. For Mexico, during 1977–2013, the volatility of trend shocks is larger than that of stationary shocks, thereby the exception. For the other cases, including Mexico during 1902–2005, the aforementioned persistence effect overwhelms the volatility effect, and/or the latter effect takes sides with stationary shocks.

Third, the asymmetric domestic cost of default amplifies the second factor, but not the first effect. The asymmetric domestic cost is a penalty imposed on debtor output only when an income shock achieves more than its unconditional mean, but is not sanctioned when a shock is below its

⁴ The volatility effects reverses if volatility is too high, as per Uribe and Schmitt-Grohé (2017). As a result, agents are exposed to large negative shocks, leading to higher default frequency and more bond spreads. Increased uncertainty induces an increase in precautionary savings and, consequently, a decrease in the desired external debt level.

unconditional mean. Generally, the more costs are imposed on defaulters, the more a sovereign hesitates to renege a debt contract, resulting in lower credit risks and more debt capacity. Asymmetric cost intensifies the volatility effect, since larger volatility indicates more chances to exceed the unconditional mean, leading to a more substantial domestic cost preventing a sovereign from defaulting. Meanwhile, as higher persistence means fewer recovery opportunities from downturns, asymmetric cost does not amplify the persistence effect. In the estimation, the asymmetric domestic cost in Mexico is severe compared to that of Argentina, thereby the cost promotes 1977–2013 Mexico as the exception for the source of financial frictions. Providing parameter estimates of sovereign default models by Bayesian structural estimation is another contribution of this paper. The calibrations differ among studies, although almost all papers investigating the Argentinian economy,⁵ reflecting a low agreement on the magnitude of default costs in the empirical studies.⁶

Fourth, although the formulations of trend and stationary shocks are similar, identification arises from sovereign default cycles not always coinciding with business cycles, which indicates the drivers of the countercyclicality should sometimes remain above their unconditional mean for a long time, even during recessions. In fact, Reinhart and Rogoff (2014) report that Argentina, during 1894–1950, did not experience sovereign default, but at least two business cycles according to Figure 1 in

⁵ The calibrations of default costs also differ among studies. For example, the annual probability of re-entry ranges from 29.3% (Mendoza and Yue, 2012) to 73.4% (Arellano, 2008), and the asymmetric domestic cost from 2.0% (Aguiar and Gopinath, 2007) to 10.0% (Alfaro and Kanczuk, 2009).

⁶ In a broad survey on sovereign defaults, Panizza et al. (2009) find the main costs of sovereign defaults are exclusion from international capital markets or trade, interest rate spikes, and large output reductions. Regarding exclusion from international trade, Rose (2005) explains that sovereign default decreases bilateral trade by around 8% over 15 years. On the other hand, Gelos et al. (2011) report the average exclusion as four years (in the 1980s) or 0–2 years (after 1980). Martinez and Sandleris (2011) find a 3.2% decrease in trade over five years. As for spikes in interest rates, Flandreau and Zumer (2004) find defaults increase spreads by around 90 basis points, whereas Borensztein and Panizza (2009) find that the effect is 250–400 basis points. The effects of domestic costs (large decline in output) are found to be 0.6% by Chuan and Sturzenegger (2005) and 0.6–2.5% by Borensztein and Panizza (2009).

García-Cicco et al. (2010). By definition, trend shocks are responsible for not only output fluctuations but also growth level, seeming rather related to business cycles. Thereby, stationary shocks rather tend to take charge of sovereign default cycles.

Overall, through the resulting financial frictions, stationary shocks dominate the countercyclicality in business cycles, although trend shocks account for volatile outputs and the exception of Mexico post-1977 arises. In other words, this paper bridges the gap between the two literatures strands.

The basic estimation strategy in this paper is almost identical to that of Gust et al. (2017), which conduct a Bayesian estimation on full-nonlinear DSGE models. To evaluate likelihoods, we must use a particle filter instead of Kalman filter, because sovereign default models are full-nonlinear.⁷ For the same reason, Gust et al. (2017) use a particle filter for the New-Keynesian model with zero lower bound. The main obstacle is that both the full-nonlinear solution and particle filter require significant computation times.

For reducing the computational burden, we use coarse grids compared with preceding studies, such as Aguiar and Gopinath (2006), but interpolate value functions. For instance, Hatchondo et al. (2010) report that a discrete state space technique (DSS) with interpolation with coarse grids solves the model faster and more accurate than a DSS with fine grids but without interpolation. The method of approximating stationary AR (1) processes is Rouwenhorst's (1995), since it is the best approximation method for highly persistent AR (1) processes according to Kopecky and Suen (2010). Moreover, the persistence of transitory productivity shocks is often above 0.9 in the case of sovereign default models.

The estimation method is the simulated tempering sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2014, 2015), instead of the random-walk Metropolis–Hastings

⁷ Almost all sovereign default models are full-nonlinear DSGE, for example, Arellano (2008), Cuadra and Sapriza (2008), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Yue (2010), Boz (2011), Mendoza and Yue (2012), and Durdu et al. (2013).

algorithm (RWMH). The utilized algorithm has some advantages over RWMH, such as propagating the particles of parameter vectors to the entire prior space. Whereas RWMH has one or several chains with single initial values, simulated tempering SMC avoids parameter draws with severe autocorrelation or caught in a local mode. The advantage is important, since there is less agreement on the parameter values of sovereign default models. There are some applications of simulated tempering SMC to DSGE models, such as the studies of Herbst and Schorfheide (2014, 2015), Gust et al. (2017), and Hirose et al. (2017), but to the best of our knowledge, this study is the first application of a simulated tempering SMC to a full nonlinear regime-switching DSGE model.

The remainder of this paper is organized as follows. In Section 2, we propose the sovereign default model. Section 3 presents an estimation strategy tailored to sovereign default models, Section 4 the estimation results, and Section 5 the sensitivity analysis. Section 6 concludes the paper.

2. Model

2.1. Model economy

We consider a sovereign default model with both stationary and nonstationary shocks, similar to those utilized by Aguiar and Gopinath (2006) and Arellano (2008). Unlike these two studies, we simultaneously analyze the effects of both transitory and permanent shocks as describes later. Another improvement is the model considering both proportional and asymmetric domestic costs.

We assume that there is a single tradable good. The economy receives a stochastic endowment stream, which has the similar formulation as the total factor productivity in Aguiar and Gopinath (2006, 2007), García-Cicco et al. (2010) and Chang and Fernández (2013):

$$Y_t = e^{z_t} \Gamma_t, \quad (1)$$

where Γ_t denotes the trend and z_t is a transitory shock. In equation (1), the endowment stream is

comprised of stationary and nonstationary productivity shocks. The transitory productivity shock, z_t , follows an AR (1):

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z. \quad (2)$$

Where $\varepsilon_t^z \sim N(0, \sigma_z^2)$. The stochastic trend is formulated as:

$$\Gamma_t = g_t \Gamma_{t-1}, \quad (3)$$

$$\ln g_t = (1 - \rho_g) (\ln \mu_g - c) + \rho_g \ln g_{t-1} + \varepsilon_t^g. \quad (4)$$

where $c = \frac{1}{2} \frac{\sigma_g^2}{(1 - \rho_g^2)}$ and $\varepsilon_t^g \sim N(0, \sigma_g^2)$. The state vector is unbounded, because the endowment stream has a trend. Detrending is discussed in subsection 2.3. Households are identical, and maximize utility according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \quad (5)$$

where $0 < \beta < 1$ is the discount factor, C_t consumption, and $u(\cdot)$ an increasing and strictly concave utility function. The utility function is assumed to display a constant coefficient of relative risk aversion, γ , as follows:

$$u(C) = \frac{(C)^{1-\gamma}}{1-\gamma}. \quad (6)$$

A benevolent government maximizes the present expected discounted value of the future utility

flows of households in equation (6). The government utilizes international borrowing to smooth consumption and alter its time path. It also buys one-period discount bonds B_{t+1} at price $q(B_{t+1}, z_t, \Gamma_t)$, which is endogenously determined, depending on the government's incentive to default, and total amounts of sovereign debt and endowment. Positive values for B_{t+1} indicate the government purchases bonds, and negative values issuing bonds in international financial markets. Earnings on the government portfolio are distributed as lump sums to households. The resource constraint in the economy when the government chooses to repay debts is:

$$C_t = Y_t + B_t - q(B_{t+1}, z_t, \Gamma_t)B_{t+1}. \quad (7)$$

The government is excluded from the international financial markets when it chooses to default. The resource constraint in the default state is:

$$C_t = Y_t^{def}, \quad (8)$$

where Y_t^{def} is the endowment when in default state.

Foreign investors are assumed to evaluate defaultable bonds in a risk-neutral manner. During every period, risk-neutral investors lend B_{t+1} to maximize expected profits, ϕ , as follows:

$$\phi = q(B_{t+1}, z_t, \Gamma_t)B_{t+1} - \frac{1 - \delta(B_{t+1}, z_t, \Gamma_t)}{1 + r}, \quad (9)$$

where $\delta(B_{t+1}, z_t, \Gamma_t)$ is the default probability, depending on debt accumulation and an aggregate shock.

2.2. Recursive formulation

Let $V^o(B_t, z_t, \Gamma_t)$ denote the government's value function before the default or repayment decisions. We define $V^c(B_t, z_t, \Gamma_t)$ as the value associated with not defaulting, and $V^d(z_t, \Gamma_t)$ as the one associated with defaulting. $V^o(B_t, z_t, \Gamma_t)$ satisfies

$$V^o(B_t, z_t, \Gamma_t) = \max_{\{c,d\}} \{V^c(B_t, z_t, \Gamma_t), V^d(z_t, \Gamma_t)\}. \quad (10)$$

The decision is given by

$$D(B_t, z_t, \Gamma_t) = \begin{cases} 1 & \text{if } V^c(B_t, z_t, \Gamma_t) < V^d(z_t, \Gamma_t). \\ 0 & \text{otherwise} \end{cases}. \quad (11)$$

An economy becomes an autarky when the government chooses default, and the value function is given by:

$$V^d(z_t, \Gamma_t) = u(Y_t^{def}) + \beta E_t \theta V^o(0, z_{t+1}, \Gamma_{t+1}) + (1 - \theta) \beta E_t V^d(z_{t+1}, \Gamma_{t+1}), \quad (12)$$

where θ is the probability the economy regains access to international financial markets. When the government decides to repay debts, the value function is given by:

$$V^c(B_t, z_t, \Gamma_t) = \max_{(B_{t+1})} \{u(C_t) + \beta E_t V^o(B_{t+1}, z_{t+1}, \Gamma_{t+1})\}. \quad (13)$$

Therefore, default probabilities $\delta(B_{t+1}, z_t, \Gamma_t)$ are given by:

$$\delta(B_{t+1}, z_t, \Gamma_t) = E_t[D_{t+1}]. \quad (14)$$

The bond price that satisfies the lender's zero-profit condition is:

$$q(B_{t+1}, z_t, \Gamma_t) = \frac{1 - E_t[1 - D_{t+1}]}{1 + r}. \quad (15)$$

2.3. Detrending and domestic cost

The state vector is unbounded, because the endowment stream has a trend. We normalize the nonstationary element following Aguiar and Gopinath (2006), dividing a variable X by $\mu_g \Gamma_{t-1}$, and denote $X/\mu_g \Gamma_{t-1}$ by x , whereas Aguiar et al. (2016) normalize their variables by the current level of technology Γ_t , as in the DSGE studies of Smets and Wouters (2007), and Chang and Fernández (2013). Importantly, the method of Aguiar and Gopinath (2006) enables us to analyze the effect of trend shocks on business cycles. The logged and detrended endowment streams of equation (1) are expressed as:

$$\begin{aligned} \ln Y_t - (\ln \mu_g + \ln \Gamma_{t-1}) &= z_t + \ln \Gamma_t - (\ln \mu_g + \ln \Gamma_{t-1}), \\ \ln y_t &= z_t + \ln g_t - \ln \mu_g. \end{aligned} \quad (16)$$

Hence, the detrended endowment stream is composed of both stationary and nonstationary productivity shocks. By contrast, the trend shock term is deleted if normalizing variables by the current level of technology, Γ_t . The budget constraints are:

$$c_t = y_t + b_t - q(b_{t+1}, z_t, g_t)b_{t+1}. \quad (17)$$

Then, the default decision is featured by:

$$D(b_t, z_t, g_t) = \begin{cases} 1 & \text{if } V^c(b_t, z_t, g_t) < V^d(z_t, g_t). \\ 0 & \text{otherwise} \end{cases}. \quad (18)$$

Further, the detrended value functions are given by:

$$V^o(b_t, z_t, g_t) = \max_{\{c, d\}} \{V^c(b_t, z_t, g_t), V^d(z_t, g_t)\}, \quad (19)$$

$$V^d(z_t, g_t) = u(y_t^{def}) + \beta E_t \theta V^o(0, z_{t+1}, g_{t+1}) + (1 - \theta) \beta E_t V^d(z_{t+1}, g_{t+1}), \quad (20)$$

$$V^c(b_t, z_t, g_t) = \max_{(b_{t+1})} \{u(c_t) + \beta E_t V^o(b_{t+1}, z_{t+1}, g_{t+1})\}. \quad (21)$$

The bond price that satisfies the lender's zero-profit condition is:

$$q(b_{t+1}, z_t, g_t) = \frac{1 - E_t[1 - D_{t+1}]}{1 + r}. \quad (22)$$

Domestic cost can be defined after normalization, since the asymmetric domestic cost requires the unconditional mean of the income process, which is a random-walk before normalization. The model includes two types of proportional and asymmetric domestic costs. Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), and Yue (2010) use only a proportional cost, whereas Arellano (2008), Cuadra and Saprizza (2008), and Cuadra et al. (2010) adopt only an asymmetric cost. However, the effects of both costs on business cycles differ. Proportional costs are immediately and always imposed to debtors during default, while asymmetric costs are only incurred when output fluctuates above the unconditional mean level. This means proportional cost always reduces the default incentive. However, asymmetric cost does not inhibit default enticement when the output is sufficiently lower than the unconditional mean. In the model,

two types of domestic costs are combined.

$$y_t^{def} = \begin{cases} (1 - \lambda_\beta)y_t & \text{if } (1 - \lambda_\beta)y_t < (1 - \lambda_\alpha)\bar{y} \\ (1 - \lambda_\alpha)\bar{y} & \text{if } (1 - \lambda_\beta)y_t \geq (1 - \lambda_\alpha)\bar{y} \end{cases}, \quad (23)$$

where \bar{y} denotes the unconditional mean of y_t .

