Supply Chain Disruptions:
Evidence from the Great East Japan Earthquake

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Abstract

Exploiting the exogenous and regional nature of the Great East Japan Earthquake of 2011, this paper provides a systematic quantification of the role of input-output linkages as a mechanism for the propagation and amplification of shocks. We document that the disruption caused by the earthquake and its aftermaths propagated upstream and downstream supply chains, affecting the direct and indirect suppliers and customers of disaster-stricken firms. We then use our empirical findings to obtain an estimate for the overall macroeconomic impact of the shock by taking these propagation effects into account. We find that the propagation of the shock over input-output linkages can account for a 1.2 percentage point decline in Japan's gross output in the year following the earthquake. We interpret these findings in the context of a general equilibrium model that takes the firm-to-firm linkages into account explicitly.

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1 Introduction

The production of goods and services in any modern economy is organized around complex, interlocking supply chains, as firms rely on a variety of different inputs for production. The sheer scale of transactions along supply chains attests to their vital role in the economy. For instance, in 2007 alone, firms in the United States spent over 12 trillion dollars on various intermediate goods and services, an amount of the same order of magnitude of the annual U.S. GDP (Streitwieser, 2009).

Due to the key role of intermediate goods in the production process, disruptions to the orderly flow of goods and services have been increasingly recognized by policymakers as a source of aggregate risk. Overlapping policy initiatives at the international (World Economic Forum, 2012), regional (European Commission, 2013), and national levels rely on the premise that firm-level or regional shocks — such as natural disasters, terrorism, or cyber attacks — can propagate through input-output linkages to a wide array of firms and industries, with potentially adverse macroeconomic impacts. For example, the U.S. National Strategy for Global Supply Chain Security issued in January 2012 is based on the premise that supply chain linkages “serve to propagate risk that arises from a local or regional disruption across a wide geographic area,” which in turn “can adversely impact global economic growth and productivity” (The White House, 2012). In parallel, a growing academic literature has explored whether the presence of supply chain linkages can translate microeconomic shocks into aggregate, business cycle fluctuations.

Despite the interest of academics and policymakers alike, evidence on the role of input-output linkages as a channel for the propagation of shocks and a source of macroeconomic risk has been scant. In large part, this shortcoming reflects the dual challenge of identifying plausible exogenous micro shocks in firm-level data and tracing their impact as they spread throughout the economy.

In this paper, we provide a systematic quantification of the role of input-output linkages as a mechanism for propagation and amplification of shocks by exploiting a large, but localized, natural disaster — namely, the Great East Japan Earthquake of 2011. Relying on information on firms’ locations, we exploit the heterogeneous exposure of Japanese firms to the earthquake to obtain measures of firm-level disturbances. We then combine this information with extensive micro-data on inter-firm transactions to trace and quantify the extent of shock propagation along supply chains. This analysis also enables us to obtain an estimate for the overall macroeconomic impact of the shock on the Japanese economy (above and beyond the impact on the firms directly exposed to the shock) by taking these propagation effects into account.

To guide our empirical analysis, we begin by developing a theoretical framework in the spirit of Long and Plosser (1983) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) that explicitly takes the inter-firm input-output linkages into account. The model, which allows for

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1 The much publicized, if anecdotal, reports of disruptions to supply chains following the 2011 earthquake were in fact one of the major triggers for the above described policy initiatives. For example, a few weeks after the earthquake, Reuters reported that “supply chain disruptions in Japan have forced at least one global automaker to delay the launch of two new models and are forcing other industries to shutter plants. … The automaker is just one of dozens, if not hundreds, of Japanese manufacturers facing disruptions to their supply chains as a result of the quake, the subsequent tsunami and a still-unresolved nuclear threat” (Reuters, 2011).
general substitution patterns both within intermediate inputs and across intermediate inputs and primary factors (such as labor), provides us with sharp predictions for the nature and the extent of propagation of microeconomic shocks in the economy. In particular, our characterization results establish that, regardless of parameter values, a negative shock to a given firm propagates downstream, reducing the output of not only the disrupted firm’s immediate customers, but also its customers’ customers and so on. We then show that firm-level shocks also propagate upstream to the firm’s direct and indirect suppliers. Unlike downstream propagation, however, a negative shock can lead to either positive or negative upstream propagation depending on the elasticity parameters. Nonetheless, our model predicts that, regardless of its sign, the intensity of upstream propagation is always weaker than that of downstream propagation. Our theoretical results also establish that both upstream and downstream propagation effects attenuate as the shock travels over the supply chain. Finally, our results provide a characterization of the conditions under which shocks may propagate “horizontally” to firms who share common customers with the distressed firms.

Proceeding to our empirical analysis, we exploit two key features of the March 2011 earthquake in Japan. First, the large-scale destruction caused by the earthquake (which was followed by a massive tsunami and the failure of the Fukushima Dai-ichi Nuclear Power Plant) had a significant negative impact on the economic performance of the affected areas: the real GDP growth rate of the four most severely affected prefectures was 3.1 percentage points lower in the 2011 Japanese fiscal year (April 2011–March 2012) than the previous year. Second, despite their large impact on the coastal areas, the earthquake and its aftermaths were essentially local, regional shocks that directly affected only a small fraction of the Japanese economy, with the four affected prefectures accounting for only 4.7% of the aggregate Japanese output. These two features, together with the exogenous nature of the earthquake, provide us with a natural experiment in which a small subset of firms were exposed to a large negative shock.2

To assess the role of input-output linkages as a mechanism for propagation of the shock throughout the economy, we use a proprietary dataset compiled by a major private credit reporting agency. The raw dataset contains information on roughly half of all private and publicly traded firms in Japan, covering almost all firms with more than five employees across all sectors of the economy. For each firm-year, we observe a set of firm-level covariates as well as the identities of the firm’s suppliers and customers, thus enabling us to construct the network of supply chain relationships for the firms in the sample. We then combine this dataset with information on firms’ headquarters locations to identify the set of firms that were directly exposed to the shock.

Based on this information, we examine whether the presence of direct and indirect input-output linkages to firms in the disaster-stricken areas had an impact on firms’ performance in the year after the earthquake. In particular, we compare the post-earthquake sales growth rates of firms at different distances — in the supply chain network sense — from the disaster area firms to a

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2Taken at face value, these figures also suggest that, solely based on the economic size of the affected areas, the earthquake can account for a $3.1 \times 0.047 \approx 0.15$ percentage point decline in aggregate real GDP growth in Japan. In comparison, the actual decline in Japan’s real GDP growth rate in the 2011 fiscal year was more than five times as large (around 0.8 percentage points).
control group of firms that were relatively more distant. We find significant evidence of both downstream and upstream propagation of the shock: supply-disrupted firms (i.e., firms with at least one earthquake-hit supplier) under-performed the control group by 2 percentage points in the year following the earthquake, while demand-disrupted firms (i.e., firms with at least one customer in the disaster area), experienced a 1.2 percentage point decline in growth compared to the control group. Our estimates also indicate that the disruption caused by the earthquake led to significant indirect propagation, not only affecting the disrupted firms’ immediate transaction partners, but also their customers’ customers, their suppliers’ suppliers, and so on. In line with our model’s prediction, we find that the intensity of both downstream and upstream propagation effects attenuate as the shock travels over the supply chain: the disaster-stricken firms’ customers’ customers experienced a 1.3 percentage point reduction in sales growth (compared to a 2 percentage point reduction for the immediate customers), while their suppliers’ suppliers underperformed the control group by 0.7 percentage points (compared to a 1.2 percentage point reduction for the immediate suppliers). We find similar qualitative effects for (indirect) suppliers and customers more distant from the source of the shock. These estimates also indicate that, consistent with our model’s prediction, the intensity of the upstream propagation effect is weaker than that of the downstream effect. Finally, we find no evidence for economically or statistically significant horizontal propagation patterns.

We then verify that our main empirical findings are not driven by pre-earthquake dynamics and are robust to a range of alternative specifications and controls. Importantly, we establish that the effects identified in our baseline results are not due to the disruption in the supply of electricity in the aftermath of the tsunami and the nuclear disaster. In particular, we use the fact that Japan’s power grid consists of two (almost perfectly) isolated sub-grids with very little possibility of power transmission across the divide and verify that all our estimates remain unchanged when we restrict our analysis to the subset of firms located in the western half of the country, which did not experience electricity disruptions.

We conclude the paper by using our empirical results on the extent of propagations to obtain an estimate for the overall macroeconomic impact of the earthquake and its aftermaths on the Japanese economy. We find that the propagation of the shock over input-output linkages can account for a decline of 1.2 percentage points in Japan’s gross output in the year following the earthquake. Our estimates also suggest that the indirect propagation of the shock to firms with no direct linkages to disaster area firms is quantitatively as important as the shock’s impact on disaster-stricken firms’ immediate transaction partners.

Overall, our empirical findings provide substantial evidence for the role of input-output linkages as an important mechanism for the propagation and amplification of shocks. They also provide a detailed picture of the nature and intensity of this propagation, suggesting that input-output linkages can play a quantitatively non-trivial role in translating firm-level disturbances into sizable fluctuations at the aggregate level.
Related Literature  Our paper is most closely related to the recent collection of papers, such as Acemoglu et al. (2012, 2016b), Baqee (2016), and Bigio and La’O (2016), that emphasizes the role of input-output linkages as a mechanism for propagation and amplification of shocks.\(^3\) The importance of this mechanism has also been increasingly recognized by the trade literature, with papers such as Johnson (2014) and di Giovanni, Levchenko, and Méjean (2015) arguing that direct trade and multinational linkages can lead to business cycle comovements across countries. Closer to our paper, Caliendo, Rossi-Hansberg, Parro, and Sarte (2016) study the role of intersectoral and interregional trade linkages in propagating disaggregated productivity changes across U.S. states.\(^4\)

Our theoretical framework is most closely related to Baqee (2016), who, using a multi-sector model similar to ours, shows that the extensive margin of firm entry can further amplify shocks. In contrast, the main focus of our theoretical results is to provide a characterization of how shocks to a given firm impact the output of other firms as a function of the economy’s input-output linkages and the corresponding elasticities. Our characterization results provide sharp predictions for the nature and extent of upstream and downstream propagation in the economy.

Despite its theoretical plausibility, credible identification of the role of input-output linkages as a propagation mechanism has remained largely unexplored. While recent empirical works, such as Foerster, Sarte, and Watson (2011), di Giovanni, Levchenko, and Méjean (2014), and Atalay (2015), investigate the role of input-output linkages in the propagation of microeconomic shocks, they invariably rely on strong identifying assumptions for backing out the shocks from data.

Two exceptions are the contemporaneous works of Barrot and Sauvagnat (2016) and Boehm, Flaaen, and Pandalai-Nayar (2016), who, in exercises similar to ours, leverage natural disasters to study the role of firm-level linkages in propagating input disruptions. Combining county-level data on the occurrence of natural disasters in the U.S. with Compustat data on the identity of customers of large and publicly traded firms, Barrot and Sauvagnat (2016) find that shocks to suppliers impose substantial output losses on their direct customers. They also document that such shocks propagate horizontally to firms that share common customers with the disrupted firms, though only when the latter produce relation-specific inputs that are not easily substitutable. Relatedly, Boehm et al. (2016) provide evidence for cross-country transmission of shocks by documenting that American affiliates of Japanese multinationals suffered large drops in output in the months following the 2011 earthquake in Japan.\(^5\) We contribute to this literature by exploiting the much more detailed nature of

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\(^3\)These works may in turn be placed in the larger literature that studies the microeconomic origins of aggregate fluctuations. The literature includes early contributions by Jovanovic (1987) and Durlauf (1993); papers such as Horvath (1998, 2000), Dupor (1999), Carvalho (2010), and Jones (2013), studying the role of linkages in propagating shocks and distortions; and Gabaix (2011), Amiti and Weinstein (2013), and Carvalho and Gabaix (2013), emphasizing the role of firm size distribution in translating micro shocks into macro fluctuations. Relatedly, Nirei (2006, 2015) argues that if firm-level investments are lumpy and strategic complements, micro shocks can have non-trivial aggregate implications, even with no heterogeneity in firm size distribution. See Carvalho (2014) and Gabaix (2016) for detailed surveys of this literature. Also see Acemoglu, Ozdaglar, and Tahbaz-Salehi (2016c), who provide a unified, reduced-form framework for the role of network interactions as a propagation mechanism.

