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Structural Household Finance

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ABSTRACT

The analysis of household finance has non-negligible implications in asset pricing literature and other areas, but empirical research on this topic is a challenging task. I construct the model to consider two kinds of heterogeneity: incomplete market and limited participation, and implement the density matching approximate Bayesian computation algorithm with the cross-sectional household portfolio survey data. I find that the estimate of relative risk aversion parameter takes a plausible value. This outcome implies that the equity premium puzzle can be due to upward bias from a specification error associated with the representative agent economy.

JEL classification: G11, C10, C80
Keywords: Approximate Bayesian Computation, Sequential Monte Carlo, Structural Estimation, Household Portfolio

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I. Introduction

Although household asset allocation behavior is disproportionately important in asset pricing and other areas (e.g. tax rate on capital gains and re-distributional effects of inflation (Doepke and Schneider (2006))), research on household finance has not developed sufficiently. According to Campbell (2006), there are two challenges with regard to household finance: how to measure the household portfolio choice precisely and how should the decision-making be modeled adequately. Additionally, I think there is a third challenge, that is, how to estimate the structural parameters in the proposed theoretical model with the data of household portfolio choice.

With respect to the first point, the most reliable survey on financial wealth in the U.S. is the Survey of Consumer Finances (SCF). The SCF is a triennial cross-sectional survey on financial wealth conducted since 1962. It has excellent coverage by both age and wealth, and the sample size of the survey is about 6,000 families. Although we do not know about the asset diversification (e.g. we know the total amount of stock, but do not know the holdings of individual stocks), we know about the asset allocation because it includes the balance of safe assets (deposits and bond holdings) and risky assets (stocks and mutual funds). The biggest challenge for an empirical analysis is that the survey does not track each household but refreshes a household sample each time. Therefore, we cannot employ dynamic panel estimation techniques to calibrate the structural parameters in the dynamic model. The situation is almost the same in different countries, including Japan.[1]

With respect to the second point, the question of how to model the household portfolio choice has mainly been discussed in the asset pricing literature. However, the research interest has not been to model individual household portfolio choice decision-making, but to explain aggregate stock market behavior. These theoretical challenges are collectively dubbed consumption-based asset pricing models (C-CAPM; e.g. Ludvigson (2015)). Formally, the C-CAPMs are built on the representative agent formulation where structural parameters are calibrated by aggregate statistics. As symbolized by the equity premium puzzle first introduced by Mehra and Prescott (1985), the standard representative agent model comes out to be failure when attempting to explain a number of facts about asset pricing (Campbell (2003)). Although various extensions (e.g. habit or recursive utility (Epstein and Zin (1989), Weil (1989)), rare event (Barro (2006), Julliard and Ghosh (2012))) were invented to improve the performance, they cannot fully resolve the equity premium puzzle.

A different strand of literature focuses on heterogeneity across households. This literature is generally classified into two groups. One group focuses on the interactions of heterogeneous agents who can partially insure against idiosyncratic risks. Since there exists an incompleteness in the insurance market and agents are not identical in wealth levels, neglected heterogeneity can alter asset pricing implication induced by the representative agent economy. The other group focuses on the fact that not everyone participates in the stock market, and therefore stock price depends only on stock market participants; on the other hand, bond price depends on all the households. This limited participation also has different asset pricing implications from the representative framework.

The first group considers the following precautionary saving mechanism. In complete insurance markets, households can completely hedge their individual risk and each consumption level is proportional to the aggregate consumption level. But, in incomplete insurance markets, the volatility of each consumption level can be higher than the aggregate, and the asset pricing mechanism can vary. Telmer (1993) and Lucas (1994) considered the general equilibrium economy with transitory idiosyncratic shocks and borrowing or short-sales constraints, and concluded that the incompleteness itself cannot affect pricing, because households who face uninsured idiosyncratic risks can hedge their risk by trading assets through the financial market (self-insurance). Aiyagari and Gertler (1991) and Heaton and Lucas (1996) considered a similar economy but with trading costs. By introducing frictions such as trading costs, households have some limitations in hedging their own risk via trading, and accumulate precautionary savings as a buffer stock (Deaton (1991)). They concluded that the equity premium puzzle can be explained only when the trading costs are set to be unrealistically high. In contrast with transitory idiosyncratic shocks, Constantinides and Duffie (1996) studied permanent idiosyncratic shock. When idiosyncratic shocks are permanent, households have less incentive to trade because such trades cannot hedge their individual risk. Accordingly, the market leads to a no-trade equilibrium, the need for all assets increases, and hence the return on all assets decreases. Although the no-trade equilibrium cannot explain the observed risk premium by itself, the puzzle can be resolved when the aggregate shock and the volatility of idiosyncratic shocks are negatively correlated. Krusell and Smith (1997) studied whether the research outcomes relied on realistic heterogeneity or not. There are two types of model setups: two infinitely lived agents [Telmer (1993), Lucas (1994), and Heaton and Lucas (1996)], or a continuum of agents [Aiyagari and Gertler (1991) and Constantinides and Duffie (1996)]. In lieu of using the two agent setup, which is easy to compute but makes it hard to match their outputs with cross-sectional observations (e.g. no trade equilibrium of Constantinides and Duffie (1996) generates unrealistic degenerate distributions.), Krusell and Smith (1997) constructed the same mechanism on the realistic richer population structure. They concluded that the puzzle is not in conformity with realistic wealth heterogeneity.

The second group focuses on limited participation, which was first stressed by Mankiw and Zeldes (1991). Mankiw and Zeldes (1991) and Attanasio, Banks, and Tanner (2002) empirically found that the consumption growth of stockholders is systematically bigger than that of non-stockholders. This might imply that the consumption growth of non-stockholders does not depend on stock returns, which is different from the assumption of the standard representative agent formulation. Therefore, the estimates based on the standard C-CAPM can lead to inconsistent estimates. Vissing-Jørgenson (2002) and Paiella (2004) studied the representative economy only with stockholders or a representative stockholder economy; meanwhile, Guvenen (2009) and Attanasio and Paiella (2011) studied the two infinitely-lived agents economy with stockholders and non-stockholders. The main difference between these papers is whether limited participation was exogenous or endogenous. Despite the differences in setup, these papers showed that accounting for participation heterogeneity can serve to reconcile theoretical outcomes with empirical evidence.

With respect to the third point, i.e. how to estimate the structural parameters of an incomplete market/limited participation model is statistically challenging. In general, an empirical test of the heterogeneous households' portfolio choice model requires disaggregated household-level portfolio panel data. However, we cannot use the household portfolio panel data for estimation because the SCF refreshes the sample every survey, as described above. Instead of using the household portfolio panel data, some studies used the household income panel data to test only the incomplete market implications. For example, Storesletten, Telmer, and Yaron (2004) used the Panel Study of Income Dynamics (PSID); on the other hand, Brav, Constantinides, and Geczy (2002), Cogley (2002), Vissing-Jørgenson (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009) used the Consumer Expenditure Survey (CEX). There are a few problems in following their estimation method, aside from their mixed implications. First, consumption data in PSID is available only for food. Thus, there is a general concern about its legitimacy as an empirical counterpart of the dynamic general equilibrium object. Second, CEX is a rotation panel which tracks each individual household only for four consecutive quarters. Because of its limited time-series dimension, most studies focused on cross-sectional moments of consumption growth and estimated the Euler equation. But, Toda and Walsh (2015) pointed out that the existence of higher-order moments is not guaranteed in general. Therefore, estimates based on these moments are not compatible. Thirdly, consumption data from household-level surveys is only available with a variety of measurement errors. When we use the Euler equation estimation, the measurement error is raised to a power and thus leads to larger specification errors. Fourth, they could not use the information on household portfolio compositions for estimation. Since the previous literature focused on whether the proposed model could explain the observed equity premium level or not, their Euler equation estimation was sufficient to test the empirical validity of the asset pricing. From the viewpoint of household finance, however, how households compose their financial portfolio is also important because it exhibits the household risk attitude. So, I should care not only whether the simulated distribution matches the empirical one, but also care whether the asset allocation policy matches the experiential one. Gomes and Michaelides (2008) also tried to match the stock allocation, but they focused only on the average share and not on the policy.