2.4. Equilibrium definition

The equilibrium is characterized by

- a. a triplet of value functions $\{V^c(b_t, z_t, g_t), V^d(z_t, g_t), V^o(b_t, z_t, g_t)\}$;
- b. rules for default $D(b_t, z_t, g_t)$ and borrowing (saving) $b(b_t, z_t, g_t)$;
- c. and a bond price function $q(b_{t+1}, z_t, g_t)$;

such that

- i. given a bond price function $q(b_{t+1}, z_t, g_t)$, policy functions $D(b_t, z_t, g_t)$ and $b(b_t, z_t, g_t)$ solve Bellman equations $\{V^c(b_t, z_t, g_t), V^d(z_t, g_t), V^o(b_t, z_t, g_t)\}$;
- ii. given policy rules $D(b_t, z_t, g_t)$ and $b(b_t, z_t, g_t)$, bond price $q(b_{t+1}, z_t, g_t)$ satisfies equation (22).

3. Solution and econometric inference

3.1. Model solution

The model is numerically solved by value function iteration with linear interpolation. The solution features adopting linear interpolation for approximating value functions and using Rouwenhorst's (1995) method for the approximating AR (1) process. The details of the algorithm are provided in the Appendix.

Hatchondo et al. (2010) show interpolation methods enable us to solve sovereign default models

faster and more accurately than the DSS with fine grids, but without interpolations. DSS discretizes the AR (1) process for productivity shocks, and confines the government to selecting the optimal level of debt from a discrete set of points. By contrast, interpolation methods allow the sovereign to choose the optimal borrowing level from a continuous set, and the resulting bond price schedule and implied spread behavior are rather accurate.

As for approximation methods of the AR (1) process, Kopecky and Suen (2010) show that Rouwenhorst's (1995) method performs better than other methods, such as Tauchen's (1986), quadrature-based method, and Adda-Cooper method, especially when the persistence of shocks is above 0.9.

3.2. Data

The target economies are Argentina and Mexico, following the studies of Aguiar and Gopinath (2006, 2007), García-Cicco et al. (2010), Chang and Fernández (2013), and several other studies on structural sovereign default models. The observables are real GDP per capita, external debt stocks, interest rates, and default states. External debt stocks are deflated by dollar expected inflation rates and divided by total population. In the main analysis, we use the deposit rates provided by the World Bank in World Development Indicators as country-specific interest rates, since they have longer data than J. P. Morgan's EMBI + spread. We calculate estimation results using EMBI + spread as robustness check. Interest rates are also deflated by dollar expected inflation rates. The default states are defined by Standard and Poor's, and we add the default events in Argentina in 1951 and 1956–1965, following Reinhart and Rogoff (2014). Data frequency is annual, due to external debt stocks being provided only with annual data.

The analyzed period is 1978–2013 (Argentina), 1977–2013 (Mexico), and 1902–2005 (Argentina and Mexico). The former period corresponds to Aguiar and Gopinath (2007) and Chang and Fernández (2013), and the latter follows García-Cicco et al. (2010), who point out that

1980–2005 only contains between one and a half and two cycles, thus possibly causing trend shocks to be more important than in reality. For 1902–2005, we use the data provided by García-Cicco et al. (2010).

3.3. State space representation

The basic framework of the estimation strategy is almost identical to that of Gust et al. (2017), who estimate a full-nonlinear New-Keynesian model with zero lower bound. The state transition equation, s_t , is the function Φ , depending on its past realization s_{t-1} and current innovations to shocks ϵ_t , given a set of parameters ϑ :

$$s_t = \Phi(s_{t-1}, \epsilon_t; \vartheta), \quad \epsilon_t \sim F_\epsilon(\cdot; \vartheta), \quad (24)$$

where

$$s_t = [b_{t+1}, q_t, D_t, z_t, g_t, b_t, D_{t-1}, z_{t-1}, g_{t-1}],$$

$$\epsilon_t = [\epsilon_t^z, \epsilon_t^g], \text{ and}$$

$$\vartheta = [\beta, \gamma, r, \rho_z, \sigma_z, \theta, \lambda_\alpha, \lambda_\beta, \mu_g, \rho_g, \sigma_g].$$

After solving for the state transition equation, we map the variables in the model to the observables.

The compact form of the measurement equation is:

$$y_t = \Psi(s_t, \epsilon_t; \vartheta) + u_t, \quad u_t \sim F_u(\cdot; \vartheta). \quad (25)$$

Bayesian inference amounts to the characterizing properties of the posterior distribution $p(\vartheta | Y_{1:T})$ proportional to the product of prior density $p(\vartheta)$ and the likelihood function $p(Y_{1:T} | \vartheta)$.

That is,

$$p(\vartheta|\mathbb{Y}_{1:T}) \propto p(\vartheta)p(\mathbb{Y}_{1:T}|\vartheta).$$

Since it is not possible to evaluate these integrals analytically using a Kalman filter, we use a bootstrap particle filter. This latter filter is a special case of sequential importance sampling with resampling, and the details of the algorithm are available in the Appendix.

The measurement equations include a one-period lag of the technology growth rate, while those of Smets and Wouters (2007) and Chang and Fernández (2013) append current technological growth rate. The reason for this is that we detrend the variables by $\mu_g \Gamma_{t-1}$, whereas they detrend the model using the present level of technology corresponding to Γ_t in this paper. The variables with trend X_t are decomposed as follows:

$$\begin{aligned} d \ln X_t &= (\ln X_t - \ln \mu_g - \ln \Gamma_{t-1}) - (\ln X_{t-1} - \ln \mu_g - \ln \Gamma_{t-2}) + \ln \mu_g + \ln \Gamma_{t-1} - \ln \mu_g - \ln \Gamma_{t-2} \\ &= \ln x_t - \ln x_{t-1} + \ln g_{t-1}. \end{aligned} \quad (26)$$

We assume bond prices and default decisions have no trend, as assumed in other DSGE model studies. The measurement equations for 1977–2013 Mexico and 1978–2013 Argentina are:

$$\begin{bmatrix} d \ln Y_t^{obs} \\ d \ln B_t^{obs} \\ q_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \ln y_t - \ln y_{t-1} \\ \ln b_t - \ln b_{t-1} \\ q_t \\ def_t \end{bmatrix} + \begin{bmatrix} \ln g_{t-1} \\ \ln g_{t-1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{b,t} \\ u_{q,t} \\ u_{def,t} \end{bmatrix}. \quad (27)$$

The measurement equation for 1902–2005 (Argentina and Mexico) are:

$$\begin{bmatrix} d\ln Y_t^{obs} \\ (tb/Y)_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \ln y_t - \ln y_{t-1} \\ (tb/y)_t \\ def_t \end{bmatrix} + \begin{bmatrix} \ln g_{t-1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{tb/y,t} \\ u_{def,t} \end{bmatrix}. \quad (28)$$

Since data on external debt and interest rates are unavailable for 1902–2005, we use the trade balance to output ratio and omit bond prices from the measurement equation.

Our measurement equations also include measurement errors. One reason for including them is to circumvent stochastic singularity, which arises when there are more observables than shocks in the model. The shocks in the model are two productivity shocks, but we consider three observables. García-Cicco et al. (2010), Schmitt-Grohé and Uribe (2012), and Chang and Fernández (2013) also adopt measurement errors for the same reason. The measurement errors are restricted to a maximum of 20% of the empirical standard deviation to avoid them absorbing variability, as discussed by An and Schorfheide (2007) and García-Cicco et al. (2010). The other reasons are to avoid degeneracy and absorb model misspecification, as Herbst and Schorfheide (2015) and Gust et al. (2017) describe.

We add a default decision to the measurement equations, since the definition of sovereign default is clear, and the agents in the economy can easily observe the credit state of the sovereign. Similarly, some studies applying a state space model add default decisions or obvious crises to their measurement equations, such as Schwaab et al. (2016) and Rose and Spiegel (2010, 2011, 2012).

3.4. Simulated tempering SMC–SMC algorithm

Apart from evaluating likelihood, we also employ a particle filter algorithm to elicit draws from the posterior. The algorithm is the simulated tempering SMC, proposed by Herbst and Schorfheide (2014, 2015), and is an estimation strategy that uses particle filter (SMC) to find a good proposal density, hence being labeled SMC–SMC. To the best of our knowledge, this paper is the first to apply such an estimation method to a full nonlinear regime-switching DSGE model.

The most important advantage of the SMC algorithm is it explores entire prior ranges, propagating particles of parameter vectors to thousands of multiple chains. It further prevents parameter draws from being caught in a local mode. The use of multiple chains has other desirable advantages, particularly when coping with complex posterior distributions involving long tails and multi-modality (Gilks et al., 1994; Liu et al., 2000; Ter Braak, 2006, Ter Braak and Vrugt, 2008; Radu et al., 2009; Vrugt et al., 2009).

All priors are uniform in securing objectiveness, such as in Fernández-Villaverde and Rubio-Ramírez (2005) and García-Cicco et al. (2010) (Table 1). The ranges cover the calibration values of the numerous studies on structural sovereign default models.⁸

The scale parameter is adjusted by approximately 25–40%, along with the tempering schedule. The number of particles for likelihood evaluation is 20,000 to obtain robust results efficiently, according to Amisano and Tristani (2010) and Malik and Pitt (2011). The total number of MH-steps is 600,000, which is sufficiently large.⁹

4. Results

4.1. Parameter estimates

Tables 2 and 3 present the means and intervals bracketed by 5% and 95% of the posterior distributions based on the estimations for post-1970s and 1902–2005, respectively. First, the persistence of stationary shocks is larger than that of trend shocks in all cases. Second, the volatility of trend shocks is larger than that of stationary shocks in Mexico. Finally, the asymmetric domestic

⁸ See, for example, Arellano (2008), Cuadra and Sapriza (2008), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Yue (2010), Boz (2011), Mendoza and Yue (2012), and Durdu et al. (2013).

⁹ García-Cicco et al. (2010) use two million iterations in their MCMC estimation, but their model has substantially more parameters than this paper. Log-linearized DSGE model estimation studies that use a basic Kalman filter for likelihood evaluation often conduct 500,000 MCMC iterations. DSGE studies that use a particle filter have fewer iterations than the Kalman filter case.

cost of Mexico is higher than that of Argentina. The economics implications of these results are described in subsection 4.2.

Another feature of the estimates is the probability of re-entry, as the proportional and asymmetric domestic cost are relatively low among preceding studies.¹⁰ The reason for the low probability of regaining access to capital markets is that both default frequency and period are measured as to be matched to the observations in the Bayesian estimation, whereas previous studies matched the moments of the frequency. As the resulting low probability of re-entry inhibits default, the domestic cost decreases to compensate for default frequency. The low domestic cost is consistent with empirical studies, such as Chuan and Sturzenegger's (2005), and with the 0.6–2.5% values by Borensztein and Panizza (2009).

The estimation intervals in this paper are narrow compared to other DSGE studies, such as Smets and Wouters (2007), García-Cicco et al. (2010), and Chang and Fernández (2013). The main reason for the tighter intervals is the measurement equations include default state. The candidates of the parameter vectors, which do not predict default, are assigned low likelihood. The power of predicting the default state is thus sensitive to the values of deep parameters, causing the intervals become to be tighter. However, the models do a poor job in forecasting default state if excluding default state from the measurement equations, and some parameter estimates range over almost entire space of priors.

4.2. Trend shocks versus stationary productivity shocks

The persistence of trend shocks is larger than in preceding studies, leading to a higher random-walk component (RWC) (Table 4). RWC is a criterion for determining the importance of

¹⁰ The calibrations of default costs also differ among studies. For example, the annual probability of re-entry ranges from 29.3% (Mendoza and Yue, 2012) to 73.4% (Arellano, 2008), and the asymmetric domestic cost is from 2.0% (Aguiar and Gopinath, 2007) to 10.0% (Alfaro and Kanczuk, 2009).

trend shocks, proposed by Aguiar and Gopinath (2007), which is the relative variance of the permanent component of productivity growth to total productivity growth. The equation for RWC is:

$$\frac{\sigma_{\Delta \ln g}^2}{\sigma_{\Delta TFP}^2} = \frac{\sigma_g^2 / (1 - \rho_g)^2}{[2/(1 + \rho_z)]\sigma_z^2 + [\sigma_g^2 / (1 - \rho_g^2)]} \quad (29)$$

A high RWC suggests trend shocks play an important and direct role on output fluctuations. Accordingly, the components of trend shocks in the nonlinear shock propagation are sizeable (Figs. 1 and 2), which is consistent with Aguiar and Gopinath (2007).

However, the financial frictions and the resulting countercyclicality in business cycles are mainly caused by stationary shocks, except for 1977–2013 Mexico. The bond price schedules show stationary shocks achieve higher bond prices (lower interest rates) (Figs. 5A, 9A, and 10A), thereby the sovereign borrowing in response to stationary rather than trend shocks, as shown by the policy functions (Figs. 6A, 9B, and 10B). Consequently, the drivers of sovereign default are stationary shocks (Figs. 1A, 2A, and 2B), and which vary with peacetime and default cycles (Figs. 3A, 4A, and 4B). During the realization of a shock is negative regardless of the type of the shock, the bond price immediately decreases (Figs. 5B and 7B), and a sovereign thus hesitates to issue bonds (Figs. 6B and 8B).

The mechanism behind why stationary shocks generate more financial frictions than trend shocks is relative persistence, that is, ρ_z is higher than ρ_g (Tables 2, 3, and 6). A highly persistent positive shock implies that income is higher today, but even higher over the subsequent period, thus strengthening the ability to access international financial markets to bring forward expected income and leading to the greater indebtedness, given the low funding rate. The more persistent a shock becomes, the more intense the effect is, as shown in Fig. 11.

