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Borda Count Method for Fiscal Policy

- A Political Economic Analysis -

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Abstract

Survey data reveals that government budgets tend to go into the red. Public Choice economists as well as public finance economists have been interested in this phenomenon. This paper presents a new explanation for this tendency from the political economic point of view; the current voting system might have a tendency to bring about a budget deficit. If policy choices only deal with the current tax rate and do not take into account the intertemporal tax rate, budget-balanced choice is difficult to be chosen. Even if voting choices take into account intertemporal aspects, we show that a budget-balanced choice is difficult to be chosen under relative majority rule. We further demonstrate that the Borda count method might overcome this issue.

JEL: D72; H41; H62

Keywords: relative majority rule, Borda count method, deficit

Remark

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1. Introduction

Macroeconomists almost always assume that the intertemporal government budget is balanced. In other words, the government budget is not necessarily balanced today but is to be balanced in the future. This assumption seems natural because, in our daily life, anyone has to reimburse her deficit at some time. However, in reality, the government's balanced-budget is not easy to be implemented. Buchanan (1987, p.471) asserts that the government budget tends to be in deficit by describing "*The most elementary prediction from public choice theory is that in the absence of moral or constitutional constraints democracies will finance some share of current public consumption from debt issue rather than from taxation and that, in consequence, spending rates will be higher than would accrue under budget balance*." Also, an IMF survey[3] reveals that 139-159 countries among 189 countries were in deficit during 2010-2015. Moreover, 112 countries had consecutively been in deficit during these six years.

There are several explanations why government budget tends to be in deficit. Orthodox tax smoothing (Barro, 1979) cannot explain this tendency. As seen above, Buchanan explains this phenomenon that debt finance is preferred to taxation finance under democracy, which is sometimes called a *fiscal illusion* (Buchanan and Wagner, 1977). Weingast et al. (1981) and Cogan (1993) explain this phenomenon in a microeconomic way that tragedy of commons brings about government deficit. Alesina and Tabellini (1990) propose a theory that the current government has an incentive to constrain the future government's activity by accumulating public debt.

The purpose of this paper is to propose another explanation. Our explanation, which is based on the study of political economy, is that current voting rules may have a tendency to bring about a non-budget-balanced choice. Firstly, if the policy choices are only for today's tax rate, not only non-budget-balanced people but also budget-balanced people who prefer future tax increases to today's tax increase may opt for no tax increase today. Moreover, even if policy choices take into account an intertemporal tax rate, relative majority rule might not reflect voter opinion. It is well-known that, although widely implemented, the relative majority rule has several pitfalls. For example, it is vulnerable to split voting. If there are several policy choices with similar ideologies, vote splits and thus these choices turn out to be difficult to obtain relative majority. As a result, even though ideology A is supported by the majority where ideology B is supported by the minority, if there are many policy choices embodying ideology A, a candidate with ideology B may win under relative majority voting rule. This effect, known as a *spoiler effect*, can be seen in many electoral campaigns, such as the 2000 U.S. Presidential Elections (The New York Times, 2004). We show that there is a possibility that, even if budget-balanced people are the majority, they may suffer from the spoiler effect.

[3] IMF Fiscal Monitor

There are several voting rules to overcome the caveats of relative majority rule. Among these, the Borda count method (Borda, 1784) and Condorcet method (Condorcet, 1785) are well-known. We will focus on the Borda count method hereafter[4]. Under the Borda count method, if there are n policy choices, each person gives n points, $n-1$ points, ⋯, 1 point respectively to each choice. Then, sum up the points for each choice. A choice that collects the highest points is elected. This count method is robust for split voting and thus overcomes the spoiler effect. This count method is not an armchair theory; this method is used in special legislative seats for ethnic minorities in Slovenia and similar methods, called the Dowdall method, is implemented in Nauru (Golder, 2005; Fraenkel and Grofman, 2014). We will see that the Borda count method works for our issue as well.

This explanation has some similarities with Alesina and Drazen (1991) where two groups of people play a war of attrition game in order for a group to shift the burden of public debt to another group. They have to pay a cost for the postponement of redemption each period, thus this war-of-attrition makes social welfare sub-optimal. In this model, people know that non-sustainable debt must be redeemed but totally rational people act sub-optimally. In our model, the majority of people know that non-sustainable debt must be redeemed, but, due to the current voting system, social consensus postpones redemption and acts sub-optimally. However, the mechanism is totally different from that in Alesina and Drazen (1991).

Our model is also similar to the agenda-setting model proposed by Romer and Rosenthal (1978) where agenda setters influence the final social consensus. Our model claims that the final social consensus depends on whether the voting agenda is only for today's tax rate or intertemporal tax rate. However, our model goes further; the final social consensus does depend on what kind of voting system is used.

The remainder of this paper is organized as follows. A simple model is presented in the next section. This simple model is generalized and individual utility function is specified in the following section. Finally, we conclude.

2. Simple model

Assume there is initial government deficit that is normalized to unity. Our interest is people's voting behavior that chooses a policy from several possible plans to reimburse government deficit in a two period model ($t = 1,2$). For the sake of simplicity, we assume that there are only three different plans on how to reimburse government deficit. Plan X reimburses government deficit only at $t = 1$ by tax increases where plan Y reimburses government deficit only at $t = 2$ by tax increases. Both plan X and plan Y are budget-balanced plans. However, plan Z is non-budget-balanced; it does not reimburse government deficit either at $t = 1$ or at $t = 2$. Denoting each plan by (τ_1, τ_2) where τ_i

[4] Condorcet method requires maximum likelihood method if there are four or more choices (Young, 1988), which is difficult to be implemented in our model.

means a tax increase at $t = i$, plan X is denoted as $(1,0)$, whereas plan Y is $(0,1)$ and plan Z is $(0,0)$. We assume people's voting behavior only depends on tax profile. Note that, if a policy is chosen, it will surely be implemented. In other words, selected policy fully binds not only this period's tax rate but also next period's tax rate.

Let there be $n_a + n_b + n_c$ people. n_a people have preference A, n_b people have preference B, and n_c people have preference C. People with preference A or B prefer balanced-budget plans. Among balanced-budget plans, people with preference A prefer plan X to plan Y (early reimbursement is preferred), where people with preference B prefer plan Y to plan X (late reimbursement is preferred). People with preference C prefer a non-balanced-budget plan and consider that plan Y is at least better than plan X[5]. These preferences are described in Figure 1, where $\mathrm{S} \succ \mathrm{T}$ meaning that S is preferred to T. We assume that balanced-budget people are greater in number than that of non-balanced-budget people; i.e. $n_a + n_b > n_c$. We also assume $n_c > n_a$ and $n_c > n_b$. An example of what we are considering is $n_a = n_b = 3$ and $n_c = 4$. Finally, we assume truthful-voting hereinafter.