In this paper, I consider two kinds of heterogeneity at the same time: wealth heterogeneity from uninsured idiosyncratic risk and limited participation. Then, I employ an inference method to estimate the parameters characterizing dynamics only with the cross-sectional household portfolio survey data. Finally, I estimate the structural parameters of the model by applying the proposed structural estimation method to the Japanese households' portfolio data from the National Survey of Family Income and Expenditure, a cross-sectional survey on the overall family budget structure.

Theoretically, one of the critical drawbacks in the model of limited participation is the outcome relying on unrealistic wealth heterogeneity, which Krusell and Smith (1997) criticized; while, the agents in the Aiyagari-style general equilibrium model are homogeneous with respect to stock market participation. So, structural estimations should be run on the unified framework, and otherwise leads to biased estimates. When considering participation heterogeneity, we need to choose to take the participation given or not, as also discussed in Heathcote, Storesletten, and Violante (2009). With respect to that point, Guvenen (2009) endogenize participation by exogenously assuming heterogeneity in the elasticity of intertemporal substitution (EIS) in consumption and Attanasio and Paiella (2011) assumed a participation cost, which was first studied by Luttmer (1999). But, Haliassos and Bertaut (1995) discussed that these factors cannot account for the participation puzzle empirically. In addition, Aiyagari and Gertler (1991)'s transaction costs mechanism can endogenize participation, but Vayanos (1998) empirically found that the costs were too small to explain the puzzle. Cao, Wang, and Zhang (2005) introduced Knightian uncertainty into the distribution of the asset payoff to endogenize the participation, but the empirical validity of the assumption remains in question. Thus, I treat participation as given following Vissing-Jørgenson (2002) and Paiella (2004), and employ the heterogeneous agents dynamic model to explain the stockholder's portfolio choice behavior. Hence, my model can be termed as the heterogeneous stockholders dynamic model.

By using the heterogeneous agents framework, we can numerically compute the stationary distribution. Since the distribution contains parametric information characterizing the dynamics, we can estimate the true posterior distributions of structural parameters by matching the equilibrium distribution with the observed distribution. Unfortunately, we cannot resolve the distribution analytically, as we cannot compute the likelihood and hence cannot employ maximum likelihood procedures. Instead of using the maximum likelihood method, I alternatively employ the likelihood-free inference procedure named Approximate Bayesian Computation (ABC). We can estimate the posterior distribution because ABC replaces the likelihood evaluation process with a summary statistics comparison process. Following the estimation framework, we can avoid the powered measurement error problem and can use the portfolio composition for estimation. Thus, we can get more robust estimates of structural parameters. Brav et al. (2002) performed a similar study to mine, which also considered incomplete markets and limited participation. But, their theory depended on Constantinides and Duffie (1996)'s unrealistic wealth heterogeneity and their estimation could not avoid the powered measurement error problem.

The remainder of this paper is organized as follows. The next section lays out the empirical facts about the Japanese household portfolio. Section 3 proposes the stochastic dynamic heterogeneous stockholders model, discusses the solution algorithm and calibration. Section 4 summarizes the estimation algorithm and empirical outcomes. Finally, section 5 concludes this paper.

II. Data

This section describes the Japanese household portfolio, following Bertaut and Starr-McCluer (2000) and Campbell (2006). In Japan, one of the most extensive surveys on financial wealth is National Survey of Family Income and Expenditure ("Zensho" in Japanese and hereafter, NSFIE). NSFIE is a quinquennial cross-sectional survey on the overall family budget structure conducted since 1959. The sample size is about 57,000 households including 4,400 one-person households for the 2009 survey. As in the U.S., panel data is not available.

There are a few studies about the Japanese household portfolio choice using cross-sectional survey data. For example, Iwaisako (2009) used the Nikkei Radar to summarize household portfolio allocation in Japan. Although the Nikkei Radar is the only survey that asks households their real estate wealth, their observations are limited to the Tokyo metropolitan district and the age composition is biased toward the young. Fujiki, Hirakata, and Shioji (2012) also discussed portfolio choice using the Survey of Household Finances (SHF), which is the equivalent of the SCF in the U.S. Certainly, these surveys ask households about qualitative items such as financial knowledge which is not available in NSFIE, though their sample sizes are much smaller than that of the NSFIE.[2] Because my paper focuses on the asset allocation between stocks and bonds, and not on the diversification and some qualitative factors, the NSFIE is the best data for my research interest.

Figure 1 presents the cross-sectional financial wealth distribution, the financial level for each percentile and histogram. The horizontal axis in the left figure shows the percentiles of the distribution and the vertical axis reports yen on a log scale. Financial wealth is defined as the sum of risky and safe assets. In this data, risky assets are made up of stocks and mutual funds while safe assets consist of deposits and bonds.

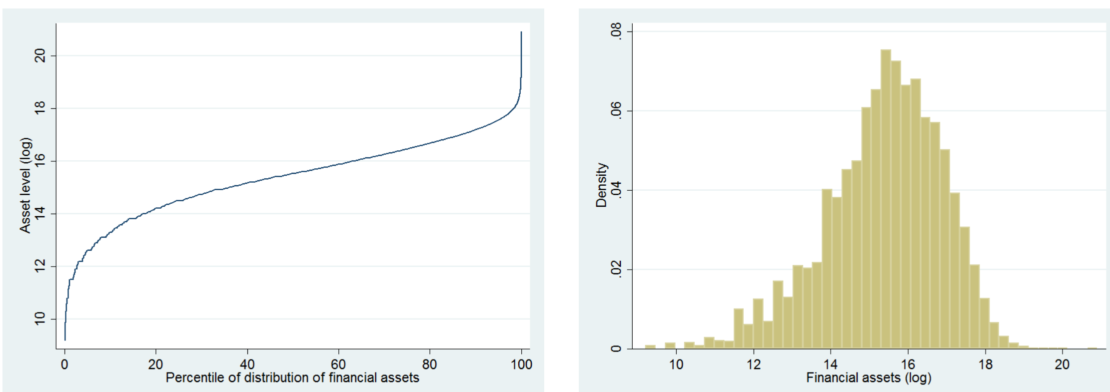

Figure 1. Japanese financial wealth distribution. The cross-sectional distribution of financial assets in the 2009 National Survey of Family Income and Expenditure.

Table I presents the summary statistics of Japanese financial wealth distribution. The median household has financial assets of 4.90 million yen and the mean has 10.65 million yen. It is clear that many households possess substantial financial assets and its distribution is highly skewed. Owing to the skewness, aggregate statistics and asset pricing highly depends on wealthy households. Thus, we cannot learn individual household financial decision making from the aggregate statistics.