However, only during 1977–2013, trend shocks in Mexico appear to account for financial

frictions more than transitory shocks, although the persistence of stationary shocks is higher than that of trend shocks (Figs. 8, 9, and Table 2). Those making an exception are the volatility effect and asymmetric cost of default. As shock volatility increases, the expected value of utility conditional on a positive realization of output increases, leading to higher debt accumulation and lower credit spreads, as shown in Figs. 12A and B. Indeed, utility conditional on a positive shock increases (Fig. 12C), because the sovereign has the option to default, which generates the lower bound of utility, meaning higher volatility expands the capacity of external debt, in turn overcoming the effect of households' risk aversion. Meanwhile, the utility on a negative shock decreases as volatility increases (Fig. 12D), since the sovereign would not borrow from foreign investors on a negative shock, implying higher volatility does not mean expanding financial capacity. The volatility of trend shocks is higher than that of stationary shocks in Mexico during both 1977–2013 and 1902–2005 (Tables 2 and 3). The volatility effect overcomes the persistence effect post-1970s Mexico, while the latter overwhelms the former during 1902–2005. For Argentina, σ_z is larger than σ_g , indicating the volatility effect intensifies more the impact of stationary shocks. Note that the volatility effects are adverse if volatility is too high, as Uribe and Schmitt-Grohé (2017) document. Agents suffer large negative shocks, leading to higher default frequencies and more credit risks. Increased uncertainty induces an increase in precautionary savings and, consequently, a fall in the desired level of external debt.

Importantly, asymmetric domestic cost amplifies the volatility effect, increasing the difference between the expected value of utility commensurate with the size of the effect, but not intensifying the persistence effect. Generally, as asymmetric domestic cost increases, the incentive for a sovereign to default declines, leading to lower default probability. The volatility effect responds to the increased asymmetric cost, since larger volatility implies more opportunity to exceed the unconditional mean, as a more substantial cost for defaulting, thus amplifying the difference between the conditional utilities on high volatility and low volatility (Fig. 13B). On the other hand, higher

persistence means a lower recovery chance from downturns (Fig. 13A). The asymmetric domestic cost of Mexico is large relative to that of Argentina, thereby 1977–2013 Mexico becomes the exception for the sources of financial frictions.

4.3. Identification: Mismatch between sovereign default and business cycles

The final important factor is that sovereign default cycles do not always match business cycles. Reinhart and Rogoff (2014) document that Argentina during 1894–1950 did not experience sovereign default, but at least two business cycles according to Figure 1 of García-Cicco et al. (2010). The mismatch means sometimes drivers of countercyclicality should remain above their unconditional means for a long period, even during recessions, since often a sovereign maintains debt repayment during downturns.

As shown in equations (2), (3) and (4), the formulations of stationary and trend shocks are similar, with both shocks having the potential to explain countercyclicality. However, identification between these two types of shocks arises from measurement equations (27) and (28). Trend shocks are responsible not only for output fluctuations but also growth level, associating them more with business cycles. Thereby, stationary shocks rather take charge of sovereign default cycles. As a result, stationary shocks remain far above the unconditional mean, even when trend shocks fall below it (Figs. 3 and 4).

4.4. Stylized facts of emerging economies

In this subsection, we verify the second moments of the models for examining the empirical regularities of business cycles in the emerging market, as implied by Aguiar and Gopinath (2006) and García-Cicco et al. (2010). Aguiar and Gopinath (2006) show that one of the advantages of introducing a trend shock is the simulated path replicates the positive correlation between interest rates and current accounts (negative correlation between bond prices and current accounts). García-Cicco et al. (2010) point out that one of the problems of a frictionless RBC is that the model

tends to generate nearly random-walk trade balances.

The moments are summarized in Table 5, demonstrating that the proposed models replicate excessive volatility in consumption, the countercyclical current account, and countercyclical interest rates (pro-cyclical bond prices). Additionally, the correlation between bond prices and the current account is negative, and the autocorrelations of trade balance–output ratios do not exhibit random-walk behaviors. The exception is the positive correlation between bond prices and current accounts in 1978–2013 Argentina. This is because, during 1978–1981, bond prices decreased sharply, while the external debt position also escalated drastically. The other periods, 1994–2000 and 2005–2013, exhibit negative correlation. Moreover, the correlation is negative in 1902–2013 Argentina, implying that the analysis of long-run data is important, as García-Cicco et al. (2010) document.

5. Robustness

5.1. Other interest rate data: EMBI + spread and U.S. interest rate

One robustness check is the estimation using the EMBI + spread and the U.S. interest rate. In main analysis, we use the deposit rate as the country-specific interest rate since longer period data are available, while many studies on emerging economies create interest rate series with EMBI + spread and the U.S. interest rate. For Mexico, the EMBI + spread during default period 1982–1990 is not available, thus we estimate only the Argentine economy. We construct the country-specific interest rate as the sum of the EMBI + spread for Argentina and the 90-day treasury-bill rate deflated by expected dollar inflation.

The results show that parameter estimates are similar to the main analysis (Table 6), and the bond price schedule and the policy function imply stationary shocks account for financial frictions more than trend shocks (Fig. 14).

5.2. Another detrending method: HP-filter

Another robustness check is the estimation on HP-filtered series. Numerous studies on sovereign default models employ an HP filter, such as Aguiar and Gopinath (2006) and Mendoza and Yue (2012), whereas we use log-differenced series assuming balanced growth. This analysis examines whether detrending method affects parameter estimates severely.

The measurement equations are different from the main analysis, since a trend is calculated using an HP-filter rather than being derived in the model. The equations are constructed similarly to those of some studies on DSGE model estimations with particle filtering using HP-filtered series, such as Fernández-Villaverde and Rubio-Ramírez (2005, 2007) and Malik and Pitt (2011). The measurement equations of the HP-filter approach are as follows (the tilde denotes deviations from trend):

$$\begin{bmatrix} \tilde{y}_t^{obs} \\ \tilde{b}_t^{obs} \\ q_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \tilde{y}_t \\ \tilde{b}_t \\ q_t \\ def_t \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{B,t} \\ u_{q,t} \\ u_{def,t} \end{bmatrix}. \quad (30)$$

The results are shown in Table 7. The probability of re-entry and domestic costs are relatively small comparing to preceding studies. Thus, we confirm that the tendency of parameter estimates is similar to the main result (Table 7). The comparison between trend and stationary shocks in bond price schedules and policy functions is not available, since there is no stochastic trend term in the HP-filter based estimation.

6. Conclusions

The major characteristics of the business cycles of emerging economies are excessive volatility in consumption, the countercyclical current account balance, countercyclical interest rates, and

frequent default at equilibrium. There are two literature strands that explain these stylized facts. The first claims that the most important sources for these characteristics are permanent productivity shocks. The second emphasizes financial frictions over nonstationary productivity shocks. However, a stochastic trend is a driver of not only business cycles, but also financial frictions. Determining whether trend or stationary shocks are the main source of financial frictions and the resulting countercyclicality of business cycles is thus essential.

We estimate sovereign default models, and investigate the properties of models and data. The main finding is that stationary shocks rather than trend shocks account for financial frictions and the resulting countercyclicality, except for post-1977 Mexico. The main determinants of these results are the persistence and volatility of shocks, asymmetric domestic cost of sovereign default, and mismatch between sovereign default and business cycles.

The results bridge the gap between the two literature strands above. First, in line with García-Cicco et al. (2010) and Chang and Fernández (2013), this paper reveals stationary shocks are the more important source of financial frictions and the resulting countercyclicality of business cycles over trend shocks. This arises from the high persistence of stationary shocks. Second, on the other hand, consistent with Aguiar and Gopinath (2007), volatility of trend shocks that is high compared to stationary shocks plays an important role in financial frictions in post-1977 Mexico, as the only exception. Third, the former effect overwhelms the latter for one-century data on Mexico, in accordance with the observations of García-Cicco et al. (2010). Additionally, the relatively large asymmetric domestic cost of default for Mexico induces the exception to occur, amplifying the volatility effect. All these findings are based on the differences of estimated parameters, which stem from the mismatch between sovereign default and business cycles.

A natural extension of this paper would be to add other important shocks in emerging economies, such as interest rate or terms of trade shocks. Uribe and Yue (2006) show that one of the major drivers of interest rate fluctuations in emerging economies is the monetary policy of the

United States. Schmitt-Grohé and Uribe (2016) and Na et al. (2014) introduce downward nominal wage rigidity into their models, and replicate defaults with large currency devaluations. The estimation framework is applicable to these important issues as well.

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Appendix

1. Solution

1.1. Value function iteration

To find the equilibrium allocations, we implement the value function iteration. Based on the implications of Hatchondo et al. (2010) for time efficiency, we adopt the one-loop algorithm that iterates simultaneously the value and bond price function:

- a. Discretize the state space evenly in b , but approximate z_t and g_t following Rouwenhorst's (1995) method.
- b. Assume the initial value of a triplet of value function $\{V^c(b_t, z_t, g_t), V^d(z_t, g_t), V^o(b_t, z_t, g_t)\}$ and bond price function $q(b_{t+1}, z_t, g_t)$.
- c. Solve the sovereign's problem and obtain policy functions $D(b_t, z_t, g_t)$, $b(b_t, z_t, g_t)$ and value functions $\{V^c(b_t, z_t, g_t), V^d(z_t, g_t), V^o(b_t, z_t, g_t)\}$.
- d. Given these policy functions, calculate a default probability $\delta(b_{t+1}, z_t, g_t)$ and the resulting bond price functions $q(b_{t+1}, z_t, g_t)$.
- e. Check for convergence whether $\max|V_{new}^o - V_{old}^o| < 1e^{-6}$. If it converges, stop. If not, go to step c.

The above process linearly interpolates $\{V^c(b_t, z_t, g_t), V^d(z_t, g_t), V^o(b_t, z_t, g_t)\}$, $q(b_{t+1}, z_t, g_t)$, and $b(b_t, z_t, g_t)$. Although Hatchondo et al. (2010) report the performances of Chebyshev collocations and cubic splines, Yamazaki (forthcoming) shows insignificant differences in the accuracies among linear, quadratic, and cubic splines. Similarly, Richter et al. (2014) also report that linear interpolation provides more accurate solutions to the full-nonlinear New-Keynesian

model than Chebyshev collocations. Judd and Solnick (1994) indicate cubic spline is superior to linear interpolations in terms of accuracy, but from the perspective of the convergence, linear interpolation performs better. The reason why linear interpolation sometimes performs better than high-order approximations is its shape-preserving property to avoid internodal oscillations among grid points (Judd and Solnick, 1994; Wang and Judd, 2000; Stachurski, 2008; Cai and Judd, 2014).

Regarding the approximation methods for the AR (1) process, Kopecky and Suen (2010) show that the Rouwenhorst method performs better than Tauchen's (1986), quadrature-based method, and Adda-Cooper method. Yamazaki (forthcoming) reports that the implications of Hatchondo et al. (2010) do not change if choosing between Tauchen (1986) and Rouwenhorst (1995), although he does not refer to their relative performance.

2. Estimation

2.1. Particle filter

Sovereign default models are full-nonlinear, meaning a particle filter is needed to evaluate likelihood, given a set of parameters. The implemented algorithm is identical to the bootstrap particle filter of Herbst and Schorfheide (2015) and Gust et al. (2017), except that Gust et al. (2017) use the adapted proposal distribution at a particular time-point to mitigate the degeneracy problem. The adaptation is attractive for statistical accuracy, but we choose to use the straight-forward particle filter to preserve the framework of the sovereign default model. We set $N_{filter} = 20,000$ to obtain robust results efficiently according to Amisano and Tristani (2010) and Malik and Pitt (2011).

Algorithm 1 (Bootstrap particle filter)

1. Initialization

Draw the initial particles from $s_0^j \sim p(s_0|\vartheta)$ and set $W_0^j = 1$.

2. Recursion

For $t = 1, \dots, T$

(a) Forecasting s_t

Propagate particles $\{s_{t-1}^j, W_{t-1}^j\}$ by simulating the state-transition equation:
 $\tilde{s}_t^j = \Phi(s_{t-1}^j, \epsilon_t^j; \vartheta)$, $\epsilon_t^j \sim F_\epsilon(\cdot; \vartheta)$.

(b) Forecasting \mathbb{Y}_t

Calculate the incremental weights:

$$\tilde{w}_t^j = p(\mathbb{Y}_t | \tilde{s}_t^j, \vartheta).$$

Approximate predictive density $p(\mathbb{Y}_t | \mathbb{Y}_{1:t-1}, \vartheta)$ as follows:

$$p(\mathbb{Y}_t | \mathbb{Y}_{1:t-1}, \vartheta) = \frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \tilde{w}_t^j W_{t-1}^j.$$

(c) Updating

Calculate the normalized weights

$$\tilde{W}_t^j = \frac{\tilde{w}_t^j W_{t-1}^j}{\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \tilde{w}_t^j W_{t-1}^j}.$$

(d) Selection

If $\hat{\rho}_t = 1$, where $\hat{\rho}_t = \mathcal{L}\{ESS_t < 0.5N\}$, where $ESS_t = N_{filter} / \left(\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} (W_t^j)^2 \right)$,

Resample the particles by multinomial resampling, $\{s_t^j\}_{j=1}^{N_{filter}} = \{\tilde{s}_t^j, \tilde{W}_t^j\}_{j=1}^{N_{filter}}$. Let

$$W_t^j = 1.$$

If $\hat{\rho}_t = 0$,

$$\text{Let } s_t^j = \tilde{s}_t^j \text{ and } W_t^j = \tilde{W}_t^j.$$

3. Likelihood approximation

The approximation of the likelihood function is given by:

$$\hat{p}(\mathbb{Y}_{1:T} | \vartheta) = \prod_{t=1}^T \left(\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \tilde{w}_t^j W_{t-1}^j \right).$$

2.2. Simulated tempering SMC

The algorithm in this paper follows the simulated tempering SMC proposed by Herbst and Schorfheide (2014, 2015). SMC algorithms were initially developed to numerically evaluate the likelihood of nonlinear state space models as discussed in subsection 2.1 of this Appendix. Additionally, SMC algorithms can be adapted to conducting posterior inference in DSGE models. The simulated tempering SMC consists of three steps: correction, selection, and mutation.

The hyperparameters of this study are the number of particles for parameter vectors $N = 2000$, number of stages $N_\phi = 300$, parameter for the tempering schedule $\lambda = 2.1$, number of blocks $N_{blocks} = 6$, number of MH steps at each stage $M = 1$, parameter controls for the weight of the proposals' mixture components $\alpha = 0.9$, and number of particles for the likelihood evaluations

$N_{filter} = 20,000$. The tempering schedule $\{\phi_n\}_{n=1}^{N_\phi}$ is determined by $\phi_n = \left(\frac{n-1}{N_\phi-1}\right)^\lambda$. As the number of stages increases, each stage requires additional likelihood evaluations. The scale parameter is adjusted by approximately 25–40%, along with the tempering schedule. The total number of likelihood estimations in the SMC algorithm is equal to $N \times N_\phi \times M = 600,000$, which is sufficiently large.