A: $\mathrm{X} \succ \mathrm{Y} \succ \mathrm{Z}$

B: $\mathrm{Y} \succ \mathrm{X} \succ \mathrm{Z}$

C: $\mathrm{Z} \succ \mathrm{Y} \succ \mathrm{X}$

Figure 1: Voting preferences for people A, B and C

As follows, we will see that balanced-budget plan is not chosen in this framework if policy choices do not take into account the intertemporal aspect or if relative majority rule is implemented.

(1) Let there be two policy choices that do not take into account the intertemporal aspect; tax increase at $t = 1$ and non-tax increase at $t = 1$. Then, people with preference B as well as preference C choose the latter choice whichever voting rule is implemented. Therefore, a non-tax increase is chosen even though balanced-budget people are the majority.

(2) Let there be three policy choices that take into account the intertemporal aspect; $(\tau_1, \tau_2) = (1,0), (0,1), (0,0)$. The first choice is voted by people with preference A, the second by people with preference B, and the third by people with preference C. Therefore, under relative majority rule, $(\tau_1, \tau_2) = (0,0)$ is chosen even though balanced-budget people are majority.

The reasons for this result are summarized in the following two points. First, if the choices only take into account today's tax rate ($t = 1$), not only non-budget-balanced people (preference C) but also balanced-budget people who prefer late reimbursement (preference B) choose the non-tax increase

[5] Identical discussion is possible when people with preference C consider that plan X is at least better than plan Y. Without loss of generality, we assume here that people with preference C consider that plan Y is at least better than plan X.

today. Since $n_b + n_c > n_a$ holds, the choice of a non-tax increase is chosen. This result holds whichever voting rule is implemented. Second, even if the choices take into account the future tax rate ($t = 2$), votes from balanced-budget people split between plan X and plan Y, which makes plan Z win under relative majority voting rule because $n_c > n_a$ and $n_c > n_b$ hold. As a result, even if balanced-budget people are the majority, the non-balanced-budget choice is chosen.

This issue is overcome by incorporating the intertemporal aspect in policy choices and implementing the Borda count method. Under the Borda count method, each person gives n points, $n-1$ point, ⋯, 1 point respectively to n choices. Take the sum of the points for each choice. A choice that collects the largest amount of points is chosen. Using the Borda count method for three policy choices (plan X, Y and Z), people with preference A give 3 points to plan X, 2 points to plan Y, and 1 point to plan Z. People with preference B give 3 points to plan Y, 2 points to plan Z, and 1 point to plan Z. People with preference C give 3 points to plan Z, 2 points to plan Y, and 1 point to plan X. This is described in Figure 2.

	X	Y	Z
A	3	2	1
B	2	3	1
C	1	2	3
Total points	$3n_a + 2n_b + n_c$	$2n_a + 3n_b + 2n_c$	$n_a + n_b + 3n_c$

Figure 2: Voting result under Borda count method

In this case, budget-balanced plan Y is chosen because $2n_a + 3n_b + 2n_c > n_a + n_b + 3n_c$ and $2n_a + 3n_b + 2n_c > 3n_a + 2n_b + n_c$. We will generalize this idea in the following section.

3. General model

As we see in the previous section, we show that the Borda count method is efficient to overcome the aforementioned issue. However, the model presented in the previous section does not specify individual utility function. Also, it considers only three policy choices; $(\tau_1, \tau_2) = (1,0), (0,1), (0,0)$. In order to generalize the previous model, we specify the voter's utility function and consider many possible policy choices hereafter.

Assume that there are k budget-balanced people and l non-budget-balanced people. If $l < k$, budget-balanced people are the majority where $l > k$ implies non-budget-balanced people are the majority[6]. Each person is indexed by $i \in [1, k + l]$. Before period 0, initial government debt that is

[6] We are interested in a case where budget-balanced people are the majority. Supporting evidence of this case is Mochida (2016). Using an Internet-based questionnaire of 1,000 answers randomly sampled from approximately 3.27

to be reimbursed by period 3 is $D > 0$, and the number of budget-balanced and non-budget-balanced people and anyone's future endowments are already common knowledge. Initial government debt might be used to finance public goods provided before period 0. In period 0, the lump-sum tax schedule is determined by voting, the detail of which is discussed later. In period 1 and period 2, each person is endowed $w_i \in (0, n_i)$ and $n_i - w_i \in (0, n_i)$ respectively and has to pay lump-sum tax, $\tau_1 \in [0, \min_i w_i)$ and $\tau_2 \in [0, \min_i (n_i - w_i))$ respectively. Note that the size of the lump-sum tax is unaffected by the size of one's endowment. Each person enjoys private consumption, $c_i^1 = w_i - \tau_1$ at period 1 and $c_i^2 = n_i - w_i - \tau_2$ at period 2 respectively, which implies that saving is prohibited to any person.

We will consider a case where tax rates τ_1 and τ_2 are restricted to non-negative integers, and budget-balanced people prefer (τ_1, τ_2) to (τ'_1, τ'_2) and non-budget-balanced people prefer (τ'_1, τ'_2) to (τ_1, τ_2) if $\frac{D}{k+l} \geq \tau_1 + \tau_2 > \tau'_1 + \tau'_2 \geq 0$. If $\tau_1 + \tau_2 = \tau'_1 + \tau'_2$, the preference depends on each person's characteristics independent from budget-balanced/non-budget-balanced. In order only to demonstrate such preferences, we specify utility functions as follows.

The government reimburses its debt at period 3 from tax revenue during period 1 to period 2; $(k + l)(\tau_1 + \tau_2)$. Since the tax schedule is solely determined by voting, the government has no objective function.