Obs	Mean	Std. Dev.	Skewness	Kurtosis	Median
48828	10.65	17.54	10.66	485.17	4.90

Table I: Summary statistics. The summary statistics of the cross-sectional financial wealth distribution (million yen for mean, standard deviation, and median values)

Figure 2 presents the participation decisions of households with different wealth positions. The horizontal axis is the same as the left side of Figure 1 and the vertical axis is the participation rate in different classes of assets. Financial assets are classified into four types: stocks, bonds, ordinary deposits, and fixed deposits. Mutual funds are classified into stocks or bonds, depending on the category of the investment asset. As found for U.S. households by Campbell (2006), most Japanese households did not participate in risky financial markets and have only deposits. A fixed deposit is similar to bonds in that both guarantee depositors or investors with a higher rate of return than that of an ordinary deposit in compensation for less liquidity. The only difference between these two is whether the principal is guaranteed or not. The ordinary deposit participation rate is almost independent of wealth level; meanwhile, fixed deposits increase with wealth level. Low participation in the stock market ($\simeq 20\%$ in aggregate) is the well-known stock holding puzzle and the implicit participation cost may be the key to solve the problem. One of the biggest challenges for the financial theory is the observed limited participation among wealthy households.

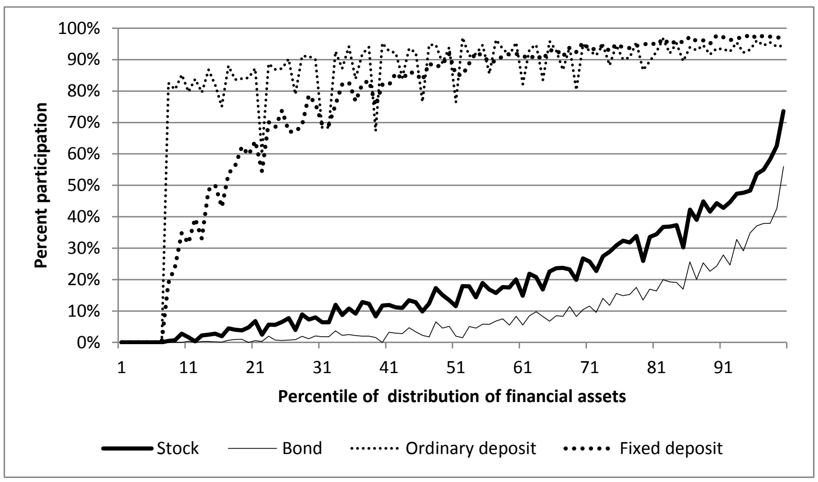

Figure 2. Participation rates by asset class. The cross-sectional distribution of asset class participation rates for the 2009 survey.

Figure 3 presents the allocation decisions of households with different wealth positions. The horizontal axis is the same as Figure 1 (left) and Figure 2, and the vertical axis shows the asset composition. The figure demonstrates that deposits play a dominant role in household financial wealth. Specifically, the share of ordinary deposits decreases with wealth level; on the other hand, the share of fixed deposits increases with wealth level up to around 60%. It can be seen that as households become wealthier, they tend to hold stock but its share is very limited. This limited share ensures the positive correlation between wealth and participation.

The NSFIE also reveals demographic factors which could affect household participation decisions and asset allocations. Age, income, sex of head($\in \{0, 1\}$, where 0 denotes women and 1 denotes men), non-labor force status($\in \{0, 1\}$), and the number of children under 18 are available in the 2009 survey.[3] Table II summarizes the effects of various factors on stock market participation and asset allocation implications without one-person households, following the specification of Campbell (2006) and Jin (2011).[4] First, I use logistic regressions to estimate the contributions of income, wealth, and demographic factors in the stock market participation decision.[5]

$$
\begin{aligned}
\theta_i^* &= \beta_0 + \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3 \ln \text{Income}_i + \beta_4 (\ln \text{Income}_i)^2 \\
&\quad + \beta_5 \ln W_i + \beta_6 (\ln W_i)^2 + \beta_7 \#\text{Children}_i + \beta_8 \text{Sex of head}_i + \beta_9 \text{Non labor force}_i + \varepsilon_i \\
d_i &= 1, \;\; if \;\; \theta_i^* > 0 \\
d_i &= 0, \;\; if \;\; \theta_i^* \leq 0 \\
Pr(d_i = 1) &= Pr(\theta_i^* > 0) = Pr(\varepsilon_i > -\beta_0 - \beta_1 ...) = F(-\beta_0 - \beta_1 ...)
\end{aligned}
$$

where θ_i^* denotes a latent optimal stock share and d_i denotes a discrete participation decision. Then, I report the OLS regression outcome of the conditional portfolio stock share on the same variables only for stockholders.

$$
\begin{aligned}
\theta_i = \beta_0 &+ \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3 \ln \text{Income}_i + \beta_4 (\ln \text{Income}_i)^2 \\
&+ \beta_5 \ln W_i + \beta_6 (\ln W_i)^2 + \beta_7 \#\text{Children}_i + \beta_8 \text{Sex of head}_i + \beta_9 \text{Non labor force}_i + \varepsilon_i
\end{aligned}
$$

The table shows that there was a strong hump-shaped age effect, positive wealth effect, and positive non-labor force effect on participation. The hump-shaped age effect implies younger households tend to buy and older households tend to sell stock. Consistent with Fujiki et al. (2012), participation is positively correlated with the wealth level, but correlation with income level is not robust in this study. The positive correlation with the non-labor force indicates that retirees tend to participate more actively in the stock market. On the whole, we cannot explain the participation behavior with only these proposed explaining variables. This observation is consistent with Haliassos and Bertaut (1995).

In terms of asset allocation, I find that a strong quadratic wealth effect is quantitatively important in explaining the conditional stock share, which is consistent with Campbell (2006). We also

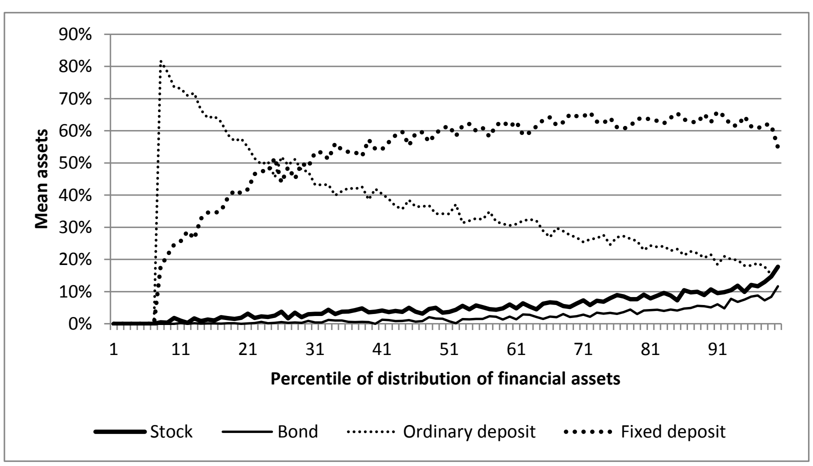

Figure 3. Asset class shares in household portfolios. The share of each asset class in the financial portfolio of households for each percentile, in the 2009 survey.

find a strong hump-shaped age effect and a positive correlation with non-labor force, though they are quantitatively less important. The quadratic wealth effect indicates that low-wealth households tend to hold stock if they participate in the stock market. On the other hand, there is a positive correlation between stock share and wealth level in the upper parts of the wealth distribution.