\(^4\)Within the finance literature, a small, but growing body of papers investigates the relationship between firms’ positions in production networks and their stock returns. For instance, Cohen and Frazzini (2008) find evidence of return predictability across economically linked firms, while Kelly, Lustig, and Van Nieuwerburgh (2013) study how concentration in the network of input-output linkages impacts firm-level return volatility. Also see Ahern (2013) and Herskovic (2015).

\(^5\)Also see Acemoglu, Akcigit, and Kerr (2016a), who investigate the propagation of various types of shocks over the U.S.
firm-level input-output linkages and obtaining a more complete picture of the propagation patterns and their macroeconomic implications. In particular, we provide evidence for the propagation of the natural disaster shock both upstream and downstream the supply chain, as well as to firms that were only indirectly linked to the disaster-stricken firms. In addition, the large scale of our study at the national level (alongside its focus on both private and publicly traded firms) enables us to provide an estimate for the overall macroeconomic impact of the earthquake shock as it propagated throughout the Japanese economy.

Our paper is also related to several recent works, such as Noy (2009), Raddatz (2009), and Strobl (2012), that study the macroeconomic impacts of natural disasters. In line with these papers, we find substantial evidence that the earthquake shock had a negative and significant effect on aggregate Japanese output. Relatedly, two recent papers by Schnell and Weinstein (2012) and Cavallo, Cavallo, and Rigobon (2014) perform comparative analyses of the impact of various natural disasters. Comparing the 2011 Great East Japan Earthquake to the 2010 earthquake in Chile, Cavallo et al. (2014) conclude that the pricing behavior and product stockout patterns across Japanese retailers are consistent with a supply-side disruption. Schnell and Weinstein (2012), on the other hand, compare the impact of the 2011 earthquake to that of the 1995 earthquake in Kobe and argue that the 2011 earthquake's much more long-lasting impact on industrial production is linked to the substantial and persistent drop in energy output in the wake of the earthquake. We provide an alternative, and complementary, explanation for the decline in Japanese output in the year immediately after the disaster, highlighting the key role of supply chain linkages as a transmission mechanism in the economy.

Supply chain disruptions have also been studied extensively by the operations management literature. On the theoretical side, papers such as Tomlin (2006, 2009), Yang, Aydin, Babich, and Beil (2009), and Bakshi and Mohan (2016) study how the extent of disruptions are shaped by firms’ sourcing and inventory decisions; Simchi-Levi et al. (2015, 2016) investigate how process flexibility and inventory management can mitigate the adverse effects of disruptions on the production process; and Gao et al. (2016) propose a risk exposure index to assess the impact of supply chain disruptions on lost sales. On the empirical side, papers such as Cachon, Randall, and Schmidt (2007), Birge and Wu (2014), Jain, Girotra, and Netessine (2016), and Osadchiy, Gaur, and Seshadri (2016) document various stylized facts on the relationship between firms’ supply chains and a variety of firm-level outcomes. We complement these studies by leveraging the exogenous nature of the natural disaster to identify firm-level shocks and provide causal evidence for the role of supply chains in transmitting shocks beyond the initially affected firms.

Finally, our paper is related to the small literature that analyzes the structure and geographical features of firm-level production networks. Atalay, Hortacsu, Roberts, and Syverson (2011) use yearly firm-level data from Compustat to characterize the buyer-supplier network of the U.S. economy, input-output network at the sectoral-level. We, in contrast, document the role of supply chain linkages in the transmission of shocks at the firm-level. Exploring the effects of business networks and inter-firm linkages in propagating business-relevant information, Cai and Szeidl (2016) run a large scale field experiment and find that business meetings increase firm sales and profits, employment, productivity, and the number of business partners.
whereas Saito, Watanabe, and Iwamura (2007), Ohnishi, Takayasu, and Takayasu (2010), and Saito (2013) offer a detailed overview of the firm-level production network in Japan. Within this literature, our work is closely related to Bernard, Moxnes, and Saito (2016), who, using the same dataset as ours, document that the opening of a high-speed train line in Japan led to the creation of new buyer-seller linkages, as well as significant improvements in firm performance.

Outline The rest of the paper is organized as follows. In Section 2, we present the theoretical framework that guides our empirical analysis. Section 3 provides a concise description of the Great East Japan Earthquake and its aftermaths. In Section 4, we describe the data and explain our procedure for constructing the network of supply chain linkages. Section 5 contains our main empirical results and Section 6 concludes. The proofs are provided in the Appendix.

2 Supply Chain Disruptions: Theoretical Framework

We start by developing a multi-firm, general equilibrium model in the spirit of Long and Plosser (1983) and Acemoglu et al. (2012) that captures how idiosyncratic firm-level shocks propagate over input-output linkages. In particular, we generalize the equilibrium characterization of Acemoglu et al. (2012) by allowing for general substitution patterns both within intermediate inputs and across intermediate inputs and primary factors of production. This generalization provides us with testable empirical predictions for the nature and the extent of propagation of microeconomic shocks in the economy.

2.1 Model

Consider a static economy consisting of \( n \) competitive firms denoted by \( \{1, 2, \ldots, n\} \), each of which producing a distinct product. Each product can be either consumed or used as an input for the production of other goods.

Firms employ CES production technologies with constant returns to scale that transform labor and intermediate goods into final products. The output of firm \( i \) is given by

\[
y_i = Z_i \left[ (1 - \mu)^{1/\sigma} l_i^{(\sigma - 1)/\sigma} + \mu M_i^{(\sigma - 1)/\sigma} \right]^{\frac{1}{\sigma - 1}},
\]

where \( \mu \) captures the material inputs’ share, \( \sigma \) represents the elasticity of substitution between labor and material inputs, \( l_i \) is the amount of labor hired by the firm, and \( Z_i \) is the corresponding productivity shock. In the above expression, \( M_i \) denotes firm \( i \)’s intermediate input bundle purchased from other firms and is given by

\[
M_i = \left[ \sum_{j=1}^{n} a_{ij} x_{ij}^{(\zeta - 1)/\zeta} \right]^{\zeta/(\zeta - 1)},
\]

where \( x_{ij} \) is the amount of good \( j \) used in the production of good \( i \) and \( \zeta \) is the elasticity of substitution between different intermediate goods. The coefficient \( a_{ij} \geq 0 \) designates the
importance of good \( j \) as an intermediate input for the production of good \( i \): a larger \( a_{ij} \) means that good \( j \) is a more important input in the production technology of firm \( i \), whereas \( a_{ij} = 0 \) if firm \( i \) does not rely on good \( j \) as an intermediate good for production. Throughout, we normalize these coefficients by assuming that \( \sum_{j=1}^{n} a_{ij} = 1 \) for all \( i \).

We summarize the inter-firm input-output linkages with matrix \( A = [a_{ij}] \), which with some abuse of terminology we refer to the economy’s input-output matrix.\(^6\) We also define the economy’s Leontief inverse as \( L = (I - \mu A)^{-1} \), whose off-diagonal \((i, j)\) element can be rewritten as

\[
\ell_{ij} = \mu a_{ij} + \mu^2 \sum_{k=1}^{n} a_{ik} a_{kj} + \ldots
\]

and accounts for the importance of firm \( j \) as a direct and indirect input-supplier to firm \( i \neq j \).

In addition to the firms, the economy is populated by a unit mass of identical consumers, who supply one unit of labor inelastically and have symmetric, logarithmic preferences over the \( n \) goods given by

\[
u(c_1, \ldots, c_n) = \sum_{i=1}^{n} \log(c_i),
\]

where \( c_i \) denotes the amount of good \( i \) consumed.

The competitive equilibrium of this economy is defined in the usual way: it consists of a collection of prices and quantities such that (i) the representative consumer maximizes her utility; (ii) all firms maximize their profits while taking the prices and the wage as given; and (iii) all \( n \) commodity markets and the labor market clear.

### 2.2 Input-Output Linkages and Propagation of Shocks

Our goal is to characterize how shocks to a given firm propagate over input-output linkages and impact the rest of the firms in the economy. Since a closed-form characterization of firms’ equilibrium outputs is not possible in general, we log-linearize the equilibrium around the point \( \epsilon_i = \log(Z_i) = 0 \) for all \( i \). Such a log-linearization provides a first-order approximation to the impact of small, firm-level productivity shocks on output. Denoting the logarithm of firm \( i \)'s output by \( \hat{y}_i \), we have the following result:

**Proposition 1.** The impact of a productivity shock to firm \( j \) on the output of firm \( i \neq j \) is equal to

\[
\frac{\partial \hat{y}_i}{\partial \epsilon_j} = \ell_{ij} + \frac{1}{\mu v_i} (\sigma - 1)(1 - \mu) \left( \sum_{k \neq i, j} v_k \epsilon_{kj} \epsilon_{ki} + v_i \epsilon_{ij} (\ell_{ii} - 1) + v_j \epsilon_{ji} (\ell_{jj} - 1) \right) + \frac{1}{\mu v_i} (1 - \zeta) \left( \sum_{k=1}^{n} ((1 - \mu)v_k + \mu) \epsilon_{kj} \epsilon_{ki} - v_i \epsilon_{ij} - v_j \epsilon_{ji} \right),
\]

where \( L = [\ell_{ij}] \) denotes the economy’s Leontief inverse and \( v_i = \sum_{k=1}^{n} \ell_{ki} \).

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\(^6\)In the special case that firms’ production technologies are Cobb-Douglas (that is, \( \zeta = \sigma = 1 \)), \( a_{ij} \) is proportional to the corresponding entry of the economy’s input-output table, measuring the value of spending on input \( j \) per dollar of production of good \( i \).
The above result illustrates that inter-firm input-output linkages (as summarized by the economy’s Leontief inverse) and the corresponding elasticity parameters play a central role in determining the nature and extent of propagation. In particular, each of the three terms on the right-hand side of (3) capture a distinct channel for how a shock to firm \( j \) impacts the output of firm \( i \).

The first term, which is equal to the corresponding element of the economy’s Leontief inverse, captures the so called “output effect”: a negative productivity shock to firm \( j \) increases good \( j \)’s equilibrium price, thus forcing \( j \)’s customers to scale back their production by reducing their demand for good \( j \). This reduction in turn increases the prices of goods produced by \( j \)’s customers and hence induces a second round of propagation to their respective customers. As such, this effect is a downstream propagation mechanism from a supplier to its customers, the customers of its customers, and so on. In fact, recall from equation (2) that \( \ell_{ij} \) is larger whenever firm \( j \) is a more important (direct or indirect) input-supplier to firm \( i \), whereas \( \ell_{ij} = 0 \) if and only if firm \( j \) is not a direct or indirect supplier of \( i \).

In addition to the output effect, shocks to firm \( j \) impact the output of firm \( i \) via two other channels. These effects, captured via the second and third terms on the right-hand side of (3), arise due to the fact that changes in input prices also affect the composition of inputs used by the firms. More specifically, the second term on the right-hand side of (3) captures how substitutability between labor and the intermediate goods bundle affects the extent of shock propagation: if labor and intermediate goods are gross substitutes, a negative shock to firm \( j \) makes utilizing labor more attractive to \( j \)’s customers (at the expense of all other inputs), thus further amplifying the effect of the shock. In fact, one can verify that the second term on the right-hand side of (3) is always non-negative for \( \sigma > 1 \).\(^7\) Note that unlike the output effect captured by the first term on the right-hand side of (3), this “labor substitution effect” may result in both upstream and downstream propagation: as long as \( \sigma \neq 1 \), a shock to firm \( j \) propagates to its (direct and indirect) suppliers as well as its customers. This can be seen by noting that the second term on the right-hand side of (3) depends on both \( \ell_{ij} \) and \( \ell_{ji} \).