Algorithm 2 (Simulated tempering SMC)

1. Initialization

Draw $\vartheta_1^i, \dots, \vartheta_1^N$ from prior $\pi(\vartheta_1)$ and set $w_1^i = N^{-1}$, $i = 1, \dots, N$.

2. Recursion

For $n = 2, \dots, N_\phi$

(a) Correction

Reweight the particles from stage $n - 1$ by defining the incremental and normalized weights by calculating incremental and normalized weights $\tilde{w}_n^i = \left(p(\mathbb{Y}|\vartheta_{n-1}^i)\right)^{\phi_n - \phi_{n-1}}$ and $\tilde{W}_n^i =$

$$\frac{\tilde{w}_n^i W_{n-1}^i}{\frac{1}{N} \sum_{i=1}^N \tilde{w}_n^i W_{n-1}^i}.$$

$E_{\pi_n}[h(\vartheta)]$ is approximated to $\tilde{h}_{n,N} = \frac{1}{N} \sum_{i=1}^N h(\vartheta_{n-1}^i) W_n^i$.

(b) Selection

Compute the effective sample size $ESS_n = \frac{N}{\left(\frac{1}{N} \sum_{i=1}^N (\tilde{W}_n^i)^2\right)}$.

If $\hat{\rho}_n = 1$, where $\hat{\rho}_n = \mathcal{L}\{ESS_n < 0.5N\}$,

Resample the particles by multinomial resampling. Draw $\{\hat{\vartheta}\}_{i=1}^N$ from a multinomial distribution characterized by support points and weights $\{\vartheta_{n-1}^i, \tilde{W}_n^i\}_{i=1}^N$. Set $W_n^i = 1$.

If $\hat{\rho}_n = 0$,

Let $\hat{\vartheta}_n^i = \vartheta_{n-1}^i$ and $W_n^i = \tilde{W}_n^i$. $E_{\pi_n}[h(\vartheta)]$ is approximated to $\hat{h}_{n,N} = \frac{1}{N} \sum_{i=1}^N h(\hat{\vartheta}_n^i) W_n^i$.

(c) Mutation

Propagate particles $\{\hat{\vartheta}_i, W_n^i\}$ using the Metropolis–Hastings algorithm with transition density $\vartheta_n^i \sim K_n(\vartheta_n|\hat{\vartheta}_n^i; \xi_n)$ and stationary $\pi_n(\vartheta)$. $E_{\pi_n}[h(\vartheta)]$ is approximated to $\bar{h}_{n,N} = \frac{1}{N} \sum_{i=1}^N h(\vartheta_n^i) W_n^i$.

3. Final importance of sampling approximation of $E_\pi[h(\vartheta)]$

When $n = N_\phi$, $\bar{h}_{N_\phi,N} = \sum_{i=1}^N h(\vartheta_{N_\phi}^i) W_{N_\phi}^i$.

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Table 1: Priors.

Parameters		Distribution	Range
β	Discount factor	Uniform	(0.0, 1.0)
γ	Risk aversion	Uniform	(0.0, 10.0)
r	Risk-free interest rate	Uniform	(0.0, 1.0)
ρ_z	Persistence of stationary productivity shocks	Uniform	(0.0, 1.0)
σ_z	SD of stationary productivity shocks	Uniform	(0.0, 10.0)
θ	Probability of re-entry	Uniform	(0.0, 1.0)
λ_α	Asymmetric domestic cost	Uniform	(0.0, 1.0)
λ_β	Proportional domestic cost	Uniform	(0.0, 1.0)
μ_g	Gross mean growth	Uniform	(1.0, 1.3)
ρ_g	Persistence of nonstationary productivity shocks	Uniform	(0.0, 1.0)
σ_g	SD of nonstationary productivity shocks	Uniform	(0.0, 10.0)

Table 2: Posteriors post-1970s.

Argentina		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.886	0.830	0.921
γ	Risk aversion	Uniform	1.675	1.499	1.900
r	Risk-free interest rate	Uniform	3.91%	3.53%	4.11%
ρ_z	Persistence of stationary shocks	Uniform	0.839	0.771	0.885
σ_z	SD of stationary shocks	Uniform	0.027	0.024	0.030
θ	Probability of re-entry	Uniform	21.40%	18.9%	22.9%
λ_α	Asymmetric domestic cost	Uniform	0.93%	0.85%	1.00%
λ_β	Proportional domestic cost	Uniform	0.046%	0.041%	0.053%
μ_g	Gross mean growth	Uniform	1.006	1.000	1.025
ρ_g	Persistence of trend shocks	Uniform	0.833	0.743	0.888
σ_g	SD of trend shocks	Uniform	0.023	0.021	0.024
Acceptance ratio			26.20%		
Marginal data density			-280.09		

Mexico		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.802	0.771	0.835
γ	Risk aversion	Uniform	3.137	2.881	3.485
r	Risk-free interest rate	Uniform	3.46%	2.85%	3.89%
ρ_z	Persistence of stationary shocks	Uniform	0.871	0.838	0.914
σ_z	SD of transitory shocks	Uniform	0.018	0.016	0.020
θ	Probability of re-entry	Uniform	11.59%	10.32%	12.48%
λ_α	Asymmetric domestic cost	Uniform	3.88%	3.49%	4.36%
λ_β	Proportional domestic cost	Uniform	2.60%	2.42%	2.76%
μ_g	Gross mean growth	Uniform	1.002	1.000	1.015
ρ_g	Persistence of trend shocks	Uniform	0.701	0.645	0.769
σ_g	SD of trend shocks	Uniform	0.022	0.020	0.024
Acceptance ratio			24.95%		
Marginal data density			-302.42		

Notes: This table shows the means and intervals bracketed by 5% and 95% of the posterior distributions. The intervals are tight, since the measurement equations include default state, assigning less likelihood on the particles of parameter vectors that could not predict default events and recoveries.

Table 3: Posteriors for 1902–2005.

Argentina		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.946	0.943	0.948
γ	Risk aversion	Uniform	2.610	2.601	2.617
r	Risk-free interest rate	Uniform	3.88%	3.87%	3.90%
ρ_z	Persistence of stationary shocks	Uniform	0.938	0.935	0.941
σ_z	SD of stationary shocks	Uniform	0.0372	0.0370	0.0373
θ	Probability of re-entry	Uniform	9.35%	9.32%	9.38%
λ_α	Asymmetric domestic cost	Uniform	1.946%	1.939%	1.952%
λ_β	Proportional domestic cost	Uniform	0.499%	0.497%	0.500%
μ_g	Gross mean growth	Uniform	1.003	1.000	1.006
ρ_g	Persistence of trend shocks	Uniform	0.890	0.887	0.893
σ_g	SD of trend shocks	Uniform	0.02835	0.02826	0.02844
Acceptance ratio			16.50%		
Marginal data density			-152.06		

Mexico		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.748	0.741	0.755
γ	Risk aversion	Uniform	3.285	3.253	3.307
r	Risk-free interest rate	Uniform	3.94%	3.90%	3.97%
ρ_z	Persistence of stationary shocks	Uniform	0.890	0.882	0.897
σ_z	SD of stationary shocks	Uniform	0.0235	0.0232	0.0237
θ	Probability of re-entry	Uniform	10.71%	10.61%	10.79%
λ_α	Asymmetric domestic cost	Uniform	5.40%	5.34%	5.44%
λ_β	Proportional domestic cost	Uniform	0.987%	0.976%	0.994%
μ_g	Gross mean growth	Uniform	1.061	1.051	1.069
ρ_g	Persistence of trend shocks	Uniform	0.883	0.875	0.891
σ_g	SD of trend shocks	Uniform	0.0329	0.0325	0.0332
Acceptance ratio			18.60%		
Marginal data density			-911.96		

Notes: This table shows the means and intervals bracketed by 5% and 95% of the posterior distributions. The intervals are tight, since the measurement equations include default state, assigning less likelihood on the particles of parameter vectors that could not predict default events and recoveries.

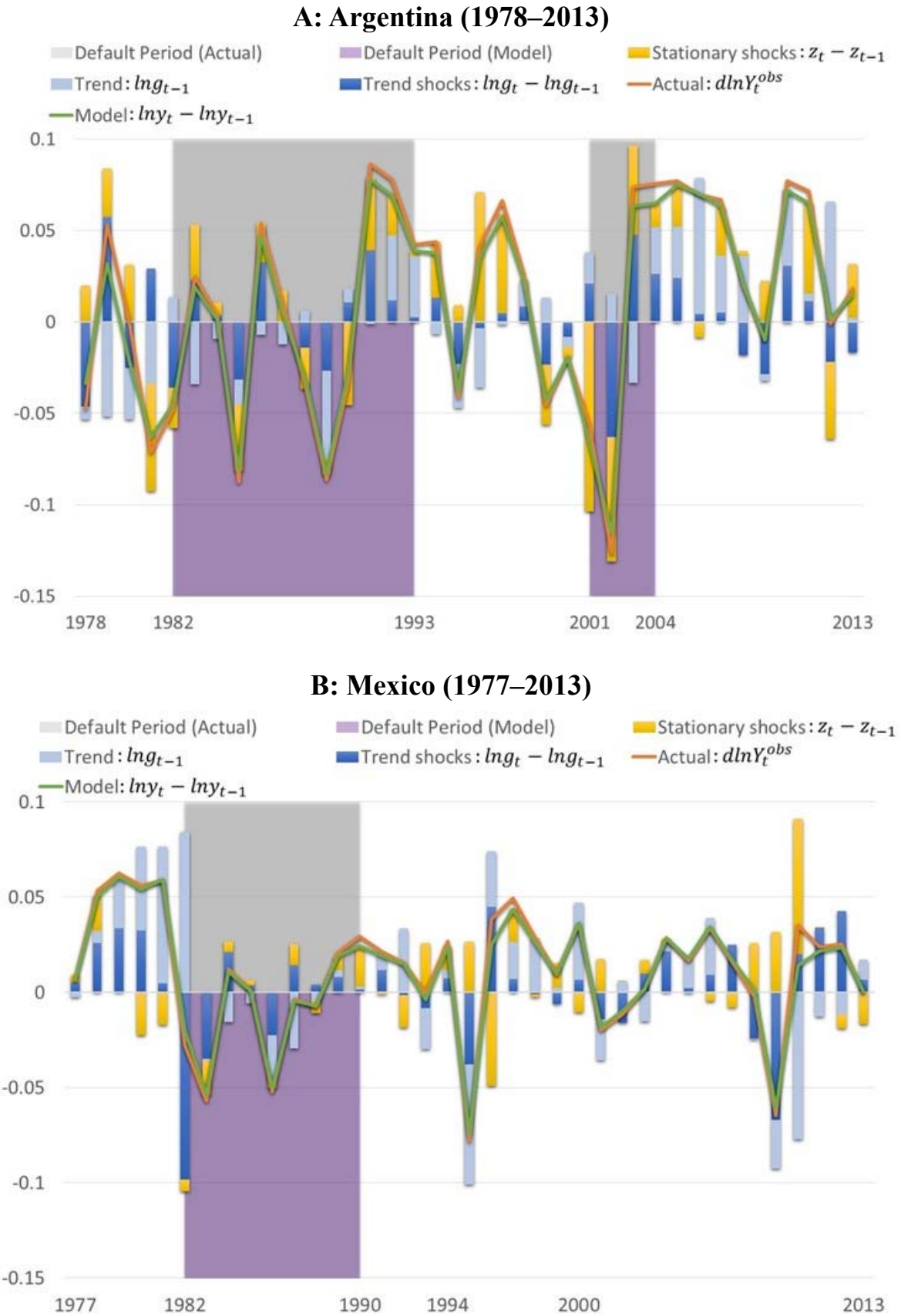
Table 4: Random-walk component.

Argentina	Proposed model 1978–2013		Proposed model 1902–2005			
	$\alpha = 0.0$	$\alpha = 0.32$	$\alpha = 0.0$	$\alpha = 0.32$		
Capital income share						
Random-walk component	0.69	0.50	0.73	0.56		

Mexico	Proposed model 1977–2013		Proposed model 1902–2005		Aguiar and Gopinath (2007)	Chang and Fernández (2013)
	$\alpha = 0.0$	$\alpha = 0.32$	$\alpha = 0.0$	$\alpha = 0.32$	$\alpha = 0.32$	$\alpha = 0.3132$
Capital income share						
Random-walk component	0.73	0.56	0.89	0.80	0.88	0.18

Notes: There is no parameter of capital income share α in the proposed models and most other structural sovereign default models assuming endowment economies. The larger α becomes, the smaller the random-walk component. Therefore, we also provide the results calibrated at $\alpha = 0.00$ and $\alpha = 0.32$, the latter being the similar value used by Aguiar and Gopinath (2007), García-Cicco et al. (2010), and Chang and Fernández (2013). The RWCs of Aguiar and Gopinath (2007) and Chang and Fernández (2013) are the result of the encompassing model. García-Cicco et al. (2010) do not provide RWCs directly, but based on their posterior, their RWCs are close to Chang and Fernández (2013).