Dependent Variable	Stock market participation Coefficients (Logit)		Portfolio stock share Coefficients (OLS)
Age	0.02**	0.03***	0.004***
	(0.01)	(0.01)	(0.00)
Age squared	-0.0002**	-0.0003***	-0.00003**
	(0.00)	(0.00)	(0.00)
Ln(income)	1.73	1.82	-0.15
	(1.12)	(1.20)	(0.11)
Ln(income) squared	-0.04	-0.04	0.01
	(0.04)	(0.04)	(0.00)
Ln(wealth)	0.24	0.24	-0.45***
	(0.22)	(0.22)	(0.05)
Ln(wealth) squared	0.02**	0.02**	0.01***
	(0.01)	(0.01)	(0.00)
Number of children	-0.01	-0.02	-0.002
	(0.02)	(0.02)	(0.00)
Sex of head	-0.08	-0.08	-0.01
	(0.05)	(0.05)	(0.01)
Non-labor force	-	0.42***	0.02***
	-	(0.04)	(0.01)
Adjusted R^2	0.17	0.17	0.23
Log-likelihood	-19350.14	-19294.03	-
Obs	45184	45184	9467

Table II: Stock market participation and asset allocation for participants: The table reports income, wealth, and demographic determinants of stock market participation by Logit analysis, and of portfolio allocation by OLS. Standard errors are reported in parentheses under the estimates. Coefficients significant at 10% are denoted by *, 5% by **, and 1% by ***. The adjusted R^2 reported in the stock market participation is McFadden's pseudo R^2.

III. Model

A. Specification

In this section, I construct the heterogeneous stockholders model in order to generate simulated outcomes that are consistent with empirical findings. Specifically, the key empirical finding is the cross-sectional distribution of household financial wealth presented in Figure 1 and the risky asset share presented in Figure 3. Participation rates presented in Figure 2 are not the research objective because I treat participation as given following Vissing-Jørgenson (2002) and Paiella (2004). Empirically, it is equivalent to using the conditional risky asset share instead of using the unconditional share.

The economy is populated by a continuum of households, who are ex-ante homogeneous and the size of which is normalized to one. Each household maximizes their lifetime expected utility subject to budget constraints, borrowing constraints, and short-selling constraints as follows:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \quad \beta \in (0,1) \tag{1}$$

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} \tag{2}$$

$$\text{s.t.} \quad c_{i,t} + b_{i,t+1} + s_{i,t+1} = w_{i,t} + R_b b_{i,t} + \tilde{R}_{i,s} s_{i,t}, \quad \forall t \tag{3}$$

$$a_{i,t} \equiv b_{i,t} + s_{i,t} \tag{4}$$

$$b_{i,t} \geq 0 \quad \forall t \tag{5}$$

$$s_{i,t} \geq 0 \quad \forall t, \tag{6}$$

where i denotes i-th household, $c_{i,t}$ denotes consumption, $b_{i,t}$ denotes risk-free asset (called as "bond") holdings, $s_{i,t}$ denotes risky asset (called as "stock") holdings, $a_{i,t}$ denotes financial wealth composed of bonds and stocks, $w_{i,t}$ denotes exogenous earnings, R_b denotes constant bond return, and $\tilde{R}_{i,s}$ denotes stochastic stock return. Households cannot sell bonds and stocks short. The utility function is assumed to be the CRRA form and σ is the coefficient of relative risk aversion (RRA). We can rewrite equation (4) using the stock share $\theta_{i,t} \in [0,1]$:

$$s_{i,t} = \theta_{i,t} a_{i,t} \tag{7}$$

$$b_{i,t} = (1-\theta_{i,t}) a_{i,t} \tag{8}$$

The earnings follow the exogenous AR(1) process given by:

$$\ln w_{i,t} = \mu(1-\rho) + \rho \ln w_{i,t-1} + \epsilon_{i,t} \tag{9}$$

$$\epsilon_{i,t} \sim \text{ i.i.d } N(0, \sigma_w^2) \tag{10}$$

The stochastic stock return $\tilde{R}_{i,s}$ independently follows the exogenous three-state Markov process defined by:

$$\tilde{R}_{i,s} = \{R_s^l, R_s^h, R_s^c\}, \quad \forall i \tag{11}$$

$$\Pi_{R_s} = \begin{pmatrix} \pi_{ll} & \pi_{lh} & \pi_{lc} \\ \pi_{hl} & \pi_{hh} & \pi_{hc} \\ \pi_{cl} & \pi_{ch} & \pi_{cc} \end{pmatrix}, \tag{12}$$

where subscript "l" indicates the state of the low price, "h" indicates the state of the high price, and "c" indicates the crisis state. The crisis state is similar to the rare event discussed by Barro (2005) and Barro (2006). In this paper, a stock market crash is defined by a stock market price decline in excess of twenty percent within the annual window, following Mishkin and White (2002).

The individual maximization problem can be expressed as the following dynamic programing problem:

$$v_i(a_i, \theta_i; w_i, R_b, R_{i,s}) = \max_{c_i, \theta_i'} \left\{ u(c_i) + \beta \mathbb{E}[v_i'(a_i', \theta_i'; w_i', R_b, R_{i,s}') | w_i, R_{i,s}] \right\} \tag{13}$$

subject to

$$c_i + b_i' + s_i' = w_i + R_b b_i + \tilde{R}_{i,s} s_i \tag{14}$$

$$s_i = \theta_i a_i \tag{15}$$

$$b_i = (1 - \theta_i) a_i \tag{16}$$

$$\theta_i \in [0, 1] \tag{17}$$

where the apostrophe $'$ denotes the next state and $v(a, \theta; w, R_b, R_s)$ denotes the value function. The Euler equation for consumption is

$$u'(c_i) = \mathbb{E}[\beta \mathcal{R}'_i u'(c_i')] \tag{18}$$

where $\mathcal{R}'_i = R_b + (R_s' - R_b)\theta_i$, and the first order condition with respect to the stock share is

$$0 = a_i \mathbb{E}[u'(c_i')(R_s' - R_b)]. \tag{19}$$

Optimal decision rules are defined by the value function and the two policy functions:

$$c = f_c(a, \theta; w, R_b, R_s) \tag{20}$$

$$\theta' = f_{\theta'}(a, \theta; w, R_b, R_s). \tag{21}$$

We can define the cross-sectional distribution of financial wealth Γ, and there exists a stationary distribution Γ^*. In my framework, however, the risk-free rate and the risk premium are set exogenously and independent of Γ^*, because the bond market consists of both stock market participants and non-participants.[6]

The most popular algorithm to solve a stochastic dynamic optimization problem is the value function iteration (VFI) approach. VFI is time-consuming and is subject to the curse of dimensionality so that it does not seem suitable to function it as the inner loop within an estimation loop. Carroll (2006) proposed a faster algorithm named the endogenous grid-points method (EGM). One of the key ideas of EGM is to rewrite the optimization problem by employing all the available resources (which we call cash on hand and define as m_i) as a one-dimensional state variable.

$$v_i(m_i) = \max_{c_i, \theta_i'} \left\{ u(c_i) + \beta \mathbb{E}(v_i'(m_i')) \right\} \tag{22}$$

$$m_i = w_i + \mathcal{R}_i a_i \tag{23}$$

Since the original EGM can only handle the problem if it has only one control variable, it is impossible to solve my model where there are two control variables. Barillas and Fernández-Villaverde (2007) combined EGM with a standard VFI which they called the generalized EGM (GEGM) to handle an optimization problem with more than one control variable. In this paper, I apply their GEGM to solve the model, which is similar to Nirei and Aoki (2009)'s two step algorithm. Algorithm 1 gives a pseudo code to implement GEGM.