Finally, the third term in Proposition 1 captures how substitutability between different intermediate inputs impacts the propagation of shocks over input-output linkages: when \( \zeta > 1 \), a negative shock to firm \( j \) induces its downstream customers to substitute away from good \( j \) to other intermediate goods. As a result, shocks to firm \( j \) not only impact \( j \)’s (direct or indirect) suppliers and customers, but can also propagate to its customers’ other suppliers, creating yet another channel for propagation.

To further clarify the implications of Proposition 1, we focus on an economy in the form of the “simple production chain” depicted in the left panel of Figure 1. In such an economy, each firm relies on the output of a single other firm for production, with firm \( i \) serving as the unique supplier to firm \( i+1 \), i.e., \( a_{i+1,i} = 1 \) for all \( i \). Despite its simplicity, this structure is rich enough to capture many of the effects discussed above.

\(^7\)This is consequence of the fact that all elements of the Leontief inverse are non-negative and that \( \ell_{kk} \geq 1 \) for all \( k \).
Figure 1. Input-Output Networks

Note: The left panel depicts the simple production chain, in which firm $i$ serves as the unique supplier to firm $i+1$; that is, $a_{i+1,i} = 1$ for all $i > 1$. Firm 1 at the top of the chain relies on labor and its own output as inputs for production ($a_{11} = 1$). The right panel depicts the Y-shaped production network, in which firm 1 uses the outputs of firms $u$ and $v$ as inputs for production; that is, $a_{1u} = a_{1v} = 1/2$.

**Downstream Propagation** We first focus on how shocks propagate downstream from a firm to its customers, the customers of its customers, and so on. We have the following corollary to Proposition 1:

**Corollary 1.** Suppose that a firm in the simple production chain is hit with a negative shock. Then,

(a) The outputs of all its downstream firms decrease.

(b) The impact on a given firm is smaller, the further downstream it is from the shock’s origin.

(c) The impact on all downstream firms intensifies as $\sigma$ increases.

Statement (a) of the above result highlights that a negative productivity shock to firm $j$, not only reduces the output of its immediate customers, but also negatively impacts its customers’ customers and so on. Thus, in this sense, such a negative shock propagates all the way downstream. Nevertheless, as statement (b) highlights, the size of this impact is diminished as the shock travels over the chain. Note that these results hold regardless of whether labor serves as a gross substitute or complement to intermediate goods in the firms’ production technology. Finally, part (c) establishes that as labor becomes a better substitute for the intermediate goods, the extent of this downstream propagation intensifies.

To see the intuition behind the above result, recall from the discussion following Proposition 1 that a negative shock to firm $j$ impacts the output of its downstream firms via two distinct channels.\footnote{Since in the simple production chain each firm relies on a single intermediate good, the channel that functions via the substitutability of different intermediate goods is inactive.} First, by increasing the price of good $j$, such a shock forces the downstream firm $i$ to scale back operations, leading to a smaller equilibrium output. This output effect is thus always negative regardless of the value of $\sigma$. The second channel, on the other hand, depends on the elasticity
parameter \( \sigma \): when labor is a gross substitute for material inputs, a negative shock to firm \( j \) not only reduces \( \dot{y}_j \) via the output effect, but also induces firm \( i + 1 \) to substitute away from good \( i \) and instead rely more heavily on labor, thus further reducing the output of firm \( i \). Therefore, when \( \sigma > 1 \), the output and substitution effects reinforce one another, leading to a stronger downstream propagation. In contrast, the two channels have opposing effects when labor and material inputs are gross complements. Nonetheless, as part (a) illustrates, the output effect always dominates the labor substitution effect for all values of \( \sigma \).

**Upstream Propagation** In addition to their impact on a firm's (direct and indirect) customers, shocks may also propagate upstream to the firm's suppliers. The following corollary to Proposition 1 characterizes this effect:

**Corollary 2.** Suppose that a firm in the simple production chain is hit with a negative shock. Then,

(a) The outputs of all its upstream firms decrease if \( \sigma > 1 \), whereas their outputs increase if \( \sigma < 1 \).

(b) The impact on a given firm is smaller the further upstream it is from the shock's origin.

(c) In a long enough chain, the impact on any upstream firm is weaker (in magnitude) than the impact on the downstream firm at the same distance from the source of the shock.

The above result thus establishes that as long as \( \sigma \neq 1 \), productivity shocks not only impact the output of the downstream firms, but also propagate upstream. In contrast to the downstream effects, however, the sign of this impact depends on whether labor and material inputs are gross substitutes or complements. In particular, as statement (a) of Corollary 2 shows, a negative shock to some firm \( j \) reduces the output of its direct and indirect suppliers if and only if labor is a gross substitute for the intermediate goods in the firms' production technology. As expected, the size of this impact is diminished as the shock travels further upstream over the chain.

To understand the intuition underlying the upstream propagation mechanism, recall from Corollary 1 that a negative shock to firm \( j \) impacts the output of its downstream customers via the output and labor substitution effects. In particular, when \( \sigma > 1 \), firms downstream to \( j \) increase their reliance on labor at the expense of good \( j \). This reduction in demand for good \( j \) in turn forces firm \( j \) to reduce its own input demand, thus manifesting itself as a negative demand shock to \( j \)'s upstream suppliers. In this sense, upstream propagation is in effect a by-product of downstream propagation.

Finally, part (c) of Corollary 2 provides a comparison between the intensity of upstream and downstream propagations and shows that shocks have a larger impact on a firm's downstream customers than on its upstream suppliers. This is due to the fact that whereas downstream propagation arises as a consequence of the substitution and output effects, the latter channel is absent in generating upstream propagation.

**Horizontal Propagation** Corollaries 1 and 2 provide a characterization of downstream and upstream propagation mechanisms in isolation. We now turn to a richer form of spillover, whereby
the simultaneous presence of the two propagation mechanisms can result in shocks to a given firm propagating to other firms which are neither its (direct or indirect) suppliers nor customers. To capture such a possibility, consider the Y-shaped production network depicted in the right panel of Figure 1, in which firm 1 relies on the outputs of firms $u$ and $v$ as intermediate goods for production. We have the following result:

**Corollary 3.** Suppose that firm $v$ is hit with a negative shock. The impact of this shock on the output of firm $u$ is decreasing in $\sigma$ and increasing in $\zeta$.

The above result shows that shocks may propagate horizontally from a firm to another even though neither firm is a direct or indirect supplier of the other. More importantly, however, it illustrates that the extent and nature of this propagation depends on the elasticity of substitution between different intermediate goods and that of between labor and material inputs. The intuition underlying Corollary 3 is along the lines of our earlier results: a negative shock to firm $v$, not only reduces the output of firm 1, but may also induce it to alter the composition of its inputs. In particular, as labor becomes a better substitute for material inputs, such a negative shock forces firm 1 to utilize labor more intensely, and as a result impacting the output of firm $u$ negatively. On the other hand, a higher $\zeta$ implies that $u$’s output is a better substitute for the good produced by firm $v$. Consequently, a negative shock to firm $v$ induces firm 1 to rely more heavily on the good produced by firm $u$, thus increasing the latter firm’s output. Corollary 3 thus illustrates that the sign of the horizontal propagation effect depends on a race between the elasticity of substitution between different material inputs, $\zeta$, and that of between labor and the intermediate input bundle, $\sigma$.

To summarize, our model implies that (i) shocks to any given firm propagate upstream and downstream, impacting the firm’s direct and indirect suppliers and customers; (ii) both propagation effects decay as shocks travel further over supply chains; and (iii) the downstream effect is quantitatively larger than the upstream effect. In addition, the model also predicts that even though a negative shock results in a negative downstream propagation regardless of parameter values, the sign of the upstream and horizontal effects depend on the corresponding elasticity parameters.

### 3 The 2011 Great East Japan Earthquake

On March 11, 2011, a magnitude 9.0 earthquake occurred off the northeast coast of Japan. This was the largest earthquake in the history of Japan and the fifth largest in the world since 1900. The earthquake brought a three-fold impact on the residents of northeast Japan: (i) the main earthquake and its aftershocks, directly responsible for much of the material damage that ensued; (ii) the resulting tsunami, which flooded 561 square kilometers of the northeast coastline; and (iii) the failure of the Fukushima Dai-ichi Nuclear Power Plant that led to the evacuation of 99,000 residents of the Fukushima prefecture.

In addition to severe damage to infrastructure, the earthquake and its aftermaths resulted in 19,418 confirmed fatalities, a further 2,592 people missing, and complete collapse of 400,305 buildings across twenty prefectures as of March 1, 2016 (Fire and Disaster Management Agency,
Figure 2 depicts the geographical distributions of casualties and demolished structures. As the figure illustrates, the impact of the shock was far from homogenous, even within the most severely affected prefectures. In particular, even though the main earthquake itself resulted in damages in some inland areas, the most severely affected areas were concentrated in the coastal regions that were exposed to the tsunami.

Not surprisingly, this localized, yet large shock had a significant negative impact on the economic performance of the affected areas. The real GDP growth rate of the four disaster-stricken prefectures in the 2011 fiscal year was $-1.7\%$, revealing weak economic performance in comparison to both their average growth rate in the previous fiscal year and aggregate Japanese GDP growth in FY 2011, which were $1.4\%$ and $1.8\%$, respectively (National Accounts of Japan, 2016). These figures also suggest that, despite its large impact on the local economy of northeast Japan, the earthquake shock cannot, in and of itself, account for the decline in Japan's aggregate GDP growth, as the four disaster-

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9As we discuss in more detail in Section 5, the area officially declared by the government as the disaster area consists of 41 municipalities within these prefectures.

10In Japan, the government's fiscal year runs from April 1 to March 31 of the following calendar year. As a result, FY 2010 contains only 20 post-earthquake days, while FY 2011 falls entirely after the earthquake.
stricken prefectures only account for roughly 4.7% of aggregate output in Japan. More specifically, solely based on the economic size of the affected areas, the earthquake can account for at most a 0.047 × (1.4 − (−1.7)) ≈ 0.15 percentage point decline in aggregate real GDP growth. However, the actual decline in Japan’s real GDP growth rate was more than five times as large, dropping from 2.6% in FY 2010 to 1.8% in FY 2011.

Concentrating on manufacturing activity provides a more detailed picture of the economic impact of the natural disaster. Figure 3 compares the monthly (year-on-year) growth rate of the Index of Industrial Production (IIP) of the disaster-stricken prefectures to that of Japan as a whole. This index, which is constructed by the Ministry of Economy, Trade and Industry of Japan (METI), measures the activity in the manufacturing and mining sectors. As the figure illustrates, the earthquake and its aftermaths resulted in a sharp, but temporary decline in the industrial production of the affected areas: the IIP in the four disaster-stricken prefectures declined on impact by over 40% relative to the previous year, followed by a partial rebound. By February 2012 (that is, one year after the earthquake), industrial production in the affected areas was about 4% lower than the corresponding level on the eve of the earthquake in February 2011. In comparison, industrial production of the entire country experienced a 15% decline in April 2011 and was back to its pre-earthquake growth rate one year after the earthquake.

Another key observation is that earthquake-hit areas were not overly specialized. The three sectors with the largest output shares in the four affected prefectures in 2010 were realty, wholesale
and retail, and other services, responsible for 10.0%, 10.9%, and 19.6% of the region's total output, respectively. These figures are comparable to the shares of the same sectors in the entire country (9.8%, 11.5%, and 19.4%, respectively). The largest difference among various sectoral shares between the four affected prefectures and Japan as a whole is in the transportation machinery sector, responsible for 2.6% and 5.8% of the two regions' respective outputs.