Each person's utility function is

$$U_i = U\left(c_i^1, c_i^2, \xi, \theta\right) = \frac{(c_i^1)^{1-\sigma}}{1-\sigma} + \frac{(c_i^2)^{1-\sigma}}{1-\sigma} - \theta v(\xi) \qquad (1)$$

where $\sigma > 0$ measures the degree of relative risk aversion and function $v(\cdot)$ measures the disutility from final government debt per capita[7]. Remaining government debt per capita at period 3 is $\xi \equiv \frac{D}{k+l} - (\tau_1 + \tau_2)$. $\theta = 1$ for budget-balanced people where $\theta = 0$ for non-budget-balanced people. We consider neither interest rate nor discount rate. An interpretation of function $v(\cdot)$ is that it measures possible future consequences caused by government debt. Therefore, it is assumed that $v(\xi)$ is a positive, differentiable and strictly increasing function when $\xi > 0$ and a weakly increasing function when $\xi \leq 0$. For technical reasons, we also assume that $v'(\xi) > 1$ when

million people in Japan, he reports that three quarters of people are budget-balanced people (immediate redemption, 33.1%; gradual redemption, 42.4%) where a quarter of people are non-budget-balanced.

[7] People do not receive utility or disutility directly from government debt. However, it is natural to assume that people may expect future tax increases after period 3, which they themselves or their descendants have to bear. Maybe people expect a future default and take into account its consequence which they themselves or their descendants will suffer if there is substantial debt outstanding. Assuming that people take into account their descendants' utility as appeared in Buchanan (1976), function $v(\cdot)$ is understood to include all these effects. It is worth noting that Alesina and Drazen (1991) also assume that postponing debt redemption is costly because of the distortion of taxation and lobbying costs. Such an effect may also be included in function $v(\cdot)$. The functional form of it may be derived by analyzing the probability of default (e.g. Cuadra et al., 2010), which is not our research focus. Similar methods can be seen in Caselli (1997) and Müller et al. (2016) where a fraction of debt defaults or probability of defaults explicitly appears in the cost function or utility function.

$\xi > 0$ hereafter[8].

Assume that anyone is given enough endowments, i.e. $\frac{D}{k+l} + 1 \le \min_i w_i$ and $\frac{D}{k+l} + 1 \le \min_i (n_i - w_i)$ in order to avoid corner solutions.

We also assume that the degree of relative risk aversion σ is small. More specifically, we assume the following condition.

Assumption 1 (sufficiently small relative risk aversion σ): We assume that, for any person i, the following inequality always holds.

$$U\left(c_i^1, c_i^2, \xi, \theta = 1\right) > U\left(\widetilde{c_i^1}, \widetilde{c_i^2}, \tilde{\xi}, \theta = 1\right) \quad (2)$$

and

$$U\left(c_i^1, c_i^2, \xi, \theta = 0\right) < U\left(\widetilde{c_i^1}, \widetilde{c_i^2}, \tilde{\xi}, \theta = 0\right) \quad (3)$$

where

$$c_i^1 = w_i - \tau_1 \ge 1 \quad (4)$$

$$c_i^2 = n_i - w_i - \tau_2 \ge 1 \quad (5)$$

$$\frac{D}{k+l} - (\tau_1 + \tau_2) = \xi \quad (6)$$

$$\widetilde{c_i^1} = w_i - \widetilde{\tau_1} \ge 1 \quad (7)$$

$$\widetilde{c_i^2} = n_i - w_i - \widetilde{\tau_2} \ge 1 \quad (8)$$

$$\frac{D}{k+l} - (\widetilde{\tau_1} + \widetilde{\tau_2}) = \tilde{\xi} \quad (9)$$

$$\tilde{\xi} - \xi \ge \frac{1}{k+l} \quad (10)$$

$$\xi, \tilde{\xi} \in \left[0, \frac{D}{k+l}\right]. \quad (11)$$

That is, for both balanced-budget-people and non-balanced-budget people, the total amount of tax (in other words, the remaining amount of government debt at period 3) is of primary importance and the allocation of it is of secondary importance. In other words, we assume that utility is close to quasi-linear utility function $U_i = U\left(c_i^1, c_i^2, \xi, \theta\right) = c_i^1 + c_i^2 - \theta v(\xi)$ and the deviation from quasi-linear utility function is only for technical purposes[9]. This condition is satisfied when the degree of relative risk aversion σ is sufficiently small.

In addition, for the sake of simplicity, we assume integer restriction as follows. As we will consider voting later, a discrete profile is easier to be dealt with than continuum profile.

Assumption 2 (step size): We assume that w_i and n_i are positive integers and τ_1, τ_2 and $\frac{D}{k+l}$

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[8] Our discussion can be extended where $v'(\xi) > 1$ holds only for $\xi > \xi_0 > 0$. In such a case, we will define $\overline{D} = D - (k+l)\xi_0$ and consider a situation where debt $\overline{D}$ (debt that is excessive) is to be reimbursed by period 3.

[9] We assume that people have a slight preference on tax allocation. In other words, some people prefer $(\tau_1, \tau_2) = (3,0)$ to $(\tau_1, \tau_2) = (2,1)$ where other people have the opposite preference. However, this preference is only of secondary importance.

(initial government debt per capita) are integers. Also, we assume that $n_i - \frac{D}{k+l}$ is an even number for all i.

Note that the integer step size is arbitrary. Therefore, if a statement requires $\frac{D}{k+l}$ to be sufficiently large, such a statement also holds when the step size is sufficiently small, and vice versa.

Due to Assumption 2, consumptions at period 1 and period 2 are always positive integers. In order to reimburse government debt whose size is D, possible tax schedules are restricted to $(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right), \left(1, \frac{D}{k+l} - 1\right), \cdots, \left(\frac{D}{k+l}, 0\right)$.

Following these settings, we propose tax allocation one most prefers.

Proposition 1: If person i is budget-balanced, she most prefers to the following tax schedule.

$$\begin{cases} (\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right) \; if \; 0 > w_i - \frac{n_i - \frac{D}{k+l}}{2} \\ (\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right) \; if \; m = w_i - \frac{n_i - \frac{D}{k+l}}{2} \in [0, \frac{D}{k+l}] \\ (\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right) \; if \; \frac{D}{k+l} < w_i - \frac{n_i - \frac{D}{k+l}}{2} \end{cases}$$

If person i is non-budget balanced, she most prefers tax allocation $(\tau_1, \tau_2) = (0,0)$.

Proof of Proposition 1: A non-budget-balanced person is assumed not to take into account the remaining government debt at period 3 into her utility at period 0. Therefore, at period 0, a non-budget-balanced person always prefers a small lump-sum tax, i.e. $(\tau_1, \tau_2) = (0,0)$.