Algorithm 1 Generalized Endogenous Grid-points Method for portfolio choice

```
Number states from n to N
Initialize θ
repeat
  for n to N do
    Compute the optimal consumption/saving policy f_c(a, θ; w(n), R_s(n)) by EGM
  end for
  Update a'
  for n to N do
    Compute the optimal allocation policy f_θ(a, θ; w(n), R_s(n)) by FOC
  end for
  Update θ
until θ converges
```

B. Calibration

In order to solve the stochastic dynamic optimization problem, we need to specify the exogenous parameter sets. In my model, the parameters to be calibrated are β (discount factor), R_b (bond return), $\{R_s^l, R_s^h, R_s^c\}$, and Π_{R_s} (stochastic stock return and its transition), and the parameters to be estimated are σ (RRA) and $\{\mu, \rho, \sigma_w\}$ (dynamic earnings process). The discount factor is set at the standard value for matching annual aggregate statistics: $\beta = 0.96$. In my model, "bond" summarizes all risk-free assets including ordinary and fixed deposits. The interest rate of a bond is calibrated with the annual yield of a one-year bond, using the data from 1980 to 2009, given by the Ministry of Finance, Japan. In the same way, "stock" summarizes all risky assets including mutual funds, real estate, and private businesses. Ideally, we should specify the covariance structure in the risky assets and compute the aggregate risky asset return and its volatility, though we cannot know how much of individual risky assets each household possesses. So, I instead presume the annual return of the Nikkei 225 to be the return of aggregate risky assets,[7] using the data from FRED. Moreover, I assume that the return of each household portfolio is independent across households, though they are somewhat correlated in reality. To classify the phase from 1980 to 2009 into three states, I first split the phase whose annual return dropped by over 20% as a crisis state, following Mishkin and White (2002). Specifically, annual returns in 1990, 1992, 1997, 2000, 2001, and 2008, dropped by over 20% and their average is about −30%. Then the residual years whose average return is 12.0% and the standard deviation is 16.9% is split into two states. Calibrated values and transitions are summarized in Table III.

Description	Symbol	Value
Discount factor	β	0.96
Risk-free rate	R_b	0.029
Risky asset return	$\tilde{R}_s$	$(-0.015, 0.255, -0.300)$
Transition	Π_{R_s}	$\begin{pmatrix} 0.4143 & 0.4143 & 0.1714 \\ 0.4143 & 0.4143 & 0.1714 \\ 0.4143 & 0.4143 & 0.1714 \end{pmatrix}$

Table III: Calibrated parameter values.

IV. Estimation

A. Method

In this paper, I try to estimate the parameters of RRA: σ and of the dynamic earnings process which each household faces: $\{\mu, \rho, \sigma_w\}$. With respect to the RRA, we find much of the empirical literature is based on the representative agent economy using Japanese data (e.g. Hamori (1992a), Hamori (1992b), Nakano and Saito (1998), and Campbell (2003)), while we cannot find any empirical studies based on the heterogeneous agents economy. Since we can observe the financial wealth distribution and limited participation in the previous sections, disregarding these two kinds of heterogeneity can lead to biased estimates.

In general, we need disaggregated household-level portfolio panel data to estimate these structural parameters in the proposed dynamic heterogeneous households' portfolio choice model. Although we cannot use the portfolio panel data as in the U.S., we can use the CEX-like household income panel data for estimation. One of the important candidates is the Family Income and Expenditure Survey (hereafter, FIES), which is a rotating panel that tracks each individual household for six consecutive months. Because we use the same data structure as CEX, (i) limited time-series dimension, (ii) non-existence of higher order moments, (iii) powered measurement error, and (iv) ignorance of portfolio composition are still problems. In addition, it is difficult to adjust the seasonality of the data, which is a problem specific to the FIES.

In order to overcome these estimation problems, I employ a simulation-based Bayesian structural estimation technique which is based on the adaptive Sequential Monte Carlo Approximate Bayesian Computation algorithm (aSMC-ABC) proposed by Del Moral, Doucet, and Jasra (2012). In general, observed cross-sectional distribution of endogenous variables can be considered as an empirical counterpart of the theoretical stationary distribution. If we compute the stationary distribution which can match the observed distribution, input parameters will be good estimates of true structural parameters. This is because a stationary distribution is a function of structural parameters. Since we cannot solve the stationary distribution analytically in general, we employ the likelihood-free simulation-based inference instead of using the maximum likelihood procedure. By employing the proposed density matching estimator, (i) the estimation outcome is independent of the time-series dimension because we use only cross-sectional statistics, (ii) the estimation outcome is independent of the existence of higher-order moments, (iii) measurement error is not powered, and (iv) we can employ the portfolio composition for estimation because we can use cross-sectional portfolio survey data.

The estimation strategy is as follows: First, we construct the theoretical model as the data generating process. In this paper, I construct the dynamic heterogeneous investors' portfolio choice model in the previous section. Then, we sample the candidates of parameters from prior distributions, and compute the equilibrium outcome based on the parameters using the data generating process. Next, we compare the outcome with the observation for each parameter proposal. I use the summary statistics of the NSFIE as the observation. Finally, we continue perturbing the proposals based on the comparison results following the aSMC-ABC algorithm until the convergence criteria are met.

Since the ABC algorithm replaces the likelihood evaluation with the comparison process, the choice of summary statistics to be compared is vital. In this paper, I use two kinds of summary statistics: statistics about the distribution and about the stock holding policy. When it comes to measuring the distance between distributions, we first come up with a two step approach that estimates the distribution for each sample at first, and then measures the distance between the estimated densities such as the Kullback-Leibler (KL) divergence. Though minimizing the KL divergence is statistically equivalent to the maximum likelihood procedure, the KL divergence cannot satisfy the properties of mathematical metrics such as the symmetric property and triangle inequality. It is not robust to the outliers, and is numerically unstable. In addition, Sugiyama, Suzuki, Kanamori, du Plessis, Liu, and Takeuchi (2013) discussed that the first density estimation process is applied without considering the second process. This separation generates a small estimation error and can result in a big error in the last process. So, I employ the L^2-distance approximation method proposed by Sugiyama et al. (2013) to minimize this kind of error. L^2-distance is a standard metric to measure the distance between distributions, defined as

$$L^2(p, p') \equiv \int (p(x) - p'(x))^2 \mathrm{d}x \tag{24}$$

from i.i.d samples $\chi := \{x_i\}_i^n$ and $\chi' := \{x'_{i'}\}_{i'}^{n'}$, where p and p' are probability density functions. I use L^2-distance as the distance metric because there are some advantages in that it satisfies the definitions of mathematical metrics, it is more robust against outliers than the KL divergence, and it can be easily estimated. Since L^2 cannot be directly computed, it is approximated by the least-squares density-difference (LSDD) estimation where the optimal bandwidth of the Gaussian kernel is computed by K-fold cross validation.(Härdle, Müller, Sperlich, and Werwatz (2004))