We end this discussion by a word on infrastructure. Even though infrastructure, such as roadways, railways, and ports, across northeast Japan was severely affected by the shock, pre-earthquake levels of activity were largely restored by late March. The one area where activity was disrupted well into the summer of 2011 was electricity supply, as several nuclear — notably the Fukushima Dai-ichi plant — and conventional power plants in northeast Japan went offline. This resulted in rolling (controlled) blackouts throughout March and a power saving edict for the summer months of 2011 that required large-lot users in Kanto and Tohoku regions to reduce power use by 15%.\footnote{We address the potential threats to our identification strategy caused by the disruption in electricity supply in Subsection 5.3.}

4 Data

Our empirical analysis relies on a proprietary dataset collected by Tokyo Shoko Research Ltd. (henceforth, TSR), which is a private credit reporting agency. Firms provide information to TSR in the course of obtaining credit reports on potential suppliers and customers or when attempting to qualify as a supplier. This information consists of a set of firm-level characteristics (such as sales figures and number of employees), as well as the identities of the firms’ suppliers and customers. The resulting (raw) database contains information on roughly one million firms in all 47 prefectures across Japan, spanning all sectors of the economy. The dataset provided to us by RIETI consists of data for the years 2010, 2011, and 2012.

Firm-Level Information TSR collects information on employment, the number of establishments, up to three (Japanese Industrial Classification 4-digit) industries the firm may belong to, sales and profits for the past two years, the resulting credit score, and a physical address for the firm’s headquarters. Each firm in the TSR database also reports the date on which its fiscal year ends.

The TSR sample is neither a census nor a representative survey, as the entry of any particular firm takes place at the request of TSR's clients. This means that TSR does not update the information on all firms on an annual basis. We therefore restrict our sample to the subset of firms for which we can (i) construct annual sales growth rate in the “post-earthquake fiscal year”, defined as the fiscal year that contains March 2011 (that is, the month of the disaster) and (ii) observe the firm-level covariates in the “pre-earthquake fiscal year”, defined as the immediate fiscal year prior to the earthquake.\footnote{Thus for example, the pre- and post-earthquake fiscal years of a firm with the fiscal year end of July 31 are, respectively, August 1, 2009–July 31, 2010 and August 1, 2010–July 31, 2011.}

This procedure leaves us with a baseline sample of 648,820 firms.\footnote{In an alternative specification, we exclude all firms with a fiscal year end in March, as the extent to which such firms’
Table 1. Firm Size Distribution

<table>
<thead>
<tr>
<th>Number of Employees</th>
<th>0–4</th>
<th>5–9</th>
<th>10–19</th>
<th>20–29</th>
<th>30–49</th>
<th>50–99</th>
<th>100–299</th>
<th>300–999</th>
<th>1000–1999</th>
<th>2000+</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSR</td>
<td>0.40</td>
<td>0.25</td>
<td>0.16</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.009</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Census</td>
<td>0.59</td>
<td>0.17</td>
<td>0.11</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.005</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notes: The table reports the fraction of firms with the number of employees in each of the respective bins. “TSR” refers to the 2010 TSR dataset. “Census” refers to the 2009 Economic Census for Business Frame.

In order to check for biases in our sample, we compare the resulting dataset with the 2009 Economic Census, which contains information on 1,805,545 firms.\(^{14}\) Table 1 reports the distribution of the number of employees in the two datasets. As the table indicates, the firm size distribution in the TSR dataset closely matches that of the census data for firms with five or more employees (though it underestimates the fraction of very small firms with four or less employees). Similarly, Figure 4 illustrates that the geographic distribution and industrial composition of firms in the TSR sample matches those of firms in the 2009 census, with the only major difference being in the fraction of firms that are active in the construction sector.

One limitation of TSR’s data on firm-level characteristics is that it only contains information on firms’ headquarters locations, as opposed to the location of their plants. Even though in principle this may create a bias in our estimates, as part of our robustness checks in Subsection 5.3 we verify that our estimates remain unchanged if we restrict our analysis to the subsample of single-plant firms (for which headquarters and plant locations coincide).

**Supply Chain Information** Each firm in the TSR dataset also provides a list of its suppliers and customers, thus enabling us to construct the network of supplier-customer linkages for the firms in the sample. Given the occurrence of earthquake in March 2011, we construct this network using the 2010 TSR sample.

The TSR dataset on supplier-customer linkages has two limitations. First, it only reports a binary measure of inter-firm supplier-customer relations: even though we observe whether one firm is another firm’s supplier or customer, we do not observe a yen measure associated with their transactions. Second, the forms used by TSR limit the number of suppliers and customers that firms can report to 24 each. Nevertheless, given that each firm in the dataset may also be reported by other firms as a transaction partner, we overcome this limitation by combining the self-reported customer and supplier relations with those reported by other firms. More specifically, we construct a firm’s transaction network by augmenting the list of suppliers (customers) reported by the firm itself with sales incurred before or after the earthquake may not be clear. We verified that our findings remain unchanged.

\(^{14}\)The census is conducted by the Statistics Bureau in the Ministry of Internal Affairs and Communications. The data is from the survey entitled “The Economic Census for Business Frame,” which identifies the basic structure of establishments and enterprises and is available at: [http://www.stat.go.jp/english/data/e-census.htm](http://www.stat.go.jp/english/data/e-census.htm).
the reports of others that state the firm as their customer (supplier). This procedure enables us to construct the list of suppliers and customers of firms that have more than 24 transaction partners per category, including very large firms that transact with several thousand firms.

In constructing the network of supply chain relationships, we discard reported transaction partners that fall outside the TSR database. Therefore, a firm may appear to have no customers because all its customers are foreign firms, domestic non-TSR firms, or non-firms (such as final demand customers or the government of Japan). Similarly, a firm may appear to have no suppliers because either all its reported suppliers are foreign or fall outside the TSR database. Throughout, we restrict the sample to the subset of firms with at least one transaction partner (being it a customer or a supplier) within the TSR database, thus discarding firms that are isolated from the rest of the network. We find no evidence of a systematic bias in the subsample of firms with at least one TSR partner.
5 Supply Chain Disruptions: Empirical Results

In this section, we use the TSR dataset to empirically examine the extent to which input-output linkages functioned as a channel for propagating the natural disaster shock throughout Japan. We organize our analysis around the three main predictions of our theoretical framework in Section 2. Recall that according to our model, (i) shocks to any given firm propagate upstream and downstream, impacting the firm's direct and indirect suppliers and customers; (ii) both propagation effects decay as shocks travel further over supply chains; and (iii) the downstream effect is quantitatively larger than the upstream effect.

We test these predictions by first identifying the set of firms that were directly exposed to the natural disaster and verifying that the shock had a significant negative impact on their performance. We then examine the evidence for the propagation of the shock over input-output linkages by comparing the differential performance of firms linked to disaster-stricken firms relative to a control group of firms with no such linkages.
5.1 Disaster Area Firms

We start by identifying the firms that were directly exposed to the triple shock of earthquake, tsunami, and nuclear disaster. Since we cannot observe the extent of firm-level disruption caused by the shock, we determine the set of disaster-stricken firms by combining information on the geographic distribution of damages with information on firms’ headquarters locations.

We identify the disaster-hit region by relying on three decrees issued by the government in the aftermath of the earthquake. The first decree, issued on April 28, 2011 by the Ministry of Land, Infrastructure, Transport and Tourism (MLIT) designated 36 municipalities as severely affected municipalities that qualified for special financial aid. The other two decrees, issued by the Prime Minister’s office on April 21 and 22, restricted entrance and residence in 12 municipalities in the aftermath of the failure of the Fukushima Dai-ichi Nuclear Power Plant. Out of the 12 municipalities constituting the evacuation zone, seven were also included in the decree issued by MLIT, thus leaving us with a total of 41 municipalities. We refer to the region covered by these 41 municipalities as the “disaster area”. We verify that the physical impact of the shocks (as measured by casualties and demolished structures) were concentrated in these municipalities. In Subsection 5.3, we verify that our results are robust to an alternative definition for the disaster area, encompassing the regions that were flooded following the tsunami.

With the definition of the disaster area in hand, we then identify the set of firms located in this area by using an address matching service provided by the Center for Spatial Information Science at the University of Tokyo and matching each firm’s headquarters address (given in TSR) to longitude and latitude data. This procedure identifies 18,728 firms in our sample. Figure 5 maps the headquarters locations of these firms.

To analyze the impact of the earthquake and tsunami on firms in the disaster area, we compare their annual real sales growth before and after the earthquake to the corresponding values for firms whose headquarters are located outside the disaster area. Since we do not have access to information on firm-specific prices, we deflate nominal sales values by the regional price indices for the respective prefectures. The results are presented in Table 2.

As the table illustrates, in the aftermath of the earthquake, the mean and median real sales growth of firms in the disaster area were significantly smaller than the corresponding values for the rest of Japanese firms. In particular, whereas disaster area firms experienced a 4.8% average decline

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15This decree was issued as a notification based on “The Act on Special Financial support to Deal with the Designated Disaster of Extreme Severity, Article 41-2.” The notification is available at http://www.mlit.go.jp/report/press/house03_hh_000054.html (in Japanese).


Table 2. Pre- and Post-Earthquake Real Sales Growth Rates

<table>
<thead>
<tr>
<th></th>
<th>Post-Earthquake</th>
<th>Pre-Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Disaster Area</td>
<td>Rest of Japan</td>
</tr>
<tr>
<td>Observations</td>
<td>18,728</td>
<td>630,092</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.048</td>
<td>-0.019</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.360</td>
<td>0.322</td>
</tr>
<tr>
<td>p33</td>
<td>-0.094</td>
<td>-0.039</td>
</tr>
<tr>
<td>p50</td>
<td>-0.025</td>
<td>-0.007</td>
</tr>
<tr>
<td>p67</td>
<td>0.014</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Notes: The table reports the summary statistics of pre- and post-earthquake sales of firms inside and outside the disaster area. Sales figures for disaster area firms are deflated by the prefecture-level GDP deflator of the four prefectures that encompass the disaster area. Sales figures for firms in the rest of Japan are deflated using GDP deflator for Japan as a whole. The quantiles are determined based on firms’ post-earthquake real sales growth rates.

in annual sales in the year after the earthquake, the average decline for firms outside the disaster area was only 1.9%, a statistically significant difference of 2.9 percentage points (with a $t$-statistic of 10.8). The table is also indicative of a significant amount of heterogeneity in firm performance. For instance, the best performing firms in the disaster area — the upper tercile in terms of post-earthquake real sales growth — are comparable to the best performing firms in the rest of Japan. The key difference resides in the bottom tercile of firm sales growth distribution: whereas the lowest performing firms in the disaster area contracted by 9.4% in real terms, the corresponding firms outside the disaster area only experienced a 3.9% decline in sales. Crucially, Table 2 also illustrates that the performance of disaster area firms did not differ from that of rest of Japanese firms in the year preceding the earthquake: the difference between average sales growth rate of firms inside and outside the disaster area was less than 0.1 percentage points, with a $t$-statistic of 0.4.

We complement this analysis by comparing other pre-earthquake characteristics of firms inside and outside the disaster area, with the results summarized in Table 3. The key observation is that, in the year preceding the earthquake, the average disaster area firm was comparable to the average firm in the rest of the country. This is true both for typical firm characteristics such as age and size (as measured by employees and sales), as well as with regards to firms’ supply chain characteristics (such as the number and average size of customers and suppliers). Importantly, this observation remains unchanged even when we disaggregate disaster area firms based on their post-earthquake performance.