For budget balanced people, due to Assumption 1, they prefer a balanced-budget tax plan, namely $\tau_1 + \tau_2 = \frac{D}{k+l}$. Among balanced-budget tax plans, one prefers smoothed consumption because the CRRA utility function is concave. Therefore,

(a) $(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right)$ is most preferred if $0 > w_i - \frac{n_i - \frac{D}{k+l}}{2}$ holds, because $c_i^1 = w_i - \tau_1 = w_i$ is still less than $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i - \frac{D}{k+l}$ and $(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right)$ is the corner solution.

(b) $(\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right)$ is most preferred if $m = w_i - \frac{n_i - \frac{D}{k+l}}{2} \in [0, \frac{D}{k+l}]$ holds, because $c_i^1 = w_i - \tau_1 = w_i - m = \frac{n_i - \frac{D}{k+l}}{2}$ is equal to $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i - \frac{D}{k+l} + m = \frac{n_i - \frac{D}{k+l}}{2}$.

(c) $(\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right)$ is most preferred if $\frac{D}{k+l} < w_i - \frac{n_i - \frac{D}{k+l}}{2}$ holds, because $c_i^1 = w_i - \tau_1 = w_i -$

$\frac{D}{k+l}$ is still greater than $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i$ and $(\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right)$ is the corner solution.

(Q.E.D.)

Still, budget-balanced person i prefers any tax schedule that satisfies $\tau_1 + \tau_2 = \frac{D}{k+l}$ to any tax schedule that does not satisfy it ($\because$ Condition of small σ).

The abovementioned utility function is given only for demonstrating people's preference where budget-balanced people prefer (τ_1, τ_2) to (τ'_1, τ'_2) and non-budget-balanced people prefer (τ'_1, τ'_2) to (τ_1, τ_2) if $\frac{D}{k+l} \geq \tau_1 + \tau_2 > \tau'_1 + \tau'_2 \geq 0$. Proposition 1 is shown only to demonstrate that people's preference between (τ_1, τ_2) and (τ'_1, τ'_2) depends on each person's characteristics (independent from budget-balanced/non-budget-balanced) if $\tau_1 + \tau_2 = \tau'_1 + \tau'_2$. The following discussion does not depend on the specification of the utility function.

Definition 1 (classification of budget-balanced people): Among budget-balanced people, let the number of people who most prefer $(\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right)$ be k_m. Note that $\sum_{m=0}^{\frac{D}{k+l}} k_m = k$ is satisfied.

We hereafter consider what kind of tax plan will be considered for voting. We assume that the budget surplus tax schedule be eliminated, namely people only consider tax schedules where total tax revenue is either equal to or smaller than current government debt.

Definition 2 (government debt per capita): Define $X \equiv \frac{D}{k+l}$.

Assumption 3 (possible tax schedule): We assume $X < \min_i w_i$ and $X < \min_i (n_i - w_i)$ are satisfied. The government provides possible tax schedules (τ_1, τ_2) that satisfy $\tau_1 + \tau_2 \leq X$, $\tau_1 \in \mathbb{N} \cup \{0\}$ and $\tau_2 \in \mathbb{N} \cup \{0\}$ and people choose their tax schedule by voting, i.e. we exclude the possibility that a budget (strictly) surplus tax schedule be adopted.

Finally, we assume people vote truthfully and do not consider strategic voting.

Let us consider several voting schemes and we can verify how the Borda count method works.

Proposition 2: Assume that the voting agenda is only about today's tax rate. Then, if $l + k_0 >$

$\max_{m\geq 1} k_m$ is satisfied, non-budget-balanced solution $\tau_1 = 0$ is chosen under relative majority voting rule. If $l + k_0 < \max_{m\geq 1} k_m$ is satisfied, $\tau_1 = 0$ is not chosen.

Proof of Proposition 2: Under relative majority rule, policy choice $\tau_1 = 0$ collects $l + k_0$ votes where $\tau_1 = m > 0$ collects k_m votes. Therefore, if $l + k_0 > \max_{m\geq 1} k_m$ is satisfied, non-budget-balanced solution $\tau_1 = 0$ is chosen even if a majority of people are budget-balanced, i.e. $l < k = \sum_{m=0}^{\frac{D}{k+l}} k_m$. It is easy to see that, if $l + k_0 < \max_{m\geq 1} k_m$, $\tau_1 = \bar{m}$ for $\bar{m} = argmax_m k_m$ is adopted. (Q.E.D.)

It can be easily seen that a budget-balanced tax plan is difficult to be adopted if the voting agenda is only for today's tax rate. Therefore, we can consider to take into account the intertemporal aspect in our voting agenda. However, we can see that the budget-balanced tax plan is still difficult to adopt under relative majority voting rule.

Proposition 3: Assume that the voting agenda is the intertemporal tax rate. Then, if $l > \max_m k_m$ is satisfied, non-budget-balanced solution $(\tau_1, \tau_2) = (0,0)$ is chosen under relative majority voting rule. If $l < \max_m k_m$ is satisfied, $(\tau_1, \tau_2) = (0,0)$ is not chosen.

Proof of Proposition 3: Each person votes for $(\tau_1, \tau_2) \in \{(\tau_1, \tau_2) | \tau_1 + \tau_2 \leq \mathrm{X},\ \tau_1 \in \mathbb{N} \cup \{0\}$ and $\tau_2 \in \mathbb{N} \cup \{0\}\}$. Then, $(0,0)$ collects l votes and $(m, \mathrm{X} - m)$ collects k_m votes respectively. If $l > \max_m k_m$ is satisfied, non-budget-balanced solution $(0,0)$ is chosen even if budget-balanced people are majority ($l < k$), in which case the majority of people prefer $(m, \mathrm{X} - m)$ whatever value m takes to non-budget-balanced solution $(0,0)$. It is easy to see that, if $l < \max_m k_m$, $(\tau_1, \tau_2) = (\bar{m}, X - \bar{m})$ for $\bar{m} = argmax_m k_m$ is adopted. (Q.E.D.)

The result of Proposition 3 results from the fact that the budget-balanced people's vote split to multiple choices, $(\tau_1, \tau_2) = (m, \mathrm{X} - m)$ for $m \in [0, X]$. In order to cope with this spoiler effect, we can see that the Borda count method works.