The second summary statistics is the stock holding policy. Because the portfolio stock share of participants is not only affected by their wealth levels, but also affected by demographic factors that I cannot model, such as age, sex of the head of the family, and employment status, I linearly modify the conditional portfolio stock share to eliminate these heterogeneities with OLS coefficients. The properties of the head of the family of the representative household are set to be employed men with average income, average age, and an average number of children. The linearly modified conditional portfolio stock share is computed as:

$$\begin{aligned}\tilde{\theta}_i = \hat{\beta}_0 + \hat{\beta}_1 \overline{Age_i} + \hat{\beta}_2 \overline{Age_i}^2 + \hat{\beta}_3 \ln \overline{\text{Income}_i} + \hat{\beta}_4 (\ln \overline{\text{Income}_i})^2 \\ + \hat{\beta}_5 \ln W_i + \hat{\beta}_6 (\ln W_i)^2 + \hat{\beta}_7 \overline{\#\text{Children}_i} + \hat{\beta}_8 \times 1 + \hat{\beta}_9 \times 0 \end{aligned} \tag{25}$$

where $\hat{\beta}_i$ is the OLS coefficient estimate for each explained variable. The modified portfolio share is summarized as a discretized grid on true financial wealth quintiles, as displayed in Figure 4. Algorithm 2 gives a pseudo code to implement the distribution matching ABC algorithm.

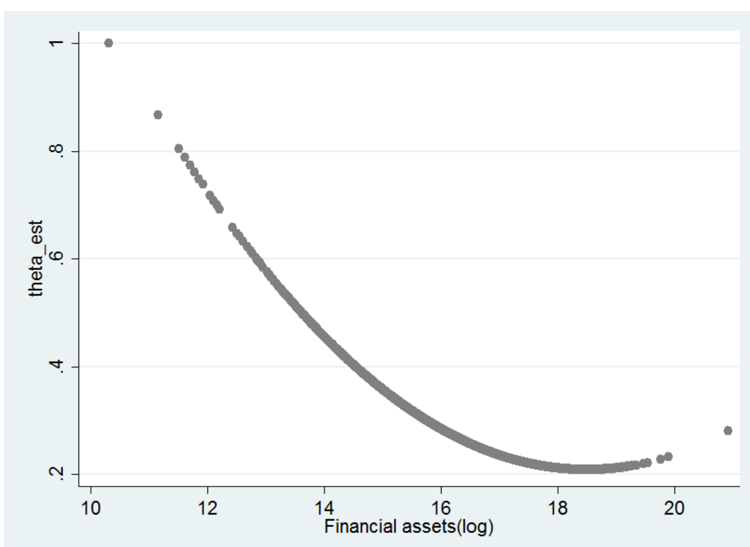

Figure 4. Modified conditional portfolio stock share. The figure shows the portfolio stock share, linearly modified with OLS coefficients.

Algorithm 2 Distribution matching ABC

Sample N parameter proposals from prior distributions
repeat
  for n to N **do**
    Simulate the model on n-th proposal, compute stationary distribution and stock share on true wealth quintiles by Algorithm 1.
    Compare summary statics of distributions.
    Compare summary statics of stock holding policies.
  end for
  Perturb the parameter proposals following normal random walk Metropolis-Hastings.
  Update the importance weight, anneal convergence criteria via computing effective sample size (ESS).
  Re-sample particles if necessary following the systematic scheme proposed by Kitagawa (1996).
until Convergence criteria are met.

B. Empirical outcome

First, I summarize the estimation settings and then show the estimation outcomes. With respect to the ABC algorithm, the number of particles is set to $N = 100$, no iteration are performed ($M = 1$), the quality index is set to $\alpha_{ABC} = .90$, convergence criterion is set to $\epsilon = .90$, and prior distributions are set to be such that $\mu \sim \mathcal{N}(15, 2)$, $\rho \sim \mathcal{BETA}(5, 2)$, $\sigma_w \sim \mathcal{IG}(3, 4)$, and $\sigma \sim \mathcal{U}(0, 50)$. Because there is no iterations, ESS is directly proportional to the number of alive particles.

The estimation results are summarized in Table IV. They imply that the RRA takes plausible value, compared to the estimates in existing literature. Accordingly, the equity premium puzzle can be interpreted as an upward bias by the specification error in the representative agent economy, which considers neither market incompleteness nor limited participation. In addition, we find that the persistence of the dynamic earnings process is around 0.85 and there is no need to assume the unit root process to mimic the observed distribution, whose outcome is consistent with Browning, Ejrnæs, and Alvarez (2010) and Gustavsson and Österholm (2014), who found strong evidence against the unit root assumption.

	μ	ρ	σ_w	σ
Prior mean	14.70	0.70	2.07	23.98
s.d.	1.85	0.16	1.44	15.50
Posterior mode	11.94	0.84	0.86	3.34
Posterior mean	12.22	0.87	1.01	3.46
Credible interval	[11.12, 13.56]	[0.80, 0.95]	[0.33, 1.75]	[2.94, 4.01]

Table IV: Summary outcomes. This table summarizes prior and posterior mean, mode, standard deviation, and 95% credible intervals.

Figure 5 compares the observed wealth distribution and the simulated distribution using posterior means and Figure 6 shows the observed stock holding shares and the simulated stock holding policy using posterior means[8], defined on true wealth quintiles. We find that the simulated policy matches the observed policy well in Figure 6, and that the simulated average conditional stock share matches the observed value in Table V. We also find that the mean of the simulated distribution (16.39) is the same as that of the observed distribution (16.44), the median of the simulated (16.54) is the same as that of the observed (16.56), and the standard deviation is less dispersed (0.77 for the simulated and 1.11 for the observed). This simulated error comes from mainly two reasons. (i) I cannot consider preference heterogeneity discussed by Krusell and Smith (1997), and (ii) the model does not care for demographic heterogeneity, such as age. Thus, if the financial wealth level depends on such a heterogeneity, that specification error would affect the simulated outcome.

Finally, I checked the validity of the calibration and estimation with the aggregate consumption-wealth ratio in Table VI. There are two reasons to use the aggregate consumption-wealth ratio. First, because the calibration targets only the financial wealth level and stock share, the ratio of the endogenous consumption level of these calibrated values is a good measure of fitness. Secondly, the aggregate consumption-wealth ratio is empirically important. Since the ratio is equivalent to the conditional expectation of difference between returns from the market portfolio and the consumption growth rate, it functions as a strong predictor of excess stock market returns (Lettau and Ludvigson (2001), Lettau and Ludvigson (2010)). As an empirical counterpart of the household consumption level, I utilize household consumption for nondurable goods in the NSFIE. Then, I compute the simulated consumption level with stationary wealth distribution, policy function and randomly generated states reflecting the stationary probability of each state. We find that the log consumption-wealth ratio of the observed and the simulated distributions are almost the same. Thus, the calibration and estimations in this study are valid.