Taken together, these findings illustrate that (i) the shock had a large negative impact on the firms in the disaster area and (ii) firms in the disaster area (including the worst-performing among them) were not substantially different from the rest of the firms in our sample based on either their pre-earthquake performance or other firm-level characteristics.
Table 3. Pre-Earthquake Characteristics of Firms Inside and Outside the Disaster Area

<table>
<thead>
<tr>
<th></th>
<th>Disaster Area</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Firms</td>
<td>Bottom Tercile</td>
<td>Middle Tercile</td>
<td>Top Tercile</td>
<td>Rest of Japan</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Log Sales</td>
<td>11.54</td>
<td>11.55</td>
<td>11.66</td>
<td>11.43</td>
<td>11.74</td>
</tr>
<tr>
<td></td>
<td>(1.52)</td>
<td>(1.50)</td>
<td>(1.55)</td>
<td>(1.51)</td>
<td>(1.64)</td>
</tr>
<tr>
<td>Log No. Employees</td>
<td>1.94</td>
<td>1.88</td>
<td>1.99</td>
<td>1.94</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(1.22)</td>
<td>(1.18)</td>
<td>(1.29)</td>
<td>(1.18)</td>
<td>(1.27)</td>
</tr>
<tr>
<td>Age</td>
<td>29.02</td>
<td>29.91</td>
<td>30.29</td>
<td>26.92</td>
<td>30.59</td>
</tr>
<tr>
<td></td>
<td>(14.92)</td>
<td>(14.72)</td>
<td>(15.34)</td>
<td>(14.48)</td>
<td>(16.04)</td>
</tr>
<tr>
<td>Log No. of Suppliers</td>
<td>1.13</td>
<td>1.13</td>
<td>1.06</td>
<td>1.20</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>(0.82)</td>
<td>(0.82)</td>
<td>(0.80)</td>
<td>(0.83)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>Log No. of Customers</td>
<td>1.17</td>
<td>1.12</td>
<td>1.14</td>
<td>1.22</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>(0.87)</td>
<td>(0.83)</td>
<td>(0.91)</td>
<td>(0.86)</td>
<td>(0.91)</td>
</tr>
<tr>
<td>Customers’ Log Sales</td>
<td>14.83</td>
<td>14.73</td>
<td>15.31</td>
<td>14.46</td>
<td>14.51</td>
</tr>
<tr>
<td></td>
<td>(2.37)</td>
<td>(2.31)</td>
<td>(2.41)</td>
<td>(2.29)</td>
<td>(2.45)</td>
</tr>
<tr>
<td></td>
<td>(2.21)</td>
<td>(2.24)</td>
<td>(2.31)</td>
<td>(2.11)</td>
<td>(2.49)</td>
</tr>
</tbody>
</table>

Notes: The table reports summary statistics of pre-earthquake characteristics of firms inside and outside the disaster area. Values are averages across firms in each category with standard deviations reported in parentheses. Column (1) reports mean and standard deviations for all firms in the disaster area. Columns (2)–(4) report mean and standard deviations for disaster area firms in, respectively, the bottom, middle, and top terciles of the distribution of the firms’ post-earthquake real sales growth rates. The last column reports mean and standard deviation for firms outside of the disaster area.

5.2 Propagation

With the set of firms directly exposed to the disaster identified, we now examine the evidence for the propagation of the shock over input-output linkages.

As a first exercise, we test our model’s predictions by quantifying the extent of propagation from the disaster area firms to their immediate upstream suppliers and downstream customers. In testing for these predictions a number of challenges come to fore. First, firm i’s output dynamics may be driven by a number of firm-level (observable and unobservable) characteristics that our theoretical framework in Section 2 does not take on board. Second, in addition to the impact due to the propagation of the shock, firm i’s output may be driven by idiosyncratic disturbances or aggregate shocks.

These considerations lead us to rely on the following specification to measure the extent of propagation from disaster area firms to their immediate supply chain partners:

\[
\Delta \log(Sales_{i,p,s}) = \alpha + \beta_{\text{down}} \cdot \text{Downstream}_i + \beta_{\text{up}} \cdot \text{Upstream}_i + \gamma' X_i + \mu_p + \lambda_s + \varepsilon_i. \tag{4}
\]

In the above specification, \(\Delta \log(Sales_{i,p,s})\) is the post-earthquake sales growth rate of firm \(i\), located in prefecture \(p\), and operating in industry \(s\); \(\text{Downstream}_i\) and \(\text{Upstream}_i\) are dummy variables that indicate whether firm \(i\) was, respectively, a customer or a supplier of a firm in the disaster area in the year prior to the earthquake; \(\mu_p\) and \(\lambda_s\) denote prefecture and three-digit industry fixed
effects that control for correlated shocks in a given prefecture or industry; and \(X_i\) denotes firm \(i\)'s pre-earthquake observable characteristics, consisting of age, number of employees, number of transaction partners, distance to the disaster area, and number of plants. The main coefficients of interest are \(\beta_{\text{up}}\) and \(\beta_{\text{down}}\), which respectively measure the differential performance of firms that were either supplied to or were supplied by disaster area firms (constituting the treatment group) relative to firms in the same industry and prefecture with no such direct supply chain linkages (constituting the control group).

Our main identifying assumption is that, conditional on firm observables \(X_i\), the presence of input-output linkages between firm \(i\) and disaster area firms is uncorrelated with any of \(i\)'s unobservable characteristics \(U_i\) that may affect its post-earthquake output dynamics; that is, \(\text{Downstream}_i, \text{Upstream}_i \perp U_i \mid X_i\). Note that this exclusion restriction is violated if supply chain partners of disaster area firms are also more likely to be affected by the shock via other channels. For instance, it may have been the case that disruptions to railways, roads, or other local infrastructure negatively impacted production of nearby firms that were also more likely to transact with disaster area firms. We address this concern by excluding from our sample all firms whose headquarters were located in the four prefectures of Aomori, Fukushima, Iwate, and Miyagi that encompass the disaster area (see Figure 5).\(^{18}\) We also note that by deploying prefecture fixed effects, we are further addressing the concern that certain areas were simply more affected than others. Finally, to control for any remaining residual effects operating at the firm-level, we include the firms’ (geographic) distance to the disaster area as one of the control variables.

Our analysis also relies on the validity of a parallel trends assumption, according to which the performance of firms in the control and treatment groups did not exhibit differential trends prior to the earthquake. We verify the validity of this assumption in Subsection 5.3 by rerunning our regressions in the year prior to the earthquake.

The first column of Table 4 reports the estimated coefficients for regression equation (4). The results indicate that the shock caused by the earthquake and its aftermaths propagated to the immediate customers of disaster area firms, a finding that is in line with the model’s prediction. In particular, we find that the post-earthquake growth rate of a typical firm immediately downstream to disaster area firms was 0.7 percentage points smaller than that of a typical firm in the same prefecture and sector with no direct linkages to disaster area firms. On the other hand, the small and statistically insignificant estimate for \(\beta_{\text{up}}\) indicates that the suppliers of firms in the disaster area did not exhibit any meaningfully different post-earthquake growth rate from the control group.

It is important to note that coefficients \(\beta_{\text{up}}\) and \(\beta_{\text{down}}\) estimated in regression (4) measure the differential performance of the partners of disaster area firms relative to the control group of firms with no direct linkages to disaster area firms. In other words, the control group includes firms that may be indirectly linked to disaster area firms, and hence, were themselves exposed to the disaster shock via these indirect linkages. As a result, the estimates reported in the first column of Table 4

\(^{18}\)A related concern is the impact of power outages that ensued the disaster: the exclusion restriction would be violated if firms transacting with disaster area firms were also more likely to be affected by power outages. We address this concern by exploiting an idiosyncrasy of Japan’s power grid in Subsection 5.3.
Table 4. Downstream and Upstream Propagation

<table>
<thead>
<tr>
<th>Post-Earthquake Sales Growth Rate</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream Distance 1</td>
<td>(-0.007^{***})</td>
<td>(-0.020^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Downstream Distance 2</td>
<td></td>
<td>(-0.013^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Downstream Distance 3</td>
<td></td>
<td>(-0.013^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Downstream Distance 4</td>
<td></td>
<td>(-0.011^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Upstream Distance 1</td>
<td>(-0.0003)</td>
<td>(-0.012^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 2</td>
<td></td>
<td>(-0.007^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Upstream Distance 3</td>
<td></td>
<td>(-0.007^{**})</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Upstream Distance 4</td>
<td></td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant</td>
<td>(-0.029^{**})</td>
<td>(-0.021^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Firm Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Prefecture FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>419,897</td>
<td>419,897</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.022</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates from regressing firms’ post-earthquake sales growth rates on various dummy variables indicating direct and indirect supplier-customer relationships with disaster area firms. The first column reports the estimated coefficients of regression (4). The second column reports the estimated coefficients of regression (5). Firm controls include the logarithm of the number of transaction partners, age, logarithm of the number of employees, distance to the disaster area, and number of plants. Robust standard errors are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

serve as lower bounds on propagation intensities.

To address this issue and assess the possibility of the indirect propagation of the shock, for each firm in the sample we construct a measure of network distance to the set of disaster area firms. More specifically, we first designate the immediate customers and suppliers of disaster area firms as, respectively, “downstream distance 1” and “upstream distance 1” firms. We then designate a firm as “downstream distance 2” if (i) it was outside the four disaster-stricken prefectures; (ii) was listed in 2010 as a customer of at least one downstream distance 1 firm; and (iii) was not a distance 1 firm itself. Using a similar recursive procedure, we identify the set of firms that were at various upstream and downstream distances from disaster area firms on the eve of the earthquake. Figure 6 depicts the geographic distribution of firms of various distances across Japan. The emerging picture clearly
illustrates that a large number of firms are at (downstream or upstream) distances 2 and 3 of disaster area firms, indicating the potential importance of indirect propagation effects.

With the above measure of network distance in hand, we extend our previous specification by introducing dummy variables for firms of various network distances to the disaster area firms. More specifically, we replace (4) with the following specification

\[
\Delta \log(Sales_{i,p,s}) = \alpha + \sum_{k=1}^{4} \beta_{\text{down}}^{(k)} \cdot \text{Downstream}_{i}^{(k)} + \sum_{k=1}^{4} \beta_{\text{up}}^{(k)} \cdot \text{Upstream}_{i}^{(k)} + \gamma X_i + \mu_p + \lambda_s + \varepsilon_i, \quad (5)
\]
where $\text{Downstream}^{(k)}_i$ and $\text{Upstream}^{(k)}_i$ represent dummy variables that indicate whether firm $i$ is, respectively, a downstream or upstream distance $k$ firm in the 2010 sample. As such, coefficients $\beta^{(k)}_{\text{down}}$ and $\beta^{(k)}_{\text{up}}$ measure the differential performance of downstream and upstream distance $k$ firms (in terms of sales growth in the year after the earthquake) relative to the control group. Note that in this specification, the control group constitutes all firms that, prior to the earthquake, were 5 or more supply chain links away from disaster area firms. We truncate the set of distance dummies at distance 4, as enlarging the treatment group further downstream or upstream would reduce the control group to a very small number of firms.

The second column of Table 4 reports the results. The estimated coefficients for all downstream variables are negative and significant, thus indicating that the disruption caused by the earthquake and its aftermaths propagated not only to disaster area firm’s immediate customers, but also all the way to firms of downstream distance 4. Moreover, consistent with the model’s prediction in Corollary 1, the intensity of this downstream propagation is non-increasing in the (network) distance to the initially disrupted firms. For instance, whereas the immediate customers of disaster area firms (i.e., downstream distance 1 firms) underperform the control group by 2.0 percentage points, the growth rate of downstream distance 2 firms is roughly 1.3 percentage points smaller than that of the control group. Note that the point estimates on the magnitude of the impact on downstream distance 1 firms is significantly higher compared to the corresponding estimates from regression (4) (reported in the first column of Table 4). As already explained, this is a consequence of the fact that the control group used in regression (4) is contaminated: it contains firms that were only indirectly linked to, but nevertheless affected by, disaster area firms. Including these firms in the control group in regression (4) thus lowers the control group’s average sales growth and results in underestimating the intensity of direct propagation to distance 1 firms.