Proposition 4: Under the Borda count method,

(1) *Almost* budget balanced tax plan is *asymptotically* chosen if balanced-budget people are majority ($k > l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small. To be precise, $\forall \varepsilon_1 > 0$, $\forall \varepsilon_2 > 0, \exists X_0$ s.t. $\forall X > X_0$[10]; $\frac{k}{l} > 1 + \varepsilon_1 \Rightarrow$tax

[10] As noted in Assumption 2, this proposition requires either the step size to be sufficiently small or government debt

plan $(\overline{\tau_1}, \overline{\tau_2})$ is chosen where $\overline{\tau_1} + \overline{\tau_2} > (1-\varepsilon_2)X$.

(2) *Almost* non-tax plan is *asymptotically* chosen if non-balanced-budget people are majority ($k < l$) and step size of government debt per capita X defined under Assumption 2 is sufficiently small. To be precise, $\forall \varepsilon_1 > 0$, $\forall \varepsilon_2 > 0, \exists X_0$ s.t. $\forall X > X_0$; $\frac{l}{k} > 1 + \varepsilon_1 \Rightarrow$tax plan $(\overline{\tau_1}, \overline{\tau_2})$ is chosen where $\overline{\tau_1} + \overline{\tau_2} < \varepsilon_2 X$.

(3) If balanced-budget people are $\frac{3}{4}$ majority or more ($k \geq 3l$), budget balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$ with $\overline{\tau_1} + \overline{\tau_2} = X$ is always chosen whatever value X takes.

(4) If non-balanced-budget people are more than $\frac{2}{3}$ majority ($2k < l$), non-tax plan $(0,0)$ is always chosen whatever value X takes.

Proof of Proposition 4: See Appendix 1

This proposition reveals that the Borda count method assures that the majority's opinion is, at least asymptotically, reflected in tax policy. If balanced-budget people are the majority and government debt per capita is sufficiently large, a tax plan close to a budget balanced tax plan is chosen asymptotically. This proposition does not say specifically which tax plan is to be chosen, but says that a tax plan that is approximately budget balanced is to be chosen. If non-balanced-budget people are the majority and government debt per capita is sufficiently large, a tax plan close to a non-tax plan is chosen asymptotically. Moreover, it is proven that, if balanced-budget people are the supermajority (75% majority), a budget balanced tax plan is always chosen and if non-balanced-budget people are the supermajority (66.7% majority), a non-tax plan is always chosen.

One may complain that, in actual political campaigns, not all possible policy choices are chosen as the agenda, and thus Proposition 4 in which all possible tax policies are on the agenda is unrealistic. Realistically, we can consider a case where some policy choices are selected as the agenda. In such a case, we can show that the majority's opinion is *almost surely* reflected in tax policy. To be precise, if the number of policy choices on the agenda is sufficiently small, the majority's opinion is almost surely reflected in tax policy (Proposition 5). If the number of policy choices on the agenda is sufficiently large, similar to Proposition 4, the opinion of the supermajority of people (75% or 66.7%) is reflected in tax policy (Proposition 6). These results reinforce our message provided in Proposition 4.

Proposition 5: Consider a case where, for sufficiently small N, N tax policies are on the agenda.

per capita X to be sufficiently large.

At least a budget balanced tax policy and non-tax policy are included on this agenda. Under the Borda count method,

(1) Budget balanced tax plan is *almost surely* chosen if balanced-budget people are majority ($k > l$), policy choices are randomly distributed, and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

To be precise, assume there are $N \geq 3$ policy choices where at least one policy is budget balanced ($\tau'_1 + \tau'_2 = X$) and one policy is non-tax plan ($(\tau_1, \tau_2) = (0,0)$) and other choices are uniformly distributed on $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. For sufficiently small N, $\forall \varepsilon > 0$, $\exists X_0$ s.t. $\forall X > X_0$; budget balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$ with $\overline{\tau_1} + \overline{\tau_2} = X$ is chosen with probability $1 - \varepsilon$ or more.

(2) A non-tax plan is *almost surely* chosen if non-balanced-budget people are majority ($k < l$), policy choices are randomly distributed, and step size of government debt per capita X defined under Assumption 2 is sufficiently small.

To be precise, assume there are $N \geq 3$ policy choices where at least one policy is budget balanced ($\tau'_1 + \tau'_2 = X$) and one policy is non-tax plan ($(\tau_1, \tau_2) = (0,0)$) and other choices are uniformly distributed on $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. For sufficiently small N, $\forall \varepsilon > 0$, $\exists X_0$ s.t. $\forall X > X_0$; non-tax plan (0,0) is chosen with probability $1 - \varepsilon$ or more.

Proof of Proposition 5: See Appendix 2

Proposition 5 holds for a sufficiently small number of policy choices on the agenda. If there is a sufficiently large number of policy choices on the agenda, the following proposition holds.

Proposition 6: Consider a case where, for sufficiently large N, N tax policies are on agenda. Assume that policy choices on the agenda include at least a budget balanced policy ($\tau'_1 + \tau'_2 = X$) and non-tax policy ($(\tau_1, \tau_2) = (0,0)$), and assume that other choices are uniformly distributed on $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. Under the Borda count method,

(1) *Almost* budget balanced tax plan is *asymptotically* and *almost surely* chosen if balanced-budget people are majority ($k > l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

(2) *Almost* non-tax plan is *asymptotically* and *almost surely* chosen if non-balanced-budget people are majority ($k < l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

(3) If balanced-budget people are $\frac{3}{4}$ majority or more ($k \geq 3l$), budget balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$

with $\overline{\tau_1} + \overline{\tau_2} = X$ is *almost surely* chosen whatever value and stem size X takes.
(4) If non-balanced-budget people are more than $\frac{2}{3}$ majority ($2k < l$), non-tax plan $(0,0)$ is *almost surely* chosen whatever value and stem size X takes.

Proof of Proposition 6: See Appendix 3

Integrating these results, the majority's opinion is *asymptotically* and *almost surely* reflected in tax policy under the Borda count method, both when N is sufficiently large and when N is sufficiently small. These results represent the superiority of the Borda count method for representing people's preferences.

4. Conclusion

The current voting system is not one-size-fits-all. If policy choices do not incorporate intertemporal aspect, voters have little chance to express their real preferences. Also, in reality, the relative majority rule is, although criticized in many ways, widely implemented. However, this voting rule is vulnerable to voting split. If there are many choices with similar ideology, these choices are difficult to be chosen under relative majority rule. Even if policy choices incorporate intertemporal aspects, the relative majority voting rule might not reflect voter preferences, which could be remedied by the Borda count method.