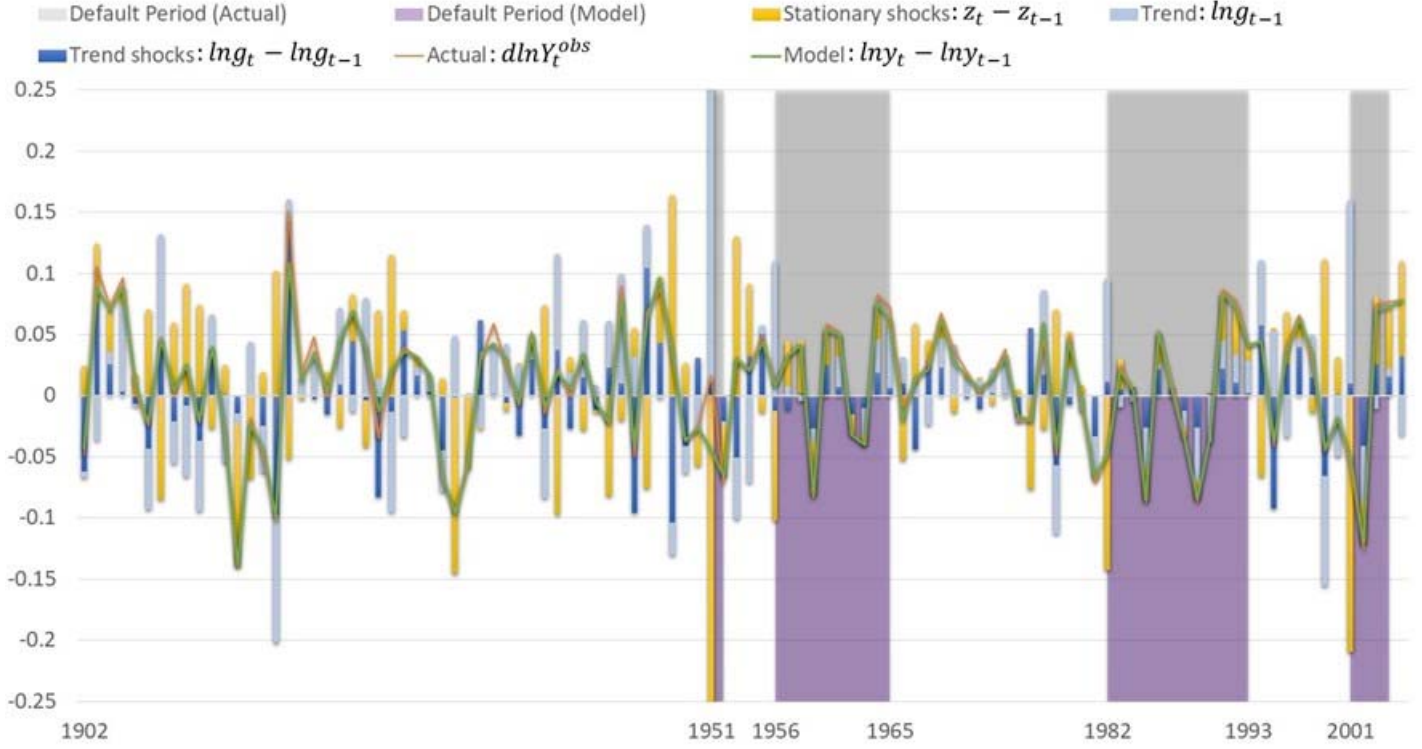
Fig. 1. The nonlinear propagations of shocks in output post 1970s.



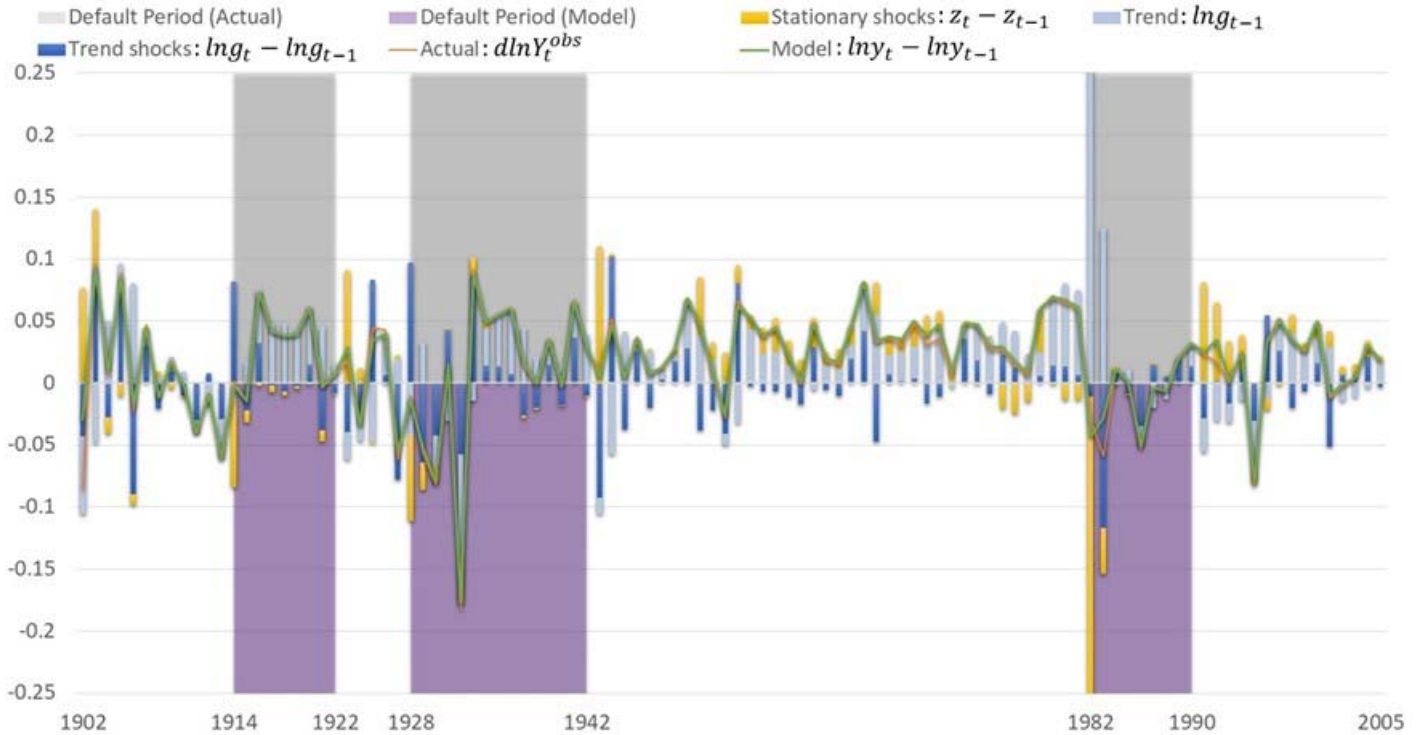
Notes: Observables are log-differenced. Output fluctuations can be fully decomposed using equation (16), but external debts and bond prices cannot be decomposed into their respective shocks because of the lack of analytical forms. Trend shocks play an important role in business cycles in both figures, reflecting high RWCs. Stationary shocks drive the defaults in Argentina, but not in Mexico, where trend shocks drive default.

Fig. 2. The nonlinear propagations of shocks in output during 1902–2005.

A: Argentina (1902–2005)

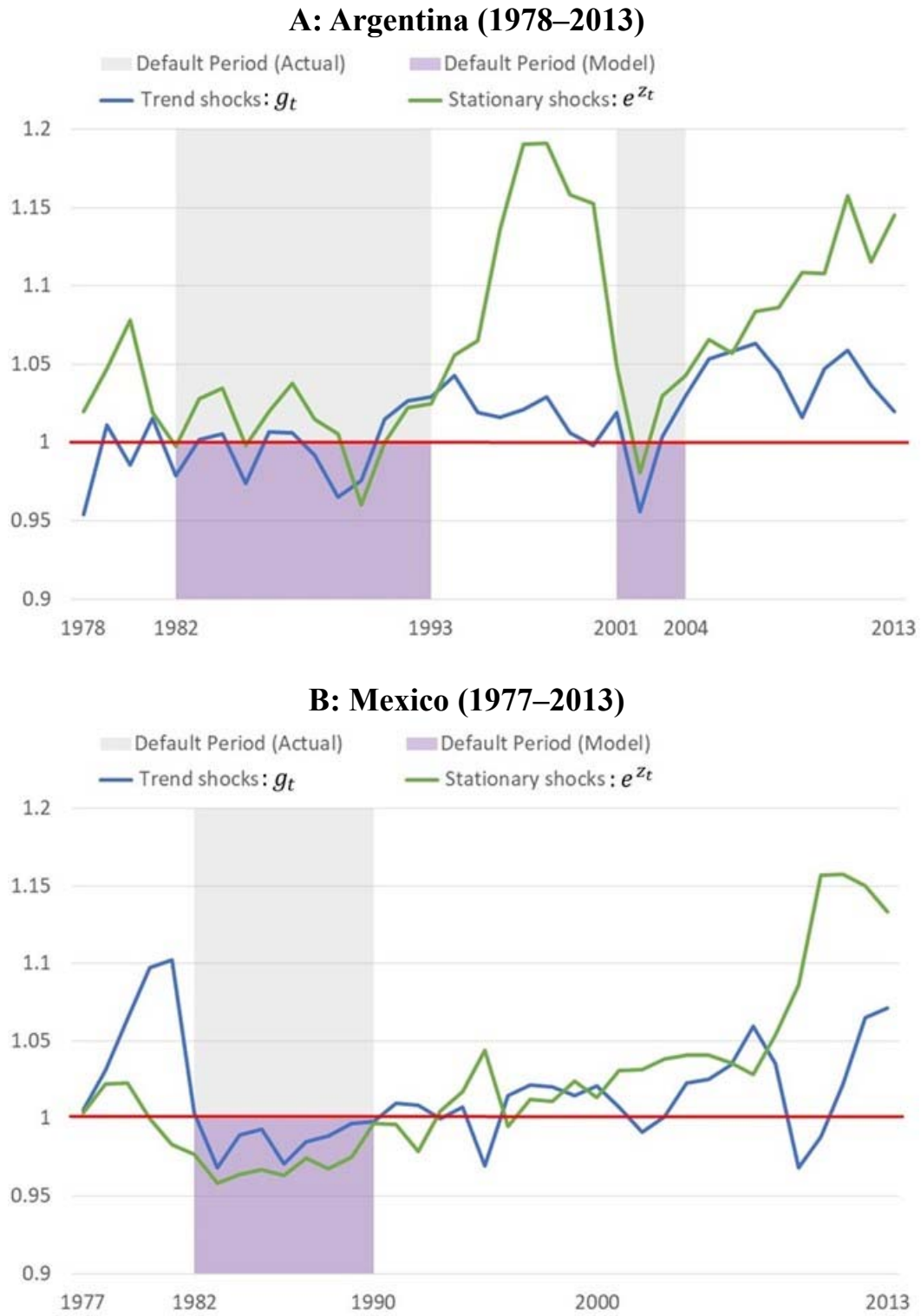


B: Mexico (1977–2013)



Notes: Observables are log-differenced. Output fluctuations can be fully decomposed using equation (16), but external debts and bond prices cannot be decomposed into their respective shocks because of the lack of analytical forms. Trend shocks play an important role in business cycles in both figures, reflecting high RWCs. Stationary shocks drive the defaults both in Argentina and Mexico. Particularly in Mexico, trend shocks are not default event drivers, unlike the 1977–2013 data.

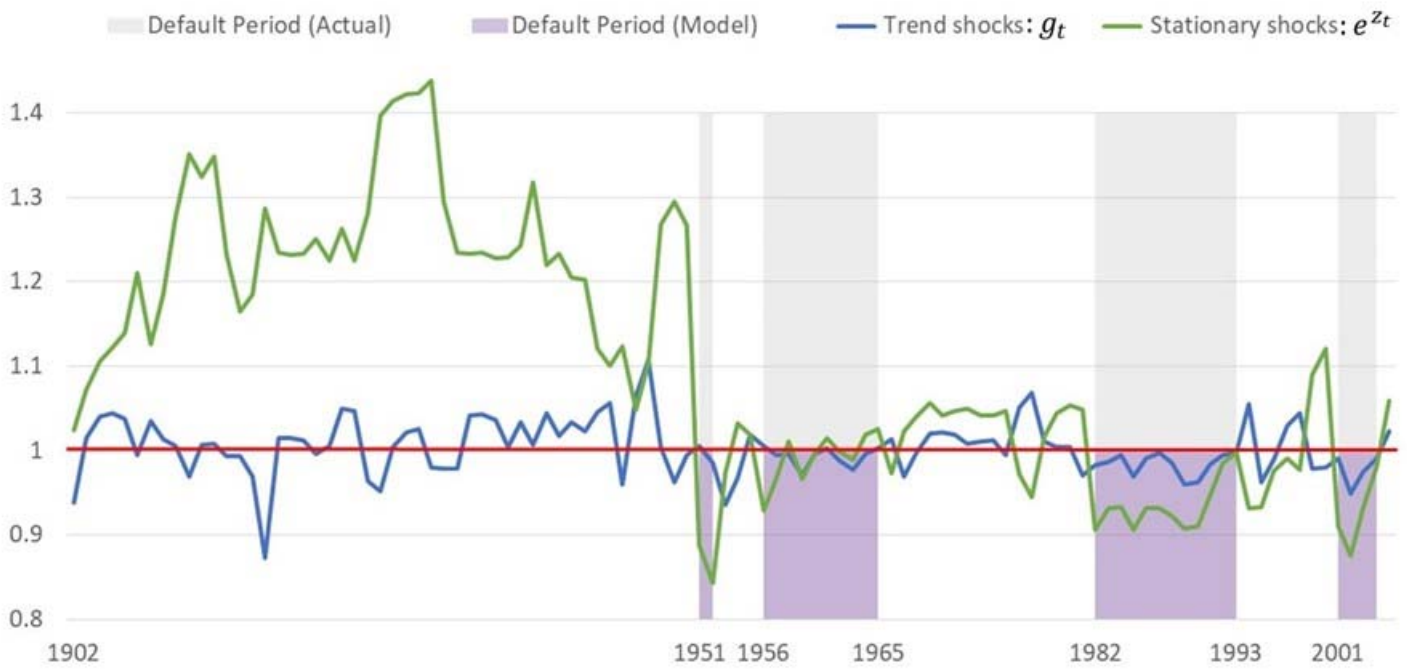
Fig. 3. Path of estimated shocks post 1970s.