The second column in Table 4 also illustrates the presence of an upstream propagation effect: the direct and indirect suppliers of disaster area firms (up to upstream distance 3) underperform the control group in the year after the earthquake. Furthermore, in line with the model’s prediction, the intensity of the propagation declines in the network distance from disaster-stricken firms. Similar to the case of downstream propagation, the disparity in the magnitude of the estimated coefficient for distance 1 firms in regressions (4) and (5) should not come as a surprise, as the control group in the former regression contains firms that were themselves affected (though not directly) by the shock to disaster area firms.

The point estimates in the second column of Table 4 also reveal that the intensity of propagation to firms downstream to the disaster area firms is always larger compared to the effect on upstream firms at the corresponding distances, that is, $|\beta^{(k)}_{\text{up}}| \leq |\beta^{(k)}_{\text{down}}|$ for all $k$. The weaker intensity of upstream propagation compared to the downstream propagation is in line with the model’s prediction in Corollary 2(c).

We conclude this discussion by noting that, viewed through the lens of our model (and Corollary 2(a) in particular), the reduction in sales growth rates of disaster area firms’ upstream suppliers compared to the control group is indicative of gross substitutability of the intermediate and primary
inputs in the firms’ production technologies.

5.3 Robustness Checks

**Placebo Test**  Our baseline estimates indicate that firms with supplier-customer linkages to disaster area firms experienced smaller sales growth rates compared to similar firms with no such linkages. This underperformance, however, may reflect the fact that customers and suppliers of disaster area firms were already on a declining trajectory pre-earthquake. To rule out such a possibility, we check our identification strategy with a placebo specification and estimate whether the existence of supplier-customer linkages to the disaster area firms predicts firms’ sales growth rates in the year prior to the earthquake. The resulting estimates are reported in Table 5. As the second column illustrates, all estimated coefficients (except for the upstream distance 2 firms) are statistically insignificant, thus indicating the validity of the parallel trends assumption.

**Electricity and Power Shortages**  The triple shocks of earthquake, tsunami, and nuclear disaster resulted in severe disruptions in the supply of electricity well into the summer of 2011, as several nuclear and conventional power plants in northeast Japan went offline, affecting the production capacity of two regional electricity providers, Tohoku Electric Power and Tokyo Electric Power Company (TEPCO). The reduction in generation capacity resulted in rolling (controlled) blackouts throughout March, followed by a power saving edict for the summer months of 2011 (when demand was higher), requiring large-lot users in Kanto and Tohoku regions to reduce power use by 15%.

The severe disruption in the supply of electricity may threaten our identification strategy if firms close to the disaster area were simultaneously (i) more likely to be affected by the power outages and (ii) more likely to have supply linkages with nearby firms inside the disaster area. Given that both conditions are likely to hold in our sample, our estimates may misattribute the disruption in the power supply to the propagation of the shocks over input-output linkages.

To address these concerns, we rely an unusual feature of Japan's power grid: while Tokyo and the rest of eastern Japan run at 50Hz frequency, the electricity in the western half of the country has a 60Hz frequency. This frequency difference partitions Japan's national grid into two halves of roughly equal sizes, with very limited capacity for power transmission across the 50Hz-60Hz divide. This idiosyncrasy of Japanese electric power system means that the adverse effects directly tied to power outages in the aftermath of the earthquake were confined to the eastern half of the country (depicted as the shaded region in Figure 7), with little impact on the supply of electricity in western Japan.

In addition, Figure 8, which plots the (seasonally adjusted) index of energy production, illustrates that while the eastern half of the country (corresponding to Tohoku, Hokkaido, and Kanto regions, all running on 50Hz electricity) experienced a sharp decline in energy production, production levels in the regions to the west of the 50Hz-60Hz frontier (that is, Chubu, Kinki, Chugoku, and Kyushu) remained roughly at the pre-earthquake levels. Together with the divided nature of Japan's

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19At the time of the earthquake, the frequency conversion capacity across the east-west frontier was 1.2 gigawatts (U.S. Energy Information Administration, 2015). For comparison, as of March 2015, the total installed generating capacity of electric utility companies in Japan was 234 gigawatts (Japan Electric Power Information Center, 2015).
Table 5. Placebo Test

<table>
<thead>
<tr>
<th>Sales Growth Rate</th>
<th>Baseline (Post-Earthquake)</th>
<th>Placebo (Pre-Earthquake)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Downstream Distance 1</td>
<td>$-0.020^{***}$</td>
<td>$-0.003$</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Downstream Distance 2</td>
<td>$-0.013^{***}$</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Downstream Distance 3</td>
<td>$-0.013^{***}$</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Downstream Distance 4</td>
<td>$-0.011^{***}$</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Upstream Distance 1</td>
<td>$-0.012^{***}$</td>
<td>$-0.001$</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Upstream Distance 2</td>
<td>$-0.007^{***}$</td>
<td>$-0.006^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 3</td>
<td>$-0.007^{**}$</td>
<td>$-0.003$</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 4</td>
<td>0.001</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant</td>
<td>$-0.021^{**}$</td>
<td>0.068^{***}</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Firm Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Prefecture FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>419,897</td>
<td>407,387</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.022</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates from regressing firms’ sales growth rates on various dummy variables indicating direct and indirect supplier-customer relationships with disaster area firms. The first column reproduces the estimates for the baseline regression (5) from Table 4 for post-earthquake sales growth rates. The second column reports the coefficients from regressing firms’ pre-earthquake sales growth rates on the supplier-customer relationship dummies. Firm controls include the logarithm of the number of transaction partners, age, logarithm of the number of employees, distance to the disaster area, and number of plants. Robust standard errors are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Thus, as our next robustness check, we rerun our baseline regression on the subsample of firms located to the west of the 50Hz-60Hz “frequency frontier”. The second column of Table 6 reports the resulting estimates. The results illustrate that the estimated coefficients remain largely unchanged, indicating that the shock to the disaster area firms propagated both upstream and downstream to firms located in the 60Hz region, with the intensity of downstream propagation stronger than that of the upstream propagation.
Figure 7. The Frequency Frontier

Note: The map depicts the partition of Japan’s electric power system into the 50Hz (lightly shaded area) and 60Hz (unshaded area) regions. The darker areas near the boundary depict regions in which users of both frequencies exist.

Figure 8. Energy Production

Source: Ministry of Economy, Trade and Industry of Japan.
Notes: The figure plots the seasonally adjusted index of energy production for March 2010–February 2012. The index is normalized to 100 for February 2011. The horizontal axis refers to months since the disaster, with 0 corresponding to February 2011. “Eastern Regions” covers Hokkaido, Tohoku, and Kanto, all of which run on 50Hz electricity. “Western Regions” covers Chubu, Kinki, Chugoku, and Kyushu, all of which run on 60Hz electricity. The two regions together cover the entire country, except for the Shikoku region in the west for which METI does not construct an energy IIP index. The two time series are obtained by aggregating regional energy IIPs weighted by the corresponding regional shares of energy production in 2009.
Table 6. Robustness Checks

<table>
<thead>
<tr>
<th>Post-Earthquake Real Sales Growth Rate</th>
<th>Baseline (1)</th>
<th>60Hz Region (2)</th>
<th>Single-Plant Firms (3)</th>
<th>Flooded Area (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream Distance 1</td>
<td>−0.020***</td>
<td>−0.028***</td>
<td>−0.019***</td>
<td>−0.037***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Downstream Distance 2</td>
<td>−0.013***</td>
<td>−0.015***</td>
<td>−0.016***</td>
<td>−0.012***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Downstream Distance 3</td>
<td>−0.013***</td>
<td>−0.015***</td>
<td>−0.015***</td>
<td>−0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Downstream Distance 4</td>
<td>−0.011***</td>
<td>−0.009*</td>
<td>−0.011**</td>
<td>−0.012***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 1</td>
<td>−0.012***</td>
<td>−0.017***</td>
<td>−0.015**</td>
<td>−0.018***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Upstream Distance 2</td>
<td>−0.007***</td>
<td>−0.008*</td>
<td>−0.009***</td>
<td>−0.005**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Upstream Distance 3</td>
<td>−0.007***</td>
<td>−0.008**</td>
<td>−0.007**</td>
<td>−0.008***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Upstream Distance 4</td>
<td>0.001</td>
<td>0.004</td>
<td>0.002</td>
<td>−0.001</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Constant</td>
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<td>0.003</td>
<td>−0.032**</td>
<td>−0.022**</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Firm Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Prefecture FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>419,897</td>
<td>231,195</td>
<td>258,035</td>
<td>419,897</td>
</tr>
<tr>
<td>R²</td>
<td>0.022</td>
<td>0.024</td>
<td>0.019</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates from regressing firms’ post-earthquake sales growth rates on various dummy variables indicating direct and indirect supplier-customer relationships with disaster area firms. Column (1) reproduces the estimates from the baseline regression (5) from Table 4. Column (2) reports the estimates for a regression with the sample restricted to the set of firms to the west of the frequency frontier. Column (3) presents the estimates for our baseline specification with the sample restricted to the set of single-plant firms. The last column presents the estimates of a regression with the disaster area redefined as the region flooded in the aftermath of the tsunami. Firm controls include the logarithm of the number of transaction partners, age, logarithm of the number of employees, distance to the disaster area, and number of plants. Robust standard errors presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

**Firms vs. Plants** Since our analysis is based on firm-level (as opposed to plant-level) data, one potential concern regarding our baseline estimates is the possibility that the treatment group — i.e., firms with headquarters outside the disaster area that are directly or indirectly linked to disaster area firms — may include multi-plant firms with plants in the disaster area. For instance, our baseline results would overestimate the intensity of downstream propagation if customers of disaster area firms are more likely to operate plants in the disaster area.

Even though the TSR data does not contain information on plant locations, it reports the
number of plants operated by each firm. We thus leverage this information and rerun our baseline specification on the subsample of single-plant firms (for which headquarters and plant locations coincide). This subsample contains roughly 60% of the firms in our sample. We follow the same sample selection criteria as in the rest of our analysis by restricting the treatment and control groups to only consist of firms that are located outside the four prefectures that constitute the broader earthquake affected area.

The results are reported in the third column of Table 6. As the table indicates, the magnitude and significance of the estimated coefficients are similar to those in our baseline regression. These results are also consistent with direct and indirect propagation of the shock to the firms upstream and downstream of disaster area firms.

**Disaster Area Definition** As a final check, we verify that our estimates are robust to the definition of the disaster area. Given that the large majority of loss of life occurred as a consequence of the tsunami, we rerun the regression in equation (5) while redefining the disaster area as regions that were flooded following the tsunami. These regions, unlike the 41 municipalities in our baseline specification, do not correspond to prefecture or municipality boundaries, and instead are defined based on aerial photos and satellite imagery of flooded areas provided by the Geospatial Information Authority of Japan. Figure 9 maps the headquarters locations of the firms located in this region.

![Figure 9. Headquarters Locations of Firms in the Flooded Region](image)

*Note:* Each dot on the map corresponds to the location of a firm in the TSR sample located in the flooded region. The dark blue curve indicates the boundary of the four prefectures (Aomori, Fukushima, Iwate, and Miyagi) that encompass the disaster area.
Once again, to avoid any possible contamination, we exclude all the firms in the four Pacific coast prefectures of Aomori, Fukushima, Iwate, and Miyagi from the regressions.

The results, reported in the last column of Table 6, are consistent with our baseline estimates, illustrating the presence of both upstream and downstream propagation to the direct and indirect customers and suppliers of the flooded area firms. Compared to the baseline specification, however, the point estimates for the impact of the shock on the distance 1 customers and suppliers are larger in magnitude (rising from 2.0 to 3.7 percentage points for downstream firms and from 1.2 to 1.8 percentage points for upstream firms). This increase in the intensity of propagation is unsurprising as the brunt of the damages was mostly concentrated in the coastal regions that were exposed to the tsunami (as documented in Figure 2).