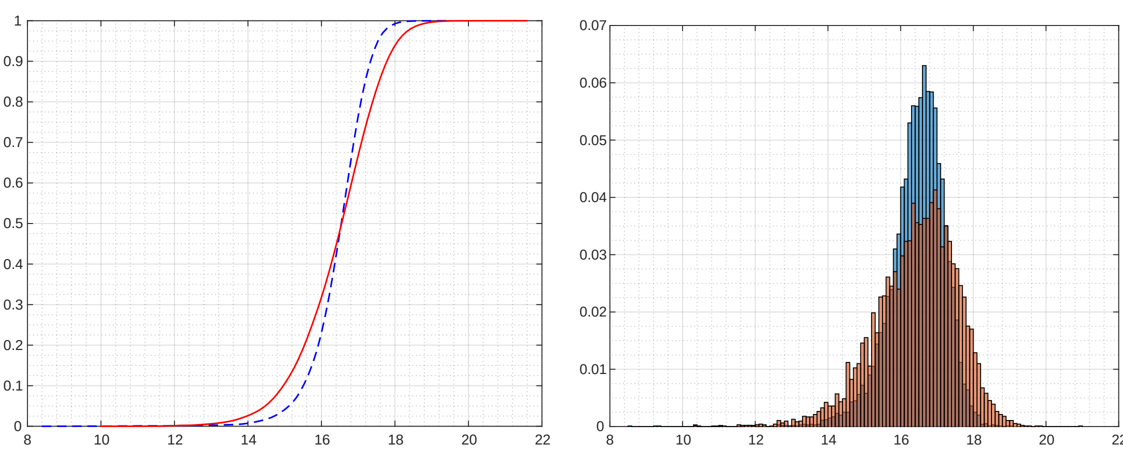

Figure 5. Distribution comparison. The figure compares the probability density functions (pdf) on the right and the cumulative distribution functions (cdf) on the left. The vertical axis measures the density and the horizontal axis measures the financial wealth level (log). The red line shows the true cdf and the blue dotted line shows the computed cdf. The red histogram shows the true asset distribution and the blue one shows the computed stationary distribution.

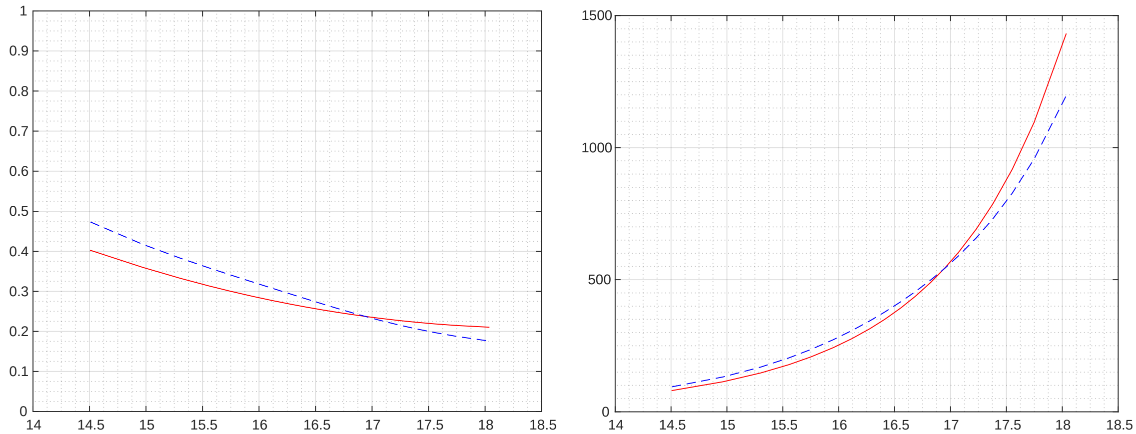

Figure 6. Stock holding policy comparison. The left figure compares the stock holding policy in each quintile and the right figure compares approximated stock holdings (10,000 yen). In the left figure, the vertical axis measures the portfolio stock share and the horizontal axis measures the quintiles; in the right figure, the vertical axis measures the approximated stock holdings and the horizontal axis measures asset quintiles. The red line denotes the observed values and the blue line shows the simulated values.

	$\overline{\theta_i}$
Observed value	0.27
Simulated value	0.28

Table V: Average conditional stock share. The table compares the observed and the simulated conditional stock shares. To compute the simulated conditional share, I generate random states following the stationary probability of each state, interpolate the simulated values of the stationary wealth distribution via stock policy functions.

	$\overline{c_i/a_i}$
Observed value	0.86
Simulated value	0.85

Table VI: Log consumption-wealth ratio. I use the household consumption for nondurable goods in the NSFIE as the observed consumption level; in order to compute the simulated consumption level, I generate the random states following stationary probability of each state, and interpolate the simulated values on the stationary wealth distribution via consumption policy functions.

V. Conclusion

This paper provides an overview of the empirical facts about portfolio allocation by Japanese households, defines a stochastic dynamic optimization problem with heterogeneous investors who decide the dynamic portfolio allocation, applies the density-matching structural estimation method to the NSFIE, and gets parametric density estimates of structural parameters. Compared to existing studies, I emphasize not only the validity of the distribution (as in Krusell and Smith (1997)), but also that of the allocation. This is because we can arrive at the empirical fact that the conditional stock share is negatively correlated with the financial wealth level. Thus, I argue the need for the model and the estimation method to consider both distribution and allocation. To test both the distributional implication and the allocation implication, I construct the model to consider two kinds of heterogeneity: incomplete market and limited participation, and implement the density matching adaptive sequential Monte Carlo approximate Bayesian computation algorithm with the cross-sectional household portfolio survey data. We find that the estimated RRA takes a plausible value relative to the estimates existing in literature. This outcome implies that the equity premium puzzle can be due to upward bias from a specification error associated with the representative agent economy and considering two kinds of heterogeneity is a key to solve the puzzle.

REFERENCES

Aiyagari, S. Rao, and Mark Gertler, 1991, Asset returns with transactions costs and uninsured individual risk, *Journal of Monetary Economics* 27, 311–331.

Attanasio, Orazio P., James Banks, and Sarah Tanner, 2002, Asset holding and consumption volatility, *Journal of Political Economy* 110, 771–792.

Attanasio, Orazio P., and Monica Paiella, 2011, Intertemporal consumption choices, transaction costs and limited participation in financial markets: reconciling data and theory, *Journal of Applied Econometrics* 26, 322–343.

Balduzzi, Pierluigi, and Tong Yao, 2007, Testing heterogeneous-agent models: an alternative aggregation approach, *Journal of Monetary Economics* 54, 369–412.

Barillas, Francisco, and Jesús Fernández-Villaverde, 2007, A generalization of the endogenous grid method, *Journal of Economic Dynamics and Control* 31, 2698–2712.

Barro, Robert, 2005, Rare events and the equity premium, NBER Working Papers 11310, National Bureau of Economic Research, Inc.

Barro, Robert, 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Bertaut, Carol, and Martha Starr-McCluer, 2000, Household portfolios in the United States, in Luigi Giso, Michael Haliassos, and Tullio Jappelli, eds., *Household Portfolios*.

Brav, Alon, George Constantinides, and Christopher C. Geczy, 2002, Asset pricing with heterogeneous consumers and limited participation: empirical evidence, *Journal of Political Economy* 110, 793–824.

Browning, Martin, Mette Ejrnæs, and Javier Alvarez, 2010, Modelling income processes with lots of heterogeneity, *Review of Economic Studies* 77, 1353–1381.

Campbell, John Y., 2003, Consumption-based asset pricing, in G.M. Constantinides, M. Harris, and R. M. Stulz, eds., *Handbook of the Economics of Finance*, Handbook of the Economics of Finance, chapter 13, 803–887.

Campbell, John Y., 2006, Household finance, *Journal of Finance* 61, 1553–1604.

Cao, H. Henry, Tan Wang, and Harold H. Zhang, 2005, Model uncertainty, limited market participation, and asset prices, *Review of Financial Studies* 18, 1219–1251.