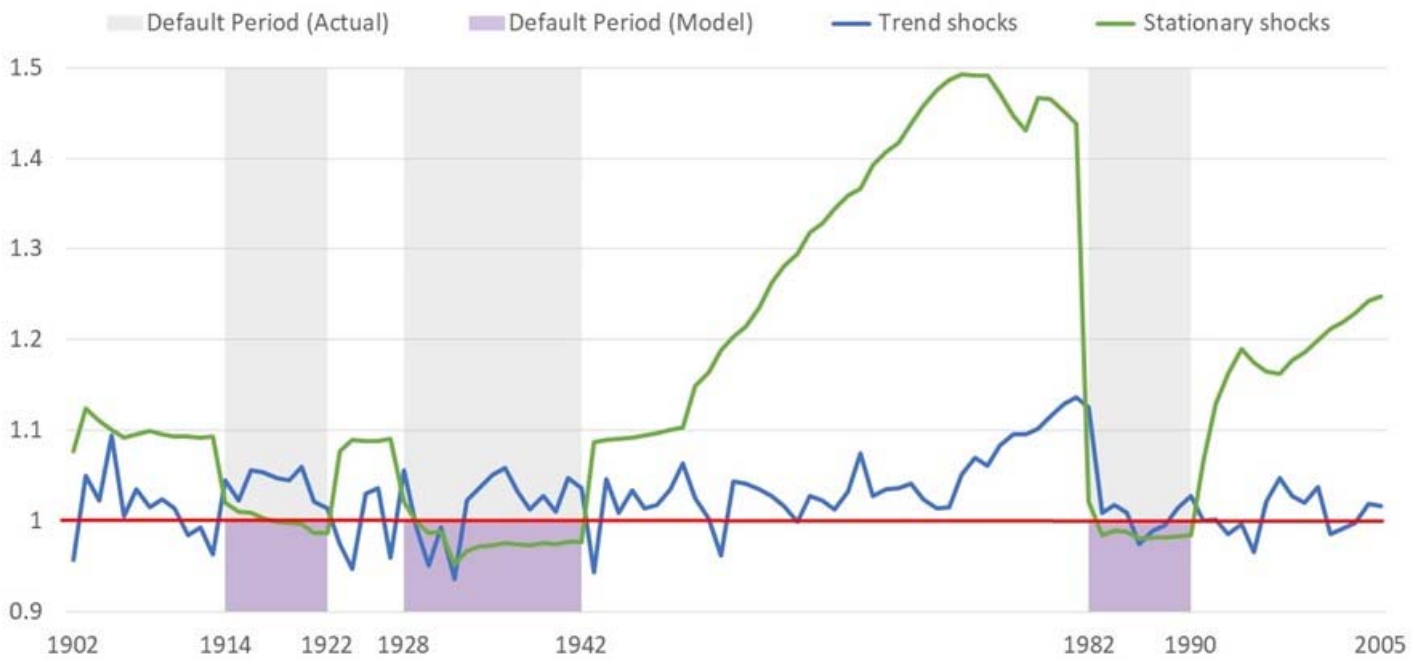
Notes: The x-axis denotes time (annual), and the y-axis the value of trend shocks g_t and stationary shocks e^{z_t} . The unconditional mean of stationary shocks is 1.00, and that of trend shocks is 1.0050 for Argentina and 1.0017 for Mexico. Regarding Argentina, stationary shocks widely deviate above the unconditional mean and suddenly drop at default events, leading to countercyclical capital flows and interest rates. For Mexico, trend shocks play this role.

Fig. 4. Path of estimated shocks during 1902–2005.

A: Argentina (1902–2005)

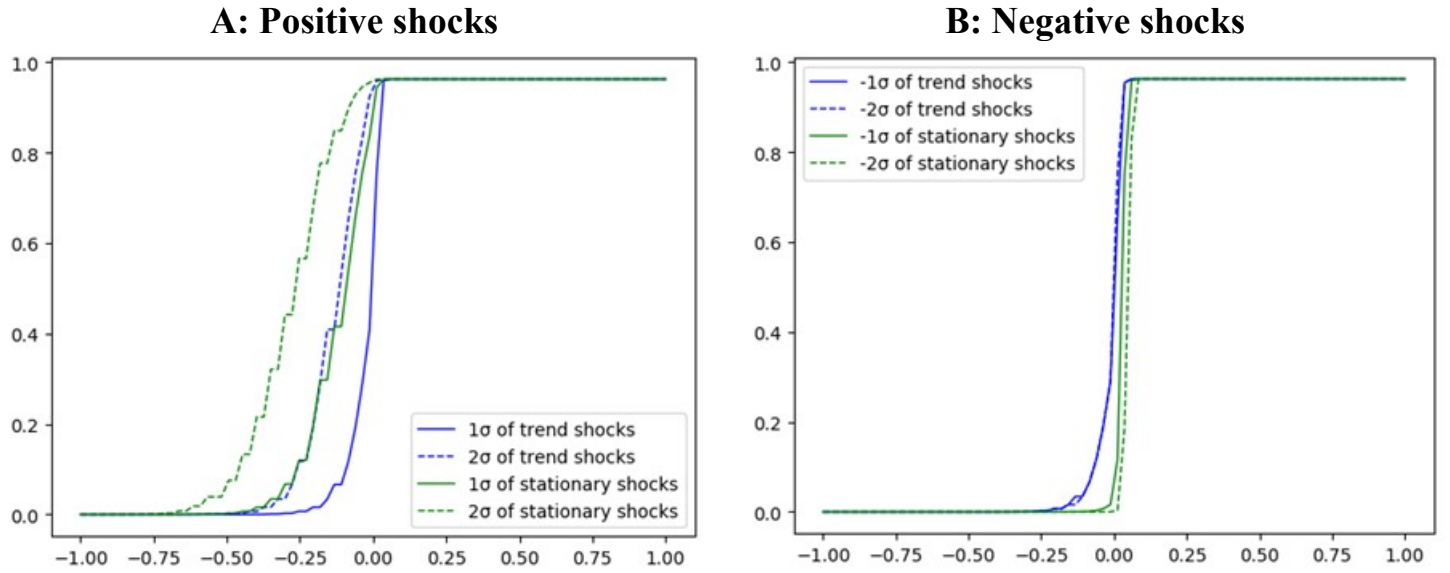


B: Mexico (1902–2005)



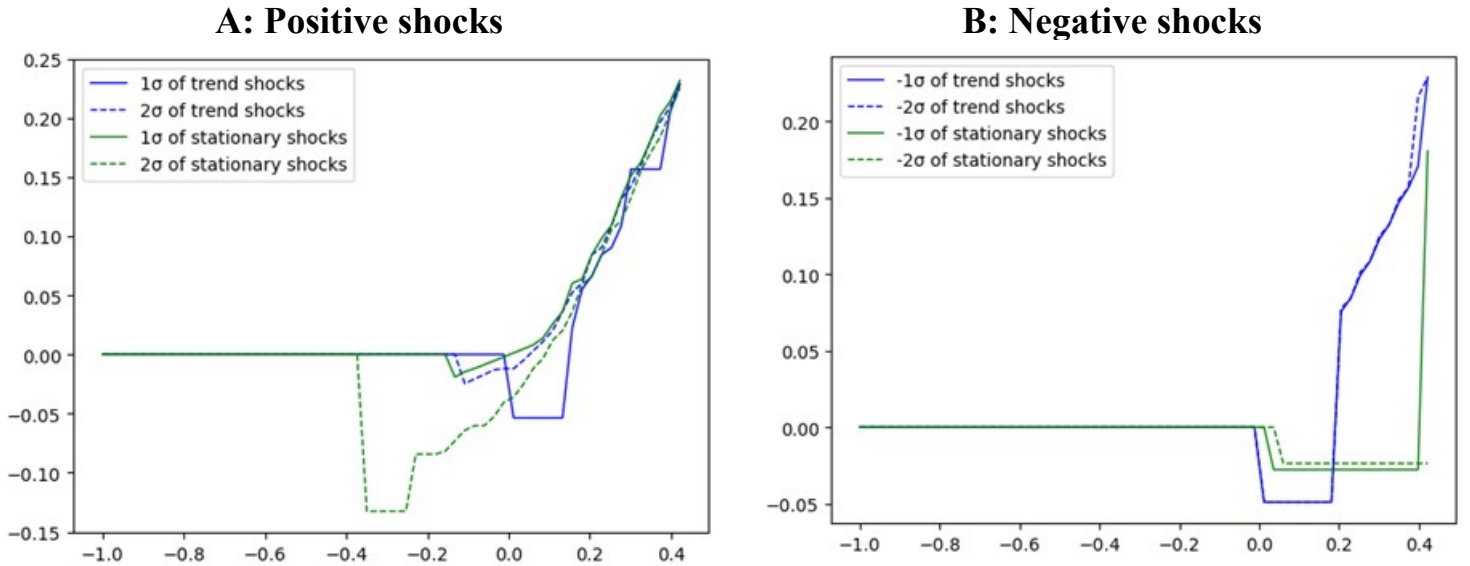
Notes: The x-axis denotes time (annual), and the y-axis the value of trend shocks g_t and stationary shocks e^{z_t} . The unconditional mean of stationary shocks is 1.00, and that of trend shocks is 1.001 for Argentina and 1.058 for Mexico. Both in Argentina and Mexico, stationary shocks widely deviate above the unconditional mean and suddenly drop at default events, leading to countercyclical capital flows and interest rates. For Mexico, stationary shocks are important, unlike in the 1977–2013 data.

Fig. 5. Bond price schedule for Argentina, 1978–2013.



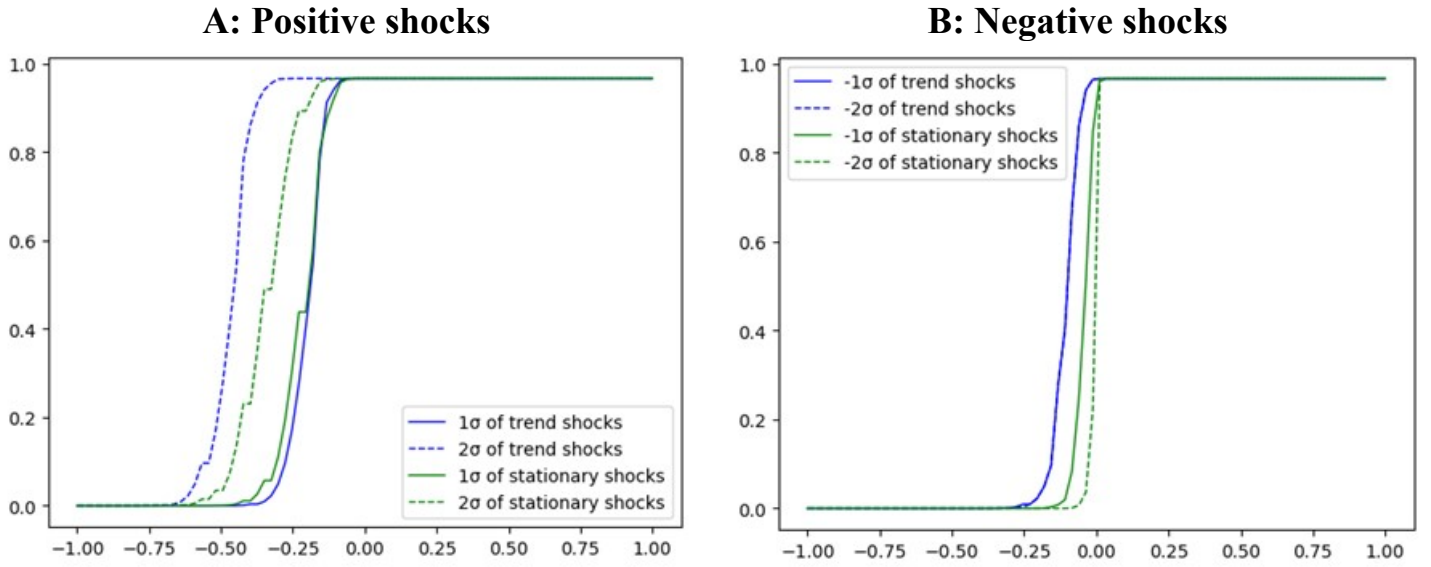
Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes the bond price. The lines are the bond schedules based on the posterior means of 1978–2013 Argentina (Table 2). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming that trend shocks lng_t are zero, and the blue lines similarly correspond to trend shocks. A positive stationary shock maintains a bond price much higher than a trend shock, whereas the effect of a negative shock is similar between the two types of shocks.

Fig. 6. Policy functions for Argentina, 1978–2013.



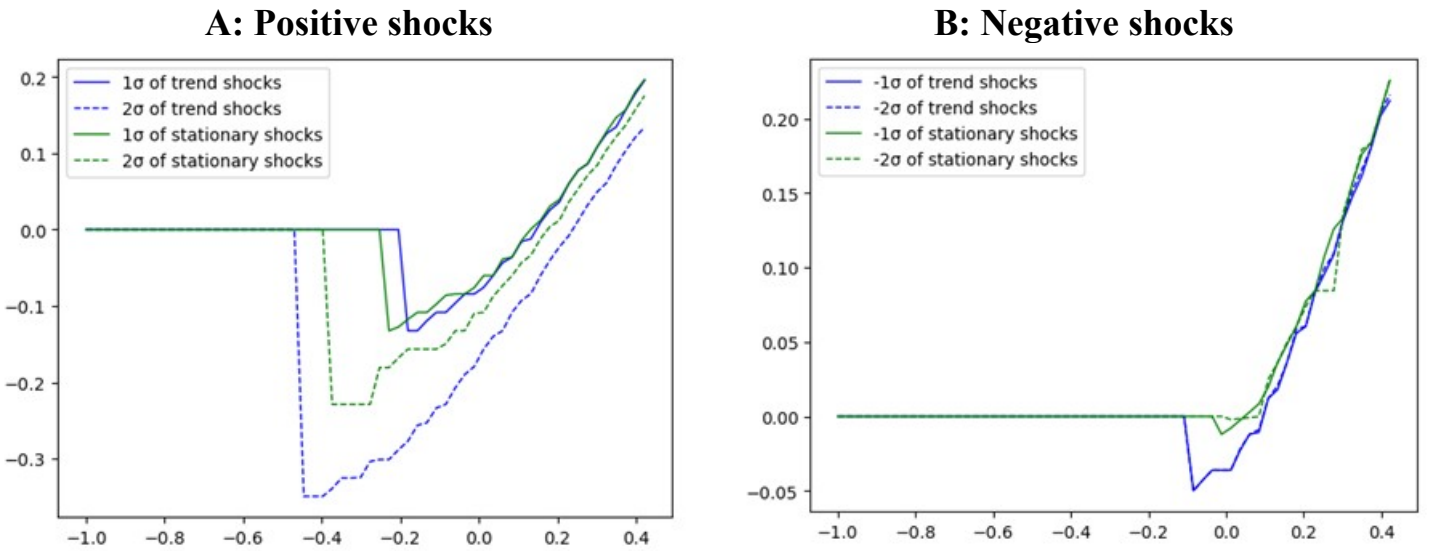
Notes: The x-axis denotes the given foreign assets b_t and a negative sign signifies the debt position. The y-axis denotes foreign assets b_{t+1} decided at time t . The lines are the policy functions based on the posterior means of 1978–2013 Argentina (Table 2). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming that trend shocks lng_t are zero, and the green lines to trend shocks in a similar way. A positive stationary shock encourages more a sovereign to borrow than a trend shock. The effect of negative shocks appears to be vague, but matters only when the sovereign has foreign assets rather than external debt.