5.4 Horizontal Propagation

Recall from our theoretical framework in Section 2 that, in addition to upstream and downstream propagation, firm-level shocks can also propagate “horizontally” to firms who share common customers with the disaster-stricken firms.

As characterized by Corollary 3, the nature and extent of such horizontal propagation depend on two distinct and potentially opposing forces. On the one hand, if a firm’s output is a good substitute for the input supplied by another firm, a negative shock to the former would lead to an increase in the latter’s output. In the context of our empirical study, this would give rise to a reallocation from disaster area firms towards firms that shared common customers with disaster-stricken firms but were not directly affected by the earthquake themselves. On the other hand, however, if the primary input is also a good substitute for the intermediate goods bundle, the negative shock to disaster area firms would manifest itself as a negative demand shock to their “co-suppliers,” as it would lead their common customers to substitute away from intermediate into primary inputs. Indeed, the (negative) patterns of upstream propagation in the aftermath of the earthquake — as documented in Table 4 — are consistent with the presence of such substitution effect.

To assess the sign and size of this horizontal propagation mechanism, we augment our baseline empirical specification (5) as follows:

$$\Delta \log(Sales_{i,p,s}) = \alpha + \beta_{\text{horiz}} \cdot \text{Horizontal}_i + \sum_{k=1}^{4} \beta_{\text{down}}^{(k)} \cdot \text{Downstream}_{i}^{(k)} + \sum_{k=1}^{4} \beta_{\text{up}}^{(k)} \cdot \text{Upstream}_{i}^{(k)} + \gamma' X_i + \mu_p + \lambda_s + \varepsilon_i, \quad (6)$$

where $\text{Horizontal}_i$ is a dummy variable indicating whether firm $i$ shared at least one common customer with disaster-area firms in the year prior to the earthquake. The coefficient $\beta_{\text{horiz}}$ measures the differential post-earthquake sales performance of such co-suppliers relative to the control group. The latter is now composed of firms that prior to the earthquake were at (network) distance 5 or larger from disaster area firms and that did not share common customers with disaster area firms. As before, we exclude all firms whose headquarters were located in the four earthquake affected prefectures in order to mitigate concerns regarding the exclusion restriction.
Table 7. Horizontal Propagation

<table>
<thead>
<tr>
<th></th>
<th>Baseline (Post-Earthquake)</th>
<th>Placebo (Pre-Earthquake)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Horizontal Co-suppliers</td>
<td>-0.002</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Downstream Distance 1</td>
<td>-0.020***</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Downstream Distance 2</td>
<td>-0.013***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Downstream Distance 3</td>
<td>-0.013***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Downstream Distance 4</td>
<td>-0.011***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Upstream Distance 1</td>
<td>-0.012***</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Upstream Distance 2</td>
<td>-0.007***</td>
<td>-0.007**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 3</td>
<td>-0.007**</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Upstream Distance 4</td>
<td>0.001</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.021**</td>
<td>0.069**</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Firm Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Prefecture FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>419,897</td>
<td>407,387</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.022</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates from regressing firms’ sales growth rates on various dummy variables indicating both co-supplier and direct or indirect supplier-customer relationships with disaster area firms. The first column gives estimates corresponding to regression (6) for post-earthquake sales growth rates. The second column reports the coefficients from regressing firms’ pre-earthquake sales growth rates on co-supplier and supplier-customer relationship dummies. Firm controls include the logarithm of the number of transaction partners, age, logarithm of the number of employees, distance to the disaster area, and number of plants. Robust standard errors are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 7 summarizes the results. Column (1) shows that horizontal effects are economically and statistically insignificant as drivers of post-earthquake firm-level growth. Note also that our previous results on downstream and upstream propagation remain unaffected upon the inclusion of co-supplier indicator variables. This suggests that the above described forces — reallocation towards co-supplying firms, on the one hand, and downstream substitution away from the intermediate inputs, on the other — largely cancel out, leaving the performance of these firms unchanged in the year after the earthquake. Viewed through the lens of Corollary 3, these results suggest that the
goods produced by co-supplier firms are gross substitutes to those of disaster area firms but that this
effect is offset by the gross substitutability of the intermediate input bundle and primary factors of
production. Finally, in Column (2) we confirm that these results are not driven by pre-earthquake
trends differentially affecting co-suppliers.

Overall, the lack of an effect on the typical co-supplying firm implies that horizontal propagation
effects were not important drivers of post-earthquake aggregate dynamics to which we now turn.\(^{20}\)

5.5 Aggregation

Our results thus far provide estimates for the nature and intensity of the propagation of the natural
disaster shock over supply chain linkages. We conclude this section by using these findings to obtain
a back-of-the-envelope estimate for the overall macroeconomic impact of the earthquake and its
aftermaths on the Japanese economy.

As a first observation, recall that our pre-earthquake sample includes 18,187 firms in the disaster
area, accounting for 1.3% of all sales in our sample. Also recall from Table 2 that disaster area
firms experienced (on average) a sales growth rate of \(-4.8\%\) in the year immediately following the
earthquake. Attributing the entire sales growth variation of these firms to the shock thus implies
that, absent any other propagation or amplification mechanism, the earthquake can account for a
\(-0.048 \times 0.013 = -0.06\) percentage point drop in gross output growth in the aggregate. Yet, based
on Indices of All Industry Activity constructed by METI, Japan’s gross output growth rate declined
by approximately 1.9 percentage points in FY 2011. This observation suggests that the earthquake
and its aftermaths cannot, in and of themselves, account for the decline in Japan’s post-earthquake
growth rate.

Next, we quantify the extent to which the propagation of the shock over input-output linkages
to firms outside the disaster area can account for this disparity. To this end, we sum the estimates
for the intensity of propagations (obtained using our baseline regression (5) and reported in the
second column of Table 4) weighted by the corresponding sales shares of firms at various network
distances from disaster area firms (reported in the last column of Table 8). Taking these weights as a
baseline and holding all else constant, we find that the direct and indirect propagation of the shock
over input-output linkages can account for a 1.2 percentage point reduction in the growth rate of
Japan’s gross output in the year following the earthquake. Comparing this figure with 0.06 indicates
that the aggregate impact of the natural disaster is one order of magnitude larger compared to a
counterfactual economy with no linkages between firms inside and outside the disaster area. It also
illustrates that the propagation of the shock to firms outside the disaster area can account for roughly
\(0.012 / 0.019 = 63\%\) of the decline in Japan’s gross output in the year following the earthquake.

A significant fraction of this decline is due to the downstream propagation of the shock to the
direct and indirect customers of disaster area firms: whereas downstream propagation can account

\(^{20}\)Note that our results regarding the insignificance of average horizontal effects do not imply that horizontal propagation
is not an important driver of output dynamics for some specific co-supplying firms. In particular, as Barrot and Sauvagnat
(2016) demonstrate, if the disrupted firm produces a relation-specific input — i.e., an input that is difficult to substitute
away from — the disrupted firm’s co-suppliers may be significantly affected.
Table 8. Distribution of Firms at Various Network Distances from Disaster Area Firms

<table>
<thead>
<tr>
<th></th>
<th>Number of Firms (1)</th>
<th>Fraction of Firms (2)</th>
<th>Sales Share (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disaster Area</td>
<td>18,187</td>
<td>0.029</td>
<td>0.013</td>
</tr>
<tr>
<td>Downstream Distance 1</td>
<td>10,309</td>
<td>0.016</td>
<td>0.303</td>
</tr>
<tr>
<td>Downstream Distance 2</td>
<td>165,177</td>
<td>0.264</td>
<td>0.341</td>
</tr>
<tr>
<td>Downstream Distance 3</td>
<td>98,863</td>
<td>0.158</td>
<td>0.056</td>
</tr>
<tr>
<td>Downstream Distance 4</td>
<td>11,296</td>
<td>0.018</td>
<td>0.003</td>
</tr>
<tr>
<td>Upstream Distance 1</td>
<td>7,903</td>
<td>0.013</td>
<td>0.074</td>
</tr>
<tr>
<td>Upstream Distance 2</td>
<td>72,675</td>
<td>0.116</td>
<td>0.048</td>
</tr>
<tr>
<td>Upstream Distance 3</td>
<td>29,371</td>
<td>0.047</td>
<td>0.005</td>
</tr>
<tr>
<td>Upstream Distance 4</td>
<td>8,374</td>
<td>0.013</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notes: This table reports the distribution of firms at various network distances from disaster area firms. The first and second columns report the number and fraction of firms. The denominator in calculating the fraction of firms for the second column is equal to 624,859, which is the total number of firms in the sample with observable pre-earthquake sales figures. The third column reports the distribution of firms’ sales shares.

for a 1.1 percentage point reduction in the growth rate of aggregate gross output, only the remaining 0.1 percentage points are attributable to upstream propagation. This disparity is due to the fact that (i) as we already argued, the intensity of downstream propagation is higher compared to that of upstream propagation and (ii) firms downstream to disaster area firms are both more numerous and larger on average compared to their upstream counterparts.

As a final remark, we note that a comparison of firms at various network distances illustrates that the indirect propagation of the shock to firms with no direct linkages to disaster area firms is quantitatively as important as the shock’s impact on disaster-stricken firms’ immediate transaction partners. In particular, out of the overall 1.2 percentage point reduction in growth rate of gross output, 0.7 percentage points were due to the shock’s propagation to disaster area firms’ immediate suppliers and customers, with the remainder attributable to the indirect propagation of the shock to firms further upstream or downstream the chain. This is due to the fact that even though the intensity of propagation decays as the shock travels over the chain, a significant fraction of firms in our sample have indirect linkages (of distance 2 or 3) to the firms in the disaster area.

6 Conclusions

At the backbone of any modern economy is an entangled web of specialized production units, ensuring the flow of goods and services from suppliers to other producers and final consumers. Precisely due to their vital role in the production process, an emerging literature has argued that
these input linkages can serve as a channel for the propagation and amplification of risk throughout the economy.

In this paper, we investigate the nature and extent of these propagation effects by using a large-scale dataset on supply chain linkages among Japanese firms together with information on firm-level exposures to the Great East Japan Earthquake in 2011. Leveraging the exogenous and localized nature of the earthquake and its aftereffects, we find strong evidence for the importance of inter-firm linkages as a shock transmission mechanism, documenting that (i) the earthquake resulted in a decline in the growth rates of disaster area firms’ downstream customers and upstream suppliers; (ii) the shock propagated beyond the immediate transaction partners of the firms located in the disaster area to firms that were only indirectly linked to disrupted firms; (iii) the propagation intensity was weaker for firms that were further away from disaster area firms in terms of supply chain distance; and (iv) the downstream effect was quantitatively larger than the upstream effect. We then use our empirical findings to obtain an estimate for the role of input-output linkages in shaping the macroeconomic impact of the earthquake and its aftereffects. We find that the propagation of the shock over input-output linkages can account for a 1.2 percentage point decline in Japan’s gross output in the year following the earthquake.

At the micro level, our findings suggest that, when faced with a supply-chain disruption, individual firms are unable to find suitable alternatives in order to completely insulate themselves from the shock (at least in the short run). This is consistent with an emerging literature (e.g., Bernard, Moxnes, and Saito (2016), Eaton, Kortum, and Kramarz (2016), and Barrot and Sauvagnat (2016)) that emphasizes the importance of search frictions and relation-specific investments along supply chains. However, it should be noted that our results are indicative of relatively small firm-level effects on average — corresponding to roughly a 2 percentage point decline in firm-level annual sales growth in our baseline specification — specially when compared to average firm-level sales growth volatility (which, as per Davis, Haltiwanger, Jarmin, and Miranda (2007), is arguably one order of magnitude larger). At the macro level, our results point to the structure of linkages as a key driver of aggregate fluctuations. In particular, they indicate that, even if average firm-level effects are not necessarily large, short firm-to-firm distances (in the supply chain sense) guarantee that disturbances can propagate to a significant fraction of firms, thus resulting in movements in the aggregates. Taken together, our findings suggest that linkages across various units within the economy may have quantitatively non-trivial implications at both micro and macro levels.