Our paper does not intend to refute the existing explanations why there is a tendency for budget deficits. Our paper intends to present another explanation that widely implemented voting systems might have a tendency to bring about budget deficits.

Our paper only considers two types of voters; balanced-budget people and non-balanced-budget people. Further research may consider other types of voters, such as semi-balanced-budget people.

A caveat of our paper is that one of our main results is valid if balanced-budget people are the majority, which is not obvious at all. Another caveat of our paper is that it assumes truthful-voting and excludes the possibility of strategic-voting. Considering the possibility of strategic-voting may polish this paper in a theoretical way.

Appendix 1 (Proof of Proposition 4)

Each person votes for $(\tau_1, \tau_2) \in \{(\tau_1, \tau_2) | \tau_1 + \tau_2 \leq \mathrm{X},\ \tau_1 \in \mathbb{N} \cup \{0\}$ and $\tau_2 \in \mathbb{N} \cup \{0\}\}$ by Borda voting.

(a) First, we calculate how many points non-tax plan $(0,0)$ collects. Non-balanced-budget people give maximum points $\frac{(X+1)(X+2)}{2}$ on non-tax plan $(0,0)$. Therefore, it collects $\frac{(X+1)(X+2)}{2} l$ points from non-balanced-budget people. Balanced-budget people give a minimum of 1 point on non-tax plan $(0,0)$. Therefore, it collects k points from balanced-budget people. In sum, non-tax plan $(0,0)$ collects $k + \frac{(X+1)(X+2)}{2} l$ points.

(b) Second, we calculate how many points budget balanced tax plans (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ collects *on average*. Non-balanced-budget people give 1 point to $X+1$ points respectively on budget balanced tax plans. Therefore, they collect $\frac{(X+1)(X+2)}{2} l$ points in total from non-balanced-budget people. Balanced-budget people give $\frac{(X+1)(X+2)}{2}$ points to $\frac{(X+1)(X+2)}{2} - X$ points respectively on budget balanced tax plans. Therefore, they collect $(X+1)\frac{(X+1)(X+2)-X}{2} k$ points in total from balanced-budget people. In sum, budget balanced tax plans collect $(X+1)\frac{(X+1)(X+2)-X}{2} k + \frac{(X+1)(X+2)}{2} l$ points in total and $\frac{(X+1)(X+2)-X}{2} k + \frac{X+2}{2} l = \frac{X^2+2X+2}{2} k + \frac{X+2}{2} l$ points *on average*.

(c) Third, we calculate how many points a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ can collect *at most*. Non-balanced-budget people strictly prefer any tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 < p$ to a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p$. Therefore, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ collects *at most* $(X+1) + X + \cdots + (p+1) = \frac{(X+p+2)(X-p+1)}{2}$ points from a non-balanced-budget person. Balanced-budget people strictly prefer any tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 > p$ to a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p$. Therefore, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ collects *at most* $1 + 2 + \cdots + (p+1) = \frac{(p+1)(p+2)}{2}$ points from a balanced-budget person. In sum, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ can collect *at most* $\frac{(p+1)(p+2)}{2} k + \frac{(X+p+2)(X-p+1)}{2} l$ points.

(d) Consider a case where balanced-budget people are majority ($k > l$). As $\frac{X^2+2X+2}{2}k + \frac{X+2}{2}l > k + \frac{(X+1)(X+2)}{2}l$ holds, non-tax plan $(0,0)$ is never chosen. If a balanced tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ is chosen, (1) and (3) are already proven. Therefore, we will focus on a case where tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ is chosen. In such a case, $\frac{(p+1)(p+2)}{2}k + \frac{(X+p+2)(X-p+1)}{2}l > \frac{X^2+2X+2}{2}k + \frac{X+2}{2}l$ is necessary. Equivalently, $\frac{X^2+2X-(p^2+p)}{2}l > \frac{X^2+2X-(p^2+3p)}{2}k \Leftrightarrow \frac{X^2+2X-(p^2+p)}{X^2+2X-(p^2+3p)} > \frac{k}{l} \Leftrightarrow 1 + \frac{2p}{X^2+2X-(p^2+3p)} > \frac{k}{l}$ is necessary. Note that $\frac{2p}{X^2+2X-(p^2+3p)} = 0$ when $p = 0$ and $\frac{2p}{X^2+2X-(p^2+3p)}$ is a strictly increasing function with p. Let $\varepsilon_1 \equiv \frac{k}{l} - 1$ and $\varepsilon_2 \equiv 1 - \frac{p}{X}$. Then, a necessary condition that a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X)$ is chosen is that $\frac{2(1-\varepsilon_2)}{(2\varepsilon_2 - \varepsilon_2^2)X + 3\varepsilon_2 - 1} > \varepsilon_1$ holds. This inequality shows that, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, sufficiently large X prevents this inequality to hold. In other words, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, if X is sufficiently large, $\tau_1 + \tau_2$ must be greater than $(1 - \varepsilon_2)X$ to be chosen. The right hand side of $\frac{2(1-\varepsilon_2)}{(2\varepsilon_2 - \varepsilon_2^2)X + 3\varepsilon_2 - 1} > \varepsilon_1$ is increasing in ε_1 and the left hand side of it is decreasing in ε_2. Thus, (1) is proven and *almost* balanced budget tax plan is always *asymptotically* chosen if government debt per capita X is sufficiently large.

(e) Continue considering a case where balanced-budget people are majority ($k > l$). (3) is proven because $\max_{p \in (0,X)} \left[1 + \frac{2p}{X^2+2X-(p^2+3p)}\right] = \left[1 + \frac{2p}{X^2+2X-(p^2+3p)}\right]_{p=X-1} = 1 + \frac{2(X-1)}{X+2} = \frac{3X}{X+2} < 3$ holds. If $\frac{k}{l} \geq 3$, the above finding implies that inequality $1 + \frac{2p}{X^2+2X-(p^2+3p)} > \frac{k}{l}$ never hold. Therefore, a balanced tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ must be chosen in this case.