Fig. 7. Bond price schedule for Mexico, 1977–2013.



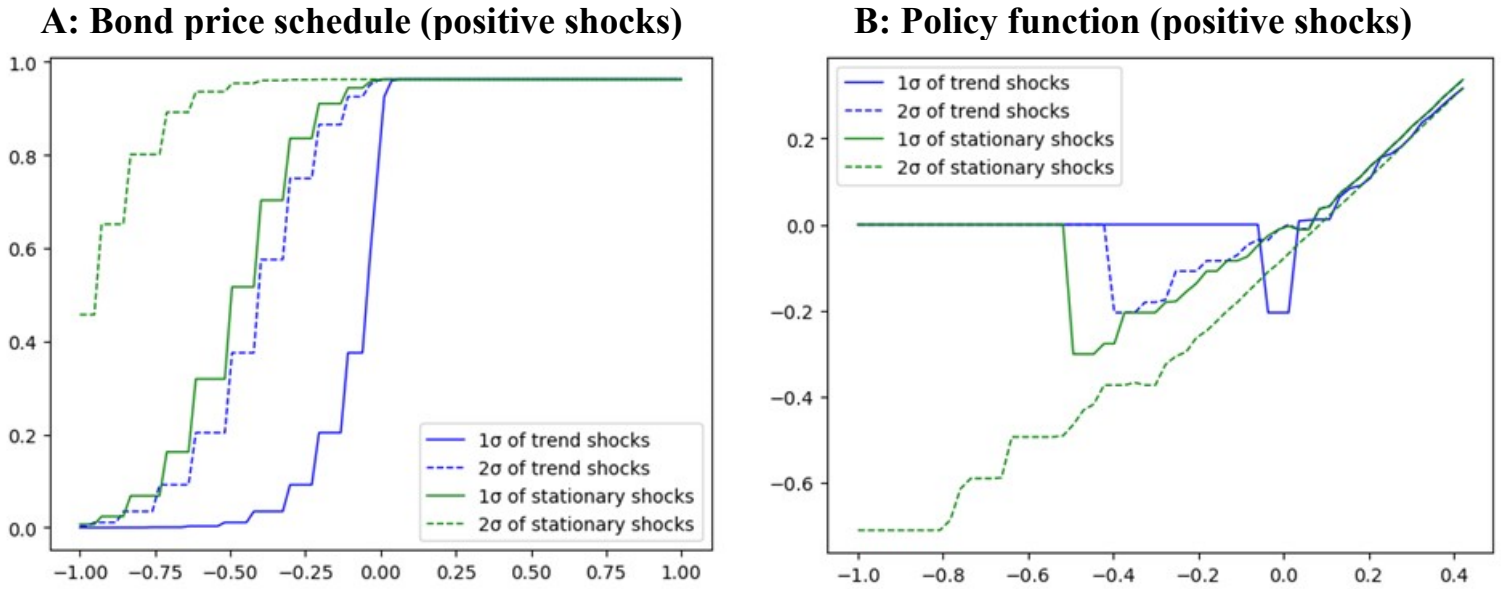
Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes the bond price. The lines are the bond schedules based on the posterior means of 1977–2013 Mexico (Table 2). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming that trend shocks lng_t are zero, and the blue lines to trend shocks in a similar way. A positive stationary shock sustains a bond price significantly higher than a trend shock, whereas the effect of negative shocks is similar between the two types of shocks.

Fig. 8. Policy functions for Mexico, 1977–2013.



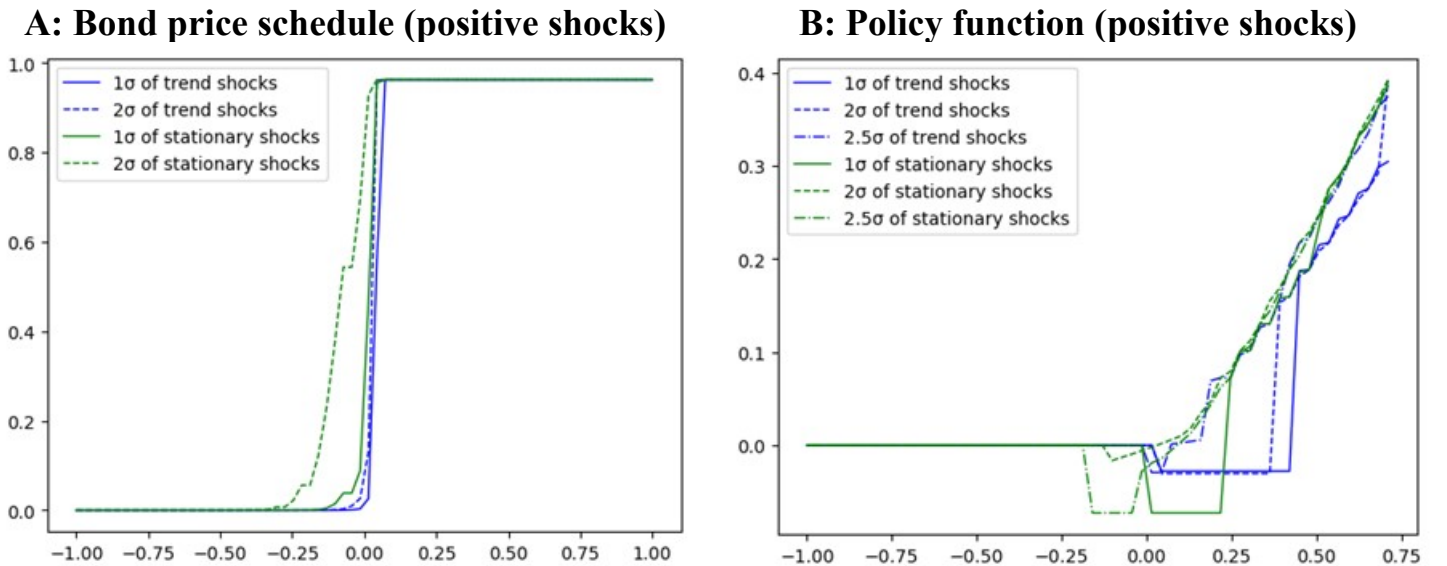
Notes: The x-axis denotes given foreign assets b_t , and a negative sign signifies the debt position. The y-axis denotes foreign assets b_{t+1} decided at time t . The lines are the policy functions based on the posterior means of 1977–2013 Mexico (Table 2). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming that trend shocks lng_t are zero, and the blue lines to trend shocks in a similar way. A positive trend shock encourages more a sovereign to borrow than a stationary shock.

Fig. 9. Bond price schedule and policy function for Argentina, 1902–2005.



Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes A: the bond price and B: foreign assets b_{t+1} decided at time t . The lines are the bond schedules, based on the posterior means of 1902–2005 Argentina (Table 3). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming trend shocks $\ln g_t$ are zero, and the blue lines to trend shocks in a similar way. A positive stationary shock sustains a bond price much higher than a trend shock. A positive stationary shock encourages more a sovereign to borrow than a trend shock.

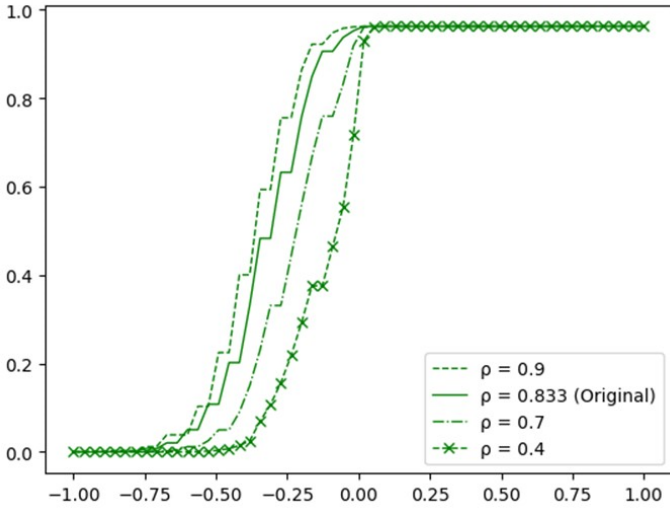
Fig. 10. Bond price schedule and policy function for Mexico, 1902–2005.



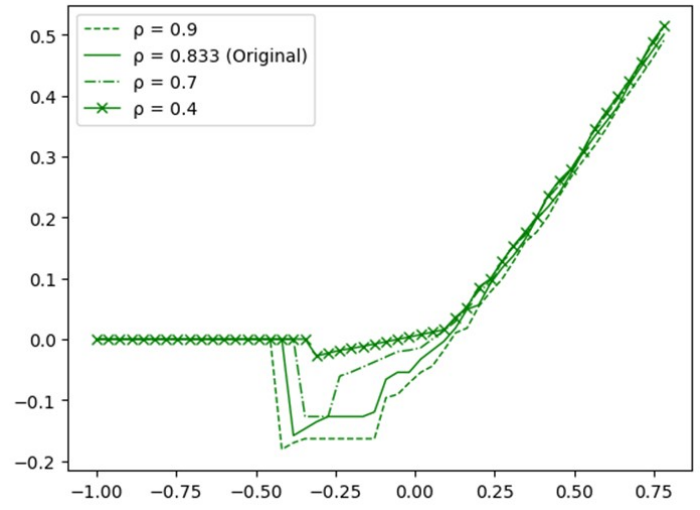
Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes A: the bond price and B: foreign assets b_{t+1} decided at time t . The lines are the bond schedules, based on the posterior means for 1902–2005 Mexico (Table 2). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming trend shocks $\ln g_t$ are zero, and the blue lines to trend shocks in a similar way. A positive stationary shock sustains a bond price higher than a trend shock, and encourages more a sovereign to borrow than a trend shock. Thereby, stationary shocks play an important role here, unlike for the 1977–2013 data.

Fig. 11. Persistence effect.

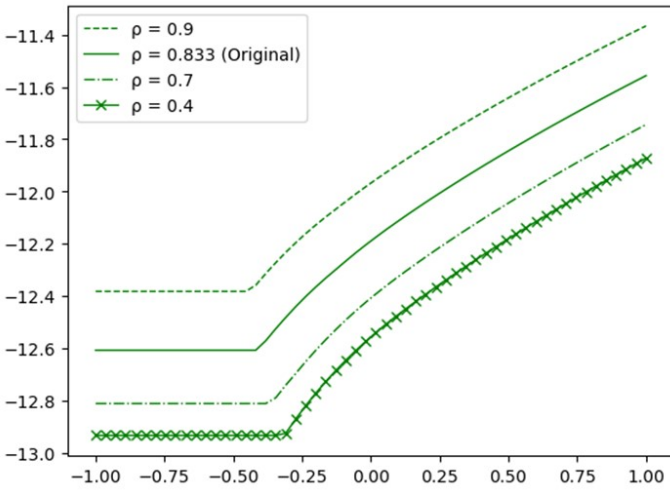
A: Bond price schedule (a positive shock)



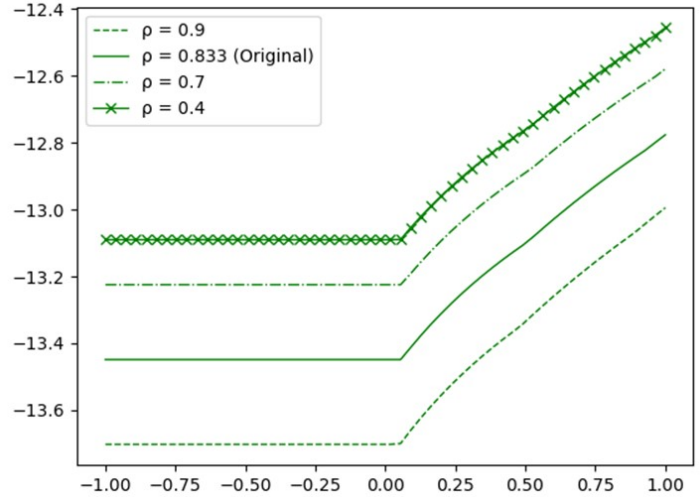
B: Policy function (a positive shock)



C: Utility conditional on a positive shock



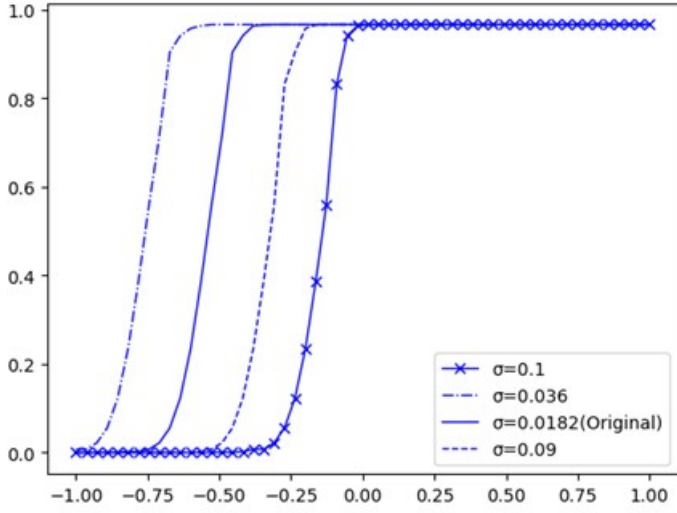
D: Utility conditional on a negative shock



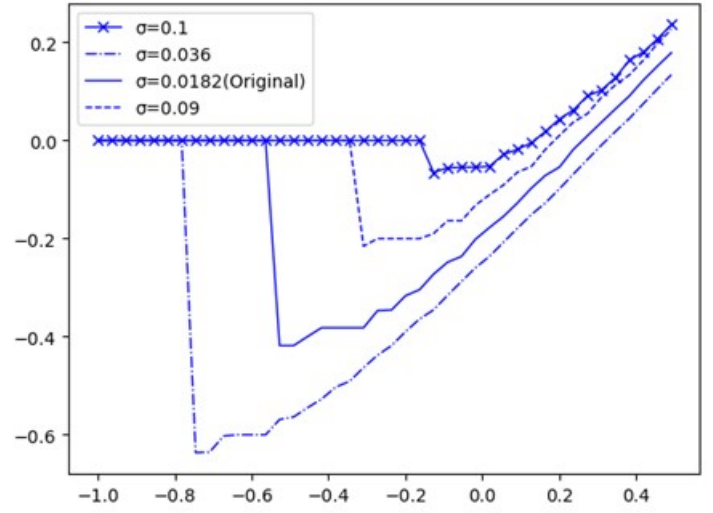
Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes A: bond prices, B: foreign assets b_{t+1} decided at time t , and C, D: conditional utility. Each value is calculated based on the posterior means for 1978–2013 Argentina (Table 2). The all lines correspond to a stationary shock z_t , assuming a trend shock lng_t is zero. The size of the shock is 2 unconditional standard deviations. As the persistence of shocks increases, A: the bond price increases, B: a sovereign borrows more, C: utility conditional on a positive shock increases, D: utility conditional on a negative shock decreases.