We view our paper as a step towards a systematic empirical investigation of the role of input-output linkages as a mechanism for propagation and amplification of shocks. Several important issues, however, remain open to future research. First, while binary information on the presence of firm-level linkages enabled us to obtain estimates for the extent to which shocks propagate, using more detailed information on the value of firm-to-firm transactions would pave the way for a more structural estimation approach. Administrative firm-level data from countries with value-added tax (such as the dataset constructed by Dhyne et al. (2015) and Magerman et al. (2016) for Belgium) would serve as the ideal dataset for such a study.
Second, even though the input-output network in our model is assumed to be exogenous, in reality, firms decide on the set of suppliers and customers that they transact with. It is reasonable to expect that the extent to which firms can form new linkages has first-order implications for the nature and intensity of shock propagation (in particular, in horizons longer than what we considered in the paper). While recent work, such as Oberfield (2013), have focused on how firm-level decisions shape productivity and the organization of production, developing a comprehensive framework for endogenous formation of networks would be crucial for the theoretical and empirical investigation of the role of input-output linkages in the propagation and amplification of shocks.
A Appendix: Proofs

Proof of Proposition 1

We start by deriving two equations that jointly determine equilibrium prices and quantities in the economy described in Subsection 2.1. The first-order conditions of firm $i$'s problem imply that

$$ l_i = (1 - \mu) y_i Z_i^{-1} p_i^\sigma $$

(7)

$$ x_{ij} = \mu a_{ij} y_j Z_j^{-1} \left( \frac{p_i}{p_j} \right)^\zeta \left( \sum_{k=1}^n a_{ik} \left( \frac{p_i}{p_k} \right)^{\zeta-1} \right)^\frac{\zeta-\sigma}{1-\zeta} $$

(8)

where we are taking the market wage as the numeraire. Plugging the expressions for $l_i$ and $x_{ij}$ into firm $i$'s production function (1), we obtain

$$ (p_i Z_i)^{1-\sigma} = 1 - \mu + \mu \left( \sum_{j=1}^n a_{ij} p_j^{1-\zeta} \right)^\frac{\zeta-\sigma}{1-\zeta} $$

(9)

On the other hand, the market clearing condition for good $i$ is given by $y_i = c_i + \sum_{j=1}^n x_{ji}$. Therefore,

$$ y_i = \frac{1}{np_i} + \mu p_i^{-\zeta} \sum_{j=1}^n a_{ji} y_j Z_j^{-1} p_j^{\sigma} \left[ \sum_{k=1}^n a_{jk} p_k^{1-\zeta} \right]^{\frac{\zeta-\sigma}{1-\zeta}} $$

(10)

where we are using equation (8) and the fact that $c_i = 1/(np_i)$. Therefore, equilibrium prices and quantities are determined by solving system of equations (9) and (10).

Since this system of equations does not have a closed-form solution in general, we next provide a first-order approximation of equilibrium prices and quantities by log-linearizing the two equations around the point $\epsilon_i = \log(Z_i) = 0$ for all firms $i$.

As a first observation, note that if whenever $\epsilon_k = 0$ for all firms $k$, then (9) implies that $p_k = 1$ for all $k$. Consequently, by equation (10), the output of firm $i$ is given by $y_i = v_i/n$, where $v_i = \sum_{k=1}^n \ell_{ki}$ denotes the $i$-th column sum of the economy’s Leontief inverse.

Now consider the small shock perturbation in which firm $j$ is hit with a small productivity shock $\epsilon_s$. Taking logarithms from both sides of (9), differentiating it with respect to $\epsilon_j$, and evaluating it at the point in which $\epsilon_k = 0$ for all $k$, we obtain

$$ \frac{\partial \hat{p}_i}{\partial \epsilon_j} + 1_{(i=j)} = \mu \sum_{k=1}^n a_{ik} \frac{\partial \hat{p}_k}{\partial \epsilon_j} $$

where $\hat{p}_k = \log(p_k)$ and $1$ denotes the indicator function. It is therefore immediate that the derivate of log prices around the point in which $\epsilon_k = 0$ for all firms $k$ is given by

$$ \frac{\partial \hat{p}_i}{\partial \epsilon_j} = -\ell_{ij} $$

(11)

The above equation thus provides a first-order approximation to the impact of a small shock to firm $j$ on the price of good $i$ in the economy.
To determine the shock’s impact on equilibrium quantities, we next log-linearize (10) around the point \( \epsilon_k = 0 \) for all \( k \). In particular, taking logarithms from both sides of (10) and differentiating it with respect to \( \epsilon_j \), we obtain

\[
v_i \frac{\partial \hat{y}_i}{\partial \epsilon_j} = \frac{\partial \hat{p}_i}{\partial \epsilon_j} - \mu \sum_{k=1}^{n} a_{ki} v_k + \mu \sum_{k=1}^{n} a_{ki} v_k \frac{\partial \hat{y}_k}{\partial \epsilon_j} + \mu (\sigma - 1) a_{ji} v_j + \mu \sigma \sum_{k=1}^{n} a_{ki} v_k + \mu (\zeta - \sigma) \sum_{k=1}^{n} a_{ki} v_k \sum_{r=1}^{n} a_{kr} \frac{\partial \hat{p}_r}{\partial \epsilon_j}.
\]

Replacing for the derivative of log equilibrium prices with respect to \( \epsilon_j \) from (11) and using the fact that \( \mu \sum_{k=1}^{n} a_{ki} v_k = v_i - 1 \) implies that

\[
v_i \frac{\partial \hat{y}_i}{\partial \epsilon_j} = (1 + \zeta (v_i - 1)) \ell_{ij} + \mu \sum_{k=1}^{n} a_{ki} v_k \frac{\partial \hat{y}_k}{\partial \epsilon_j} + \mu (\sigma - 1) a_{ji} v_j - \mu \sigma \sum_{k=1}^{n} a_{ki} v_k \ell_{kj} - \mu (\zeta - \sigma) \sum_{k=1}^{n} a_{ki} v_k \sum_{r=1}^{n} a_{kr} \ell_{rj},
\]

On the other hand, the fact that \( \mu AL = L - I \) implies that \( \mu \sum_{r=1}^{n} a_{kr} \ell_{rj} = \ell_{kj} - 1_{\{k=j\}} \). Therefore, the above expression can be simplified as

\[
v_i \frac{\partial \hat{y}_i}{\partial \epsilon_j} - \mu \sum_{k=1}^{n} a_{ki} v_k = \left( v_i \ell_{ij} - \mu \sum_{k=1}^{n} a_{ki} v_k \ell_{kj} \right) + (\mu - 1)(1 - \mu) \left( \sum_{k=1}^{n} a_{ki} v_k \ell_{kj} - a_{ji} v_j \right)
\]

Multiplying both sides of the above equation by \( \ell_{ji} \), summing over all firms \( i \), and dividing by \( v_i \) implies

\[
\frac{\partial \hat{y}_i}{\partial \epsilon_j} = \ell_{ij} + \frac{1}{\mu v_i} (\sigma - 1)(1 - \mu) \left( v_j 1_{\{i=j\}} + \sum_{k=1}^{n} v_k \ell_{kj} \ell_{ki} - v_i \ell_{ij} - v_j \ell_{ji} \right)
\]

which reduces to (3) whenever \( i \neq j \).

\[\square\]

**Proof of Corollary 1**

**Proof of part (a)** By Proposition 1, the impact of a shock to firm \( j \) on firm \( i > j \) further downstream in the production chain is given by

\[
\frac{\partial \hat{y}_i}{\partial \epsilon_j} = \mu^{i-j} + (\sigma - 1) \left( \frac{\mu^i - \mu^n}{1 + \mu^i} \right),
\]

which simplifies to

\[
\frac{\partial \hat{y}_i}{\partial \epsilon_j} = \frac{1}{1 + \mu} \left[ \mu^{i-j} + \mu^{n-j+1} + \sigma (\mu^{i-j+1} - \mu^{n-j+1}) \right].
\]

It is immediate that the right-hand side of the above expression is positive for all values of \( \sigma \).

\[\square\]
Proof of part (b)  Recall that the impact of a shock to firm \( j \) on the output of firm \( i > j \) is given by (12). Furthermore, note that the right-hand side of (12) is decreasing in \( i \). Therefore, the impact of the shock on firm \( i \) is smaller the further downstream firm \( i \) is with respect to firm \( j \) (i.e., the larger \( i \) is).

Proof of part (c)  The fact that \( \mu < 1 \) guarantees that the right-hand side of (12) is increasing in \( \sigma \). Therefore, the impact of the shock to firm \( j \) on all downstream firms \( i > j \) is increasing in \( \sigma \).

Proof of Corollary 2

Proof of part (a)  By Proposition 1, the impact of a shock to firm \( j \) on firm \( i < j \) further upstream in the production chain is given by

\[
\frac{\partial \hat{y}_i}{\partial \epsilon_j} = (\sigma - 1) \left( \frac{\mu^i - \mu^n}{1 + \mu} \right) \left( \frac{1 - \mu^{n-j+1}}{\mu^{i-1} - \mu^n} \right). \tag{13}
\]

It is therefore immediate that as long as firm \( j \) is not at the bottom of the production chain (that is, \( j \neq n \)), the expression on the right-hand side of (13) has the same sign as \( \sigma - 1 \). Consequently, as long as \( \sigma > 1 \), a negative shock to firm \( j \) reduces the output of firm \( i \). On the other hand, if \( \sigma < 1 \), a negative shock to firm \( j \) increases the output of firm \( i \).

Proof of part (b)  Equation (13) implies that \(|\partial \hat{y}_i/\partial \epsilon_j|\) is increasing in \( i \). Therefore, the (absolute value of the) impact of the shock to firm \( j \) on firm \( i \) is smaller the further upstream firm \( i \) is (that is, the smaller \( i \) is).

Proof of part (c)  To compare the extent of upstream and downstream propagation, consider two firms indexed \( j + d \) and \( j - d \) that are, respectively, downstream and upstream to firm \( j \). Note that the two firms are equidistance from firm \( j \) in the production chain. Equation (12) implies that, in a long enough chain (that is, as \( n \to \infty \)), the magnitude of the impact of a shock to firm \( j \) on the downstream firm \( j + d \) is given by

\[
\left| \frac{\partial \hat{y}_{j+d}}{\partial \epsilon_j} \right| = \mu^d \left( \frac{1 + \mu \sigma}{1 + \mu} \right).
\]

Similarly, equation (13) implies that the magnitude of the impact of the shock on firm \( j - d \) is

\[
\left| \frac{\partial \hat{y}_{j-d}}{\partial \epsilon_j} \right| = \mu^{d+1} \left| \frac{\sigma - 1}{1 + \mu} \right|.
\]

Comparing the above two expressions, it is immediate that \(|\partial \hat{y}_{j+d}/\partial \epsilon_j| > |\partial \hat{y}_{j-d}/\partial \epsilon_j|\) for all positive integer values of \( d \). Therefore, the downstream effect is always stronger than the upstream effect in magnitude.
Proof of Corollary 3

Consider the Y-shaped production network depicted in the right panel of Figure 1. Proposition 1 implies that the impact of a shock to firm $v$ on the output of firm $u$ is given by

$$\frac{\partial \hat{y}_u}{\partial \epsilon_v} = \frac{\mu(1 - \mu^n)}{2(1 - \mu)(2 - \mu - \mu^{n+1})} \left[ (1 - \zeta) + (\sigma - 1) \left( \frac{1 - \mu^{n+1}}{1 + \mu} \right) \right].$$

From the above equation, it is immediate that the impact of the shock to firm $v$ on the output of firm $u$ is increasing in $\sigma$ but decreasing in $\zeta$. 

$\Box$
References