(f) Next, consider a case where non-balanced-budget people are majority ($k < l$). If a non-tax plan $(0,0)$ is chosen, (2) is already proven. Therefore, we will focus on a case where tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ is chosen. In such a case, $\frac{(p+1)(p+2)}{2}k + \frac{(X+p+2)(X-p+1)}{2}l > k + \frac{(X+1)(X+2)}{2}l$ is necessary. Equivalently, $\frac{p^2+3p}{2}k > \frac{p^2+p}{2}l \Leftrightarrow \frac{l}{k} < 1 + \frac{2}{p+1}$ is necessary. Note that $\frac{2}{p+1}$ is a strictly decreasing function with p.

Let $\varepsilon_1 \equiv \frac{l}{k} - 1$ and $\varepsilon_2 \equiv \frac{p}{X}$. Then, a necessary condition that a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ is chosen is that $\varepsilon_1 < \frac{2}{\varepsilon_2 X + 1}$ holds. This inequality shows that, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, sufficiently large X prevents this inequality to hold. In other words, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, if X is sufficiently large, $\tau_1 + \tau_2$ must be smaller than $\varepsilon_2 X$ to be chosen. The right-hand side of $\varepsilon_1 < \frac{2}{\varepsilon_2 X + 1}$ is decreasing in ε_2 and the left-hand side of it is increasing in ε_1. Thus, (2) is proven and *almost* non-tax plan is always *asymptotically* chosen if government debt per capita X is sufficiently large.

(g) Continue considering a case where non-balanced-budget people are majority ($k < l$). (4) is proven because $\max_{p \in (0,X]} \left[1 + \frac{2}{p+1}\right] = \left[1 + \frac{2}{p+1}\right]_{p=1} = 2$ holds. If $\frac{l}{k} > 2$, the above finding implies that inequality $\frac{l}{k} < 1 + \frac{2}{p+1}$ never hold. Therefore, non-tax plan $(0,0)$ must be chosen in this case.

(Q.E.D.)

Appendix 2 (Proof of Proposition 5)

Let p satisfy $p \in \mathbb{N}$ and $0 < p \leq X$. Let $T \subset \{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}\}$ be N tax policies on the agenda and Δ_p be number of tax policies on the agenda which satisfy $\tau_1 + \tau_2 = p$. Note that $\sum_{p=1}^{X} \Delta_p = N - 1$ holds because there is a tax policy $(0,0)$ which is not included in $\sum_{p=1}^{X} \Delta_p = N - 1$. It is easily proven that $\mathrm{Prob}\{\exists p, \Delta_p \geq 2\} \xrightarrow[X \to \infty]{} +0$[11]. Therefore, it is almost sure that either $\Delta_p = 0$ or $\Delta_p = 1$ holds for all p $(0 < p \leq X)$.

[11] [sketch of proof] Consider a case where a budget balanced tax policy $({\tau'}_1, {\tau'}_2)$ and non-tax policy $(0,0)$ are given a priori and other $N - 2$ policy choices(τ_1^1, τ_2^1), (τ_1^2, τ_2^2), $\cdots$, $(\tau_1^{N-2}, \tau_2^{N-2})$ are allocated one by one. There are $E = \frac{(X+1)(X+2)}{2} - 2$ choices to be selected. Before allocating these choices, $\forall p \in (0, X)$; $\Delta_p = 0$ and $\Delta_X = 1$ hold. The first policy does not make any Δ_p greater than one with probability $\frac{E-X}{E}$ (Among E choices, only X budget balanced choices make Δ_p to be greater than 1.). The second choice does not make any Δ_p greater than one with probability $\frac{(E-1)-X-(X-1)}{E-1}$ or more. The third choice does not make any Δ_p greater than one with probability $\frac{(E-2)-X-(X-1)-(X-2)}{E-2}$ or more.$\cdots$ N-2th choice does not make any Δ_p greater than one with probability $\frac{\{E-(N-3)\}-X-(X-1)-(X-2)-\cdots-\{X-(N-3)\}}{E-(N-3)}$ or more. Therefore, as N is assumed to be sufficiently small, $1 - \mathrm{Prob}\{\exists p, \Delta_p \geq 2\} \geq \frac{E-X}{E} \cdot \frac{(E-1)-X-(X-1)}{E-1} \cdot \frac{(E-2)-X-(X-1)-(X-2)}{E-2} \cdots \frac{\{E-(N-3)\}-X-(X-1)-(X-2)-\cdots-\{X-(N-3)\}}{E-(N-3)} \xrightarrow[\substack{N\ small \\ X \to \infty}]{} 1$ and thus $\mathrm{Prob}\{\exists p, \Delta_p \geq 2\}$ is asymptotically zero if X is sufficiently large.

The budget balanced tax policy collects N points from budget-balanced people in total and 1 point from non-budget-balanced people in total. Therefore, the budget balanced tax policy collects $Nk + l$ points.

Consider any tax policy with $\tau_1 + \tau_2 = p < X$. Since it is almost sure that either $\Delta_p = 0$ or $\Delta_p = 1$ holds for all p $(0 < p < X)$, if this tax policy is t^{th} preferred by budget balanced people, it is $N + 1 - t^{\text{th}}$ preferred by non-budget-balanced people. Therefore, it collects $(N + 1 - t)k + tl$ points. It is obvious that $t \geq 2$ holds.

As a special case, non-tax policy collects $k + Nl$ points.

As $Nk + l > (N + 1 - t)k + tl$ always holds for $k > l$, statement (1) holds. Also, as $Nk + l < k + Nl$ as well as $(N + 1 - t)k + tl < k + Nl$ holds for $t \leq N - 1$ when $k < l$, statement (2) holds.

(Q.E.D.)

Appendix 3 (Proof of Proposition 6)

Assume that N is sufficiently large so that law of large numbers can be applied. Then, it is *almost sure* that there are *approximately* $\frac{N}{\frac{(X+1)(X+2)}{2}}(Q + 1) = \frac{2(Q+1)N}{(X+1)(X+2)}$ policies that satisfy $\tau_1 + \tau_2 = Q$.

As a special case, substituting $Q = X$ and we can show that there are approximately $\frac{2N}{X+2}$ budget balanced policies.

(a) First, we calculate how many points non-tax plan $(0,0)$ collects. Non-balanced-budget people give maximum points N on non-tax plan $(0,0)$. Therefore, it collects Nl points from non-balanced-budget people. Balanced-budget people give minimum 1 point on non-tax plan $(0,0)$. Therefore, it collects k points from balanced-budget people. In sum, non-tax plan $(0,0)$ collects $k + Nl$ points.