Fig. 12. Volatility effect.

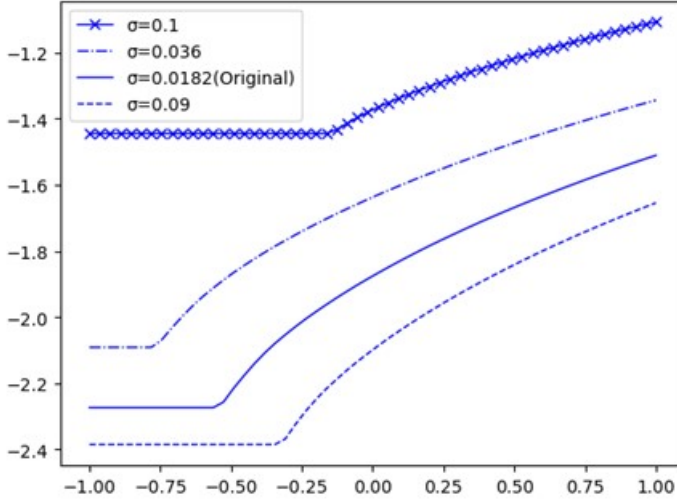
A: Bond price schedule (a positive shock)



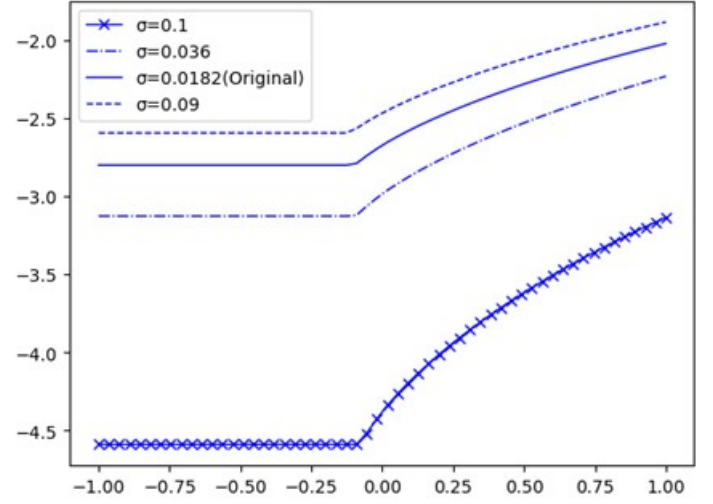
B: Policy function (a positive shock)



C: Utility conditional on a positive shock

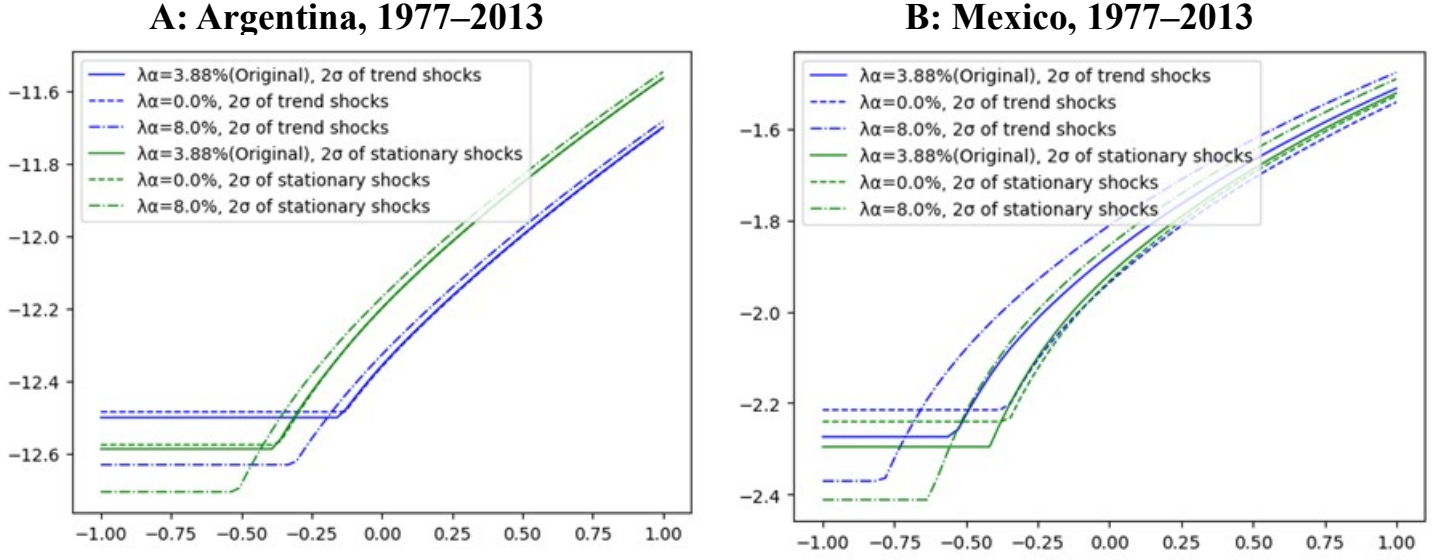


D: Utility conditional on a negative shock



Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes A: bond prices, B: foreign assets b_{t+1} decided at time t , and C, D: conditional utility. Each value is calculated based on the posterior means of 1977–2013 Mexico (Table 2). The all lines correspond to a positive trend shock $\ln g_t$, assuming a stationary shock z_t is zero. The size of the shock is 2 unconditional standard deviations. As the volatility of trend shocks increases, A: the bond price increases, B: a sovereign borrows more, C: utility conditional on a positive shock increases, D: utility conditional on a negative shock decreases. However, if the volatility becomes too high (e.g., $\sigma_g = 0.1$), A: the bond price decreases, B: a sovereign borrows less.

Fig. 13. Amplification effect of asymmetric domestic cost.



Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes utility conditional on a shock of two unconditional standard deviations. Each value is calculated based on the posterior means of A: 1978–2013 Argentina and B: 1977–2013 Mexico (Table 2). The blue lines correspond to a trend shock $\ln g_t$, assuming a stationary shock z_t is zero. The green lines correspond to a trend shock in a similar way. As the asymmetric domestic cost of default increases, utility conditional on a positive shock shifts to the lower left. The difference of maximum debt level (difference of the kink) between a trend and stationary shock expands in panel B, since larger volatility implies quicker recovery from recessions, resulting in higher default cost. On the other hand, the difference does not expand, and the kink shifts proportionally in the panel A, since high persistence indicates slow recovery from recessions.

Table 5: Second moments.

Argentina	Data	Proposed model
$\sigma_{\Delta y}$	5.74	5.46
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.50	1.47
$\sigma_{tb/y}$	3.87	4.10
$Corr(\Delta y, tb/y)$	-0.215	-0.253
$Corr(\Delta y, q)$	0.497	0.314
$Corr(tb/y, q)$	-0.173	0.128
$Serial\ corr(tb/y)$	0.674	0.091

Mexico	Data	Proposed model
$\sigma_{\Delta y}$	3.38	3.22
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.28	2.68
$\sigma_{tb/y}$	3.29	5.76
$Corr(\Delta y, tb/y)$	-0.454	-0.453
$Corr(\Delta y, q)$	0.318	0.371
$Corr(tb/y, q)$	-0.741	-0.165
$Serial\ corr(tb/y)$	0.751	0.220

Argentina	Data	Proposed model
$\sigma_{\Delta y}$	5.37	5.14
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.41	1.26
$\sigma_{tb/y}$	5.14	3.54
$Corr(\Delta y, tb/y)$	-0.05	-0.112
$Serial\ corr(tb/y)$	0.577	0.685

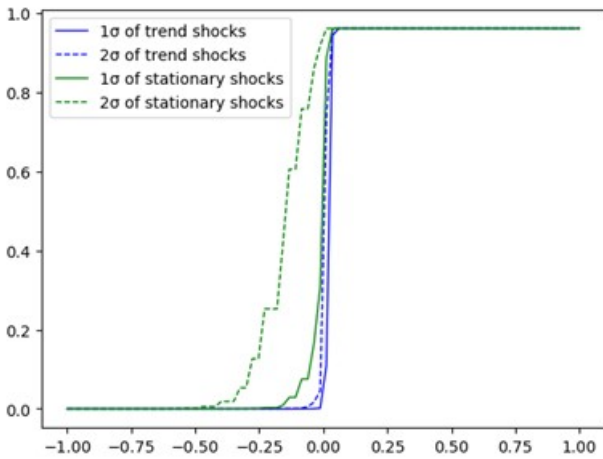
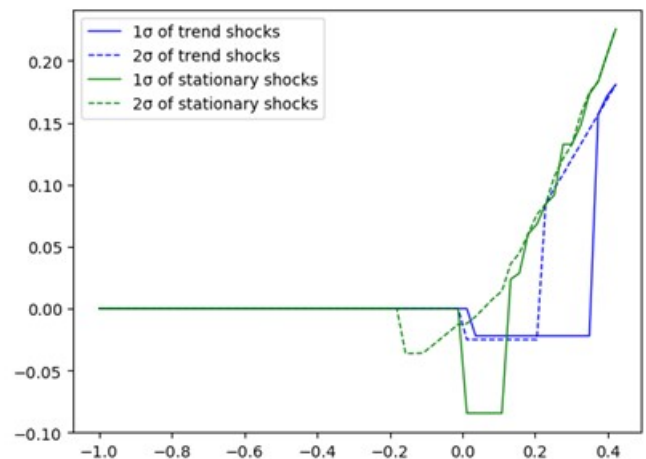
Mexico	Data	Proposed model
$\sigma_{\Delta y}$	4.26	4.14
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.45	1.13
$\sigma_{tb/y}$	4.22	1.86
$Corr(\Delta y, tb/y)$	-0.183	-0.094
$Serial\ corr(tb/y)$	0.722	0.717

Notes: Standard deviations are reported in percentage points. The moments are calculated based on the prediction series of the particle filter with 20,000 particles pertaining to the posterior mean of Tables 2 and 3, respectively. The moments match the regularities of business cycles in emerging economies.

Table 6: Posteriors of the analysis using EMBI + spread.

Argentina		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.872	0.833	0.891
γ	Risk aversion	Uniform	1.853	1.812	1.875
r	Risk-free interest rate	Uniform	4.09%	3.91%	4.16%
ρ_z	Persistence of stationary shocks	Uniform	0.927	0.910	0.939
σ_z	SD of stationary shocks	Uniform	0.026	0.026	0.027
θ	Probability of re-entry	Uniform	22.48%	22.04%	22.81%
λ_α	Asymmetric domestic cost	Uniform	1.14%	1.12%	1.15%
λ_β	Proportional domestic cost	Uniform	0.52%	0.51%	0.53%
μ_g	Gross mean growth	Uniform	1.033	1.020	1.038
ρ_g	Persistence of trend shocks	Uniform	0.812	0.780	0.827
σ_g	SD of trend shocks	Uniform	0.026	0.025	0.026
Acceptance ratio			17.55%		
Marginal data density			-263.99		

Notes: This table shows the means and intervals bracketed by 5% and 95% of the posterior distributions. The intervals are tight, since the measurement equations include default state, assigning less likelihood on the particles of parameter vectors that could not predict default events and recoveries.

Fig. 14. Bond price schedule and policy function for Argentina, 1983–2013, based on the estimation using EMBI+ spread.**A: Bond price schedule (positive shocks)****B: Policy function (positive shocks)**

Notes: The x-axis denotes foreign assets, and a negative sign signifies the debt position. The y-axis denotes A: bond prices and B: foreign assets b_{t+1} decided at time t , based on the posterior means of 1983–2013 Argentina (Table 6). The standard deviations are unconditional. The green lines correspond to stationary shocks z_t , assuming trend shocks lng_t are zero, and the blue lines to trend shocks in a similar way. A positive stationary shock maintains a bond price higher than a trend shock. A positive stationary shock encourages more a sovereign to borrow than a trend shock.

Table 7: Posteriors of the analysis using HP-filtering.

Argentina		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.897	0.883	0.907
γ	Risk aversion	Uniform	2.132	2.053	2.181
r	Risk-free interest rate	Uniform	4.39%	4.24%	4.66%
ρ_z	Persistence of transitory shock	Uniform	0.889	0.852	0.900
σ_z	SD of stationary shock	Uniform	0.028	0.028	0.029
θ	Probability of re-entry	Uniform	21.46%	20.70%	22.04%
λ_α	Asymmetric domestic costs	Uniform	3.15%	2.94%	3.25%
λ_β	Proportional domestic costs	Uniform	0.026%	0.025%	0.028%
Acceptance ratio			18.3%		
Marginal data density			-158.37		
Mexico		Priors	Mean	5%	95%
β	Discount factor	Uniform	0.901	0.837	0.927
γ	Risk aversion	Uniform	1.858	1.605	1.940
r	Risk-free interest rate	Uniform	4.11%	3.84%	4.25%
ρ_z	Persistence of transitory shock	Uniform	0.870	0.774	0.900
σ_z	SD of stationary shock	Uniform	0.018	0.017	0.018
θ	Probability of re-entry	Uniform	9.12%	7.89%	13.24%
λ_α	Asymmetric domestic costs	Uniform	2.79%	2.67%	2.90%
λ_β	Proportional domestic costs	Uniform	0.018%	0.015%	0.019%
Acceptance ratio			20.50%		
Marginal data density			-353.61		

Notes: This table shows the means and intervals bracketed by 5% and 95% of the posterior distributions. The intervals are tight, since the measurement equations include default state, assigning less likelihood on the particles of parameter vectors that could not predict default events and recoveries.