Carroll, Christopher D., 2006, The method of endogenous gridpoints for solving dynamic stochastic optimization problems, *Economics Letters* 91, 312–320.

Cogley, Timothy, 2002, Idiosyncratic risk and the equity premium: evidence from the consumer expenditure survey, *Journal of Monetary Economics* 49, 309–334.

Constantinides, George M, and Darrell Duffie, 1996, Asset pricing with heterogeneous consumers, *Journal of Political Economy* 104, 219–40.

Deaton, Angus, 1991, Saving and liquidity constraints, *Econometrica* 59, 1221–48.

Del Moral, Pierre, Arnaud Doucet, and Ajay Jasra, 2012, An adaptive sequential Monte Carlo method for approximate Bayesian computation, *Statistics and Computing* 22, 1009–1020.

Doepke, Matthias, and Martin Schneider, 2006, Inflation and the redistribution of nominal wealth, *Journal of Political Economy* 114, 1069–1097.

Epstein, Larry G, and Stanley E Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57, 937–69.

Fujiki, Hiroshi, Naohisa Hirakata, and Etsuro Shioji, 2012, Aging and household stockholdings: evidence from Japanese household survey data, IMES Discussion Paper Series 12-E-17, Institute for Monetary and Economic Studies, Bank of Japan.

Gomes, Francisco, and Alexander Michaelides, 2008, Asset pricing with limited risk sharing and heterogeneous agents, *Review of Financial Studies* 21, 415–448.

Gustavsson, Magnus, and Pär Österholm, 2014, Does the labor-income process contain a unit root? evidence from individual-specific time series, *Journal of Economic Dynamics and Control* 47, 152–167.

Guvenen, Fatih, 2009, A parsimonious macroeconomic model for asset pricing, *Econometrica* 77, 1711–1750.

Haliassos, Michael, and Carol C Bertaut, 1995, Why do so few hold stocks?, *Economic Journal* 105, 1110–29.

Hamori, Shigeyuki, 1992a, On the structural stability of preference parameters obtained from Japanese financial market data, *Economics Letters* 40, 459–464.

Hamori, Shigeyuki, 1992b, Test of C-CAPM for Japan: 1980-1988, *Economics Letters* 38, 67–72.

Härdle, Wolfgang, Marlene Müller, Stefan Sperlich, and Axel Werwatz, 2004, *Nonparametric and Semiparametric Models* (Springer).

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, 2009, Quantitative macroeconomics with heterogeneous households, *Annual Review of Economics* 1, 319–354.

Heaton, John, and Deborah J Lucas, 1996, Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy* 104, 443–87.

Iwaisako, Tokuo, 2009, Household portfolios in Japan, *Japan and the World Economy* 21, 373–382.

Jin, Fangyi, 2011, Revisiting the composition puzzles of the household portfolio: new evidence, *Review of Financial Economics* 20, 63–73.

Julliard, Christian, and Anisha Ghosh, 2012, Can rare events explain the equity premium puzzle?, *Review of Financial Studies* 25, 3037–3076.

Kitagawa, Genshiro, 1996, Monte carlo filter and smoother for non-gaussian nonlinear state space models, *Journal of Computational and Graphical Statistics* 5, 1–25.

Kocherlakota, Narayana, and Luigi Pistaferri, 2009, Asset pricing implications of Pareto optimality with private information, *Journal of Political Economy* 117, 555–590.

Krusell, Per, and Anthony A. Smith, 1997, Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns, *Macroeconomic Dynamics* 1, 387–422.

Lettau, Martin, and Sydney Ludvigson, 2001, Consumption, aggregate wealth, and expected stock returns, *Journal of Finance* 56, 815–849.

Lettau, Martin, and Sydney Ludvigson, 2010, Measuring and modeling variation in the risk-return tradeoff, in Y. Ait-Sahalia, and L. P. Hansen, eds., *Handbook of Financial Econometrics*, volume 1, 617–690 (Elsevier Science B.V.).

Lucas, Deborah J., 1994, Asset pricing with undiversifiable income risk and short sales constraints: deepening the equity premium puzzle, *Journal of Monetary Economics* 34, 325–341.

Ludvigson, Sydney, 2015, Advances in consumption-based asset pricing: empirical tests, in Milton Harris George M. Constantinides, and Rene M. Stulz, eds., *Handbook of the Economics of Finance*, 799–906.

Luttmer, E., 1999, What level of fixed costs can reconcile asset returns and consumption choices?, *Journal of Political Economy* 107, 969–997.

Mankiw, N. Gregory, and Stephen P. Zeldes, 1991, The consumption of stockholders and nonstockholders, *Journal of Financial Economics* 29, 97–112.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: a puzzle, *Journal of Monetary Economics* 15, 145–161.

Mishkin, Frederic S., and Eugene N. White, 2002, U.S. stock market crashes and their aftermath: implications for monetary policy, NBER Working Papers 8992, National Bureau of Economic Research, Inc.

Nakano, Katsura, and Makoto Saito, 1998, Asset pricing in Japan, *Journal of the Japanese and International Economies* 12, 151–166.

Nirei, Makoto, and Shuhei Aoki, 2009, Pareto distribution of income in neoclassical growth models, IIR Working Paper 09-05, Institute of Innovation Research, Hitotsubashi University.

Paiella, M., 2004, Heterogeneity in financial market participation: appraising its implications for the C-CAPM, *Review of Finance* 8, 445–480.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, 2004, Cyclical dynamics in idiosyncratic labor market risk, *Journal of Political Economy* 112, 695–717.

Sugiyama, M., T. Suzuki, T. Kanamori, M. C. du Plessis, S. Liu, and I. Takeuchi, 2013, Density-difference estimation, *Neural Computation* 25, 2734–2775.

Telmer, Chris I, 1993, Asset-pricing puzzles and incomplete markets, *Journal of Finance* 48, 1803–32.

Toda, Alexis Akira, and Kieran Walsh, 2015, The double power law in consumption and implications for testing Euler equations, *Journal of Political Economy* 123, 1177–1200.

Vayanos, Dimitri, 1998, Transaction costs and asset prices: a dynamic equilibrium model, *Review of Financial Studies* 11, 1–58.

Vissing-Jørgenson, A., 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* 110, 825–853.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Notes

[1]There are a very few exceptions such as Italy and the Netherlands.

[2]The sample size of the NSFIE is about 57,000 households (about 53,000 households with two or more people). On the other hand, the Nikkei Radar surveys from 1,500 to 3,000 households; the SHF targets 8,000 households of two or more people and 4,032 households responded for the 2010 survey.

[3]"Family units" is also available, though it is highly correlated with number of children and can cause the multicollinearity, hence I dropped it from explanatory variables in this analysis.

[4]Since it is difficult to identify the age effect due to the cohort effect, I assume the cohort effect to be zero, following previous literature.

[5]In order to check the robustness of the estimation outcome, I use the truncated data limited between the 1_{st} and 99_{th} percentiles of the cross-sectional financial wealth distribution, but the results are similar.

[6]To endogenously determine the risk-free rate, we need to employ the general equilibrium framework to incorporate non-participants like Guvenen (2009) or Attanasio and Paiella (2011), though their doubtful theoretical assumption may cause a serious specification error.

[7]I do not consider the dividend contribution to the return in the experiment.

[8]The simulated stock holding policy is a weighted sum of the policy functions where the weights reflect stationary probability of each state.