(b) Second, we calculate how many points budget balanced tax plans (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ collects *on average*. Non-balanced-budget people give 1 point to $\frac{2N}{X+2}$ points respectively on budget balanced tax plans. Therefore, they collect $\frac{\frac{2N}{X+2}\left(\frac{2N}{X+2}+1\right)}{2}l$ points in total from non-balanced-budget people. Balanced-budget people give N points to $\frac{\frac{2N}{X+2}\left(2N-\frac{2N}{X+2}+1\right)}{2}$ points respectively on budget balanced tax plans. Therefore, they collect $\frac{\frac{2N}{X+2}\left(2N-\frac{2N}{X+2}+1\right)}{2}k$ points in total from balanced-budget people. In sum, budget balanced tax plans collect $\frac{N}{X+2}\left\{\left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l\right\}$ points in total and $\left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l$

points *on average*.

(c) Third, we calculate how many points a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ can collect *at most*. Non-balanced-budget people strictly prefer any tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 < p$ to a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p$. Therefore, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ collects *at most* $\sum_{Q'=Q}^{X} \frac{2(Q'+1)N}{(X+1)(X+2)} = \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}$ points from a non-balanced-budget person. Balanced-budget people strictly prefer any tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 > p$ to a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p$. Therefore, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ collects *at most* $\sum_{Q'=0}^{Q} \frac{2(Q'+1)N}{(X+1)(X+2)} = \frac{(Q+1)(Q+2)N}{(X+1)(X+2)}$ points from a balanced-budget person. In sum, a tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = p \in (0, X]$ can collect *at most* $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l$ points.

(d) Consider a case where balanced-budget people are majority ($k > l$). As $\left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l > k + Nl$ holds, non-tax plan (0,0) is never chosen. If a balanced tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ is chosen, (1) is already proven. Therefore, we will focus on a case where tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = Q \in (0, X)$ is chosen. In such a case, $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l > \left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l$ is necessary. Equivalently, $\frac{k}{l} < \frac{2N(X+1)(X+2)-(X+1)(X+2)-2N(X+1)-2NQ(Q+1)}{2N(X+1)(X+2)+(X+1)(X+2)-2N(X+1)-2N(Q+1)(Q+2)}$ is necessary. Let $\varepsilon_1 \equiv \frac{k}{l} - 1$ and $\varepsilon_2 \equiv 1 - \frac{Q}{X}$. Whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for sufficiently large N, sufficiently large X prevents this inequality to hold[12]. In other words, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for sufficiently large N, if X is sufficiently large, $\tau_1 + \tau_2$ must be greater than $(1 - \varepsilon_2)X$ to be chosen. Thus, (1) is proven.

(e) Next, consider a case where non-balanced-budget people are majority ($k < l$). If a non-tax plan (0,0) is chosen, (2) is already proven. Therefore, we will focus on a case where tax plan (τ_1, τ_2) with $\tau_1 + \tau_2 = Q \in (0, X]$ is chosen. In such a case, $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l > k + Nl$ is necessary. Equivalently,

[12] Say, N takes its maximum value $\frac{(X+1)(X+2)}{2}$ Then, the inequality is deduced to $\frac{k}{l} < \frac{X^2+2X-Q^2-Q}{X^2+2X-Q^2-3Q}$. Since $\varepsilon_2 \equiv 1 - \frac{Q}{X}$, this inequality is equivalent to $\frac{k}{l} < \frac{(2\varepsilon_2-\varepsilon_2^2)X+1+\varepsilon_2}{(2\varepsilon_2-\varepsilon_2^2)X-1+3\varepsilon_2}$. Sufficiently large X prevents this inequality to hold. Identical discussion holds for sufficiently large N.

$\frac{k}{l} > \frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)}$ is necessary. Let $\varepsilon_1 \equiv \frac{l}{k} - 1$ and $\varepsilon_2 \equiv \frac{Q}{X}$. Whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for sufficiently large N, sufficiently large X prevents this inequality to hold[13]. In other words, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for sufficiently large N, if X is sufficiently large, $\tau_1 + \tau_2$ must be smaller than $\varepsilon_2 X$ to be chosen. Thus, (2) is proven.

(f) The sufficient condition that a budget balanced tax policy is adopted is $\forall Q \in (0, X); \left(N - \frac{N}{X+2} + \frac{1}{2}\right) k + \left(\frac{N}{X+2} + \frac{1}{2}\right) l > \frac{(Q+1)(Q+2)N}{(X+1)(X+2)} k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)} l \wedge \left(N - \frac{N}{X+2} + \frac{1}{2}\right) k + \left(\frac{N}{X+2} + \frac{1}{2}\right) l > k + Nl$. By simple calculation, this condition is equivalent to $\forall Q \in (0, X); \frac{k}{l} > \frac{2N(X+1)(X+2)-(X+1)(X+2)-2N(X+1)-2NQ(Q+1)}{2N(X+1)(X+2)+(X+1)(X+2)-2N(X+1)-2N(Q+1)(Q+2)} \wedge k > l$. For sufficiently large N, this condition is equivalent to $\frac{k}{l} > \frac{2N(3X+1)-(X+1)(X+2)}{2N(X+1)+(X+1)(X+2)} \wedge k > l$ and thus $\frac{k}{l} \geq 3$ is a sufficient condition that a budget balanced tax policy is chosen. Therefore, (3) is proven.

(g) The sufficient condition that non-tax policy is adopted is $\forall Q \in (0, X]; \frac{(Q+1)(Q+2)N}{(X+1)(X+2)} k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)} l < k + Nl$. By simple calculation, this condition is equivalent to $\forall Q \in (0, X]; \frac{k}{l} < \frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)}$. Since $\min_{Q \geq 1, N, X} \frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)} = \frac{1}{2}$ holds, $\frac{k}{l} < \frac{1}{2}$ is a sufficient condition that non-tax policy is chosen. Therefore, (4) is proven.

(Q.E.D.)

[13] Say, N takes its maximum value $\frac{(X+1)(X+2)}{2}$ Then, the inequality is deduced to $\frac{k}{l} > \frac{Q+1}{Q+3}$. Since $\varepsilon_2 \equiv \frac{Q}{X}$, this inequality is equivalent to $\frac{k}{l} > \frac{\varepsilon_2 X+1}{\varepsilon_2 X+3}$. Sufficiently large X prevents this inequality to hold. Identical discussion holds for sufficiently large N.

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