PRI Discussion Paper Series (No.15A-07)

Vote with their donations: An explanation about crowding-in of government provision of public goods

Ryo Ishida

Visiting Scholar, Policy Research Institute, Ministry of Finance Japan

October 2015

Research Department Policy Research Institute, MOF

3-1-1 Kasumigaseki, Chiyoda-ku, Tokyo 100-8940, Japan

TEL 03-3581-4111

Version of August 3, 2015

Vote with their donations: An explanation about crowding-in of government provision of public goods

Ryo Ishida[1]

Abstract

This paper considers a mechanism where providers of public goods reflect donors' preferences for public goods. When asking individuals and private companies to contribute for a certain public good, it is widely known that the total contributions result in under-provision. Among the many countermeasures for this problem, some fundraisers adopt a measure to reflect large donors' preferences for the characteristics of public goods. In such a case, private contribution is enhanced because there is additional incentive to donate. We formalized such a measure theoretically and proved that this measure surely enhances private contributions. Moreover, we find that government direct subsidy may not only crowd-out but also even crowd-in private contribution under this framework. If fundraisers reflect the major donors' preference, the influence of one's donation is leveraged by government direct provision. This element enhances private contributions. If this effect dominates the innate crowding-out effect, government direct subsidy may enhance private contribution. This mechanism is a novel explanation for both crowding-out and crowding-in under an identical framework.

Keywords: private provision, public goods, crowding out, crowding in, voting
JEL classification: H23, H41, H44

Remarks: The author thanks Miles Kimball, Joel Slemrod and Joshua Hausman for their detailed advice and suggestions, and thanks Dean Yang and Takashi Oshio for their helpful comments. The author also thanks participants in workshops of Ministry of Finance Japan and Hosei University, including Takahiro Hattori, Daisuke Ishikawa, Go Kotera, Masahiko Nakazawa, Nobuhiko Nakazawa, Makoto Nirei, Kazumasa Oguro, Tomoaki Sakamoto, Kiyoshi Takata, Nobuyuki Uda and Kazufumi Yamana, for their comments.
The views expressed herein are those of the author and do not necessarily reflect the opinions of the organizations to which the author belongs. Any remaining errors are the sole responsibility of the author.

---

[1] Visiting Scholar, Policy Research Institute, Ministry of Finance Japan
Email: rrishida112358 at gmail.com

1. Introduction

1.1 The remedy for the under-provision problem of public goods

It is a painful issue to have under-provision of public goods when the externality of public goods is not internalized. Even though it is socially desirable to build bridges or street lights, for example, the private sector's provision of such public goods is less than the optimal because individuals have an incentive to free-ride other people's contributions (Bergstrom et al., 1986).

Theoretically speaking, this problem is largely solved by a decentralized mechanism (e.g. Groves and Ledyard, 1977; Walker, 1981) without relying on coercive governmental power. However, such a mechanism is difficult to implement in the real world. In reality, instead of adopting such first best (but overly complicated) mechanism, a government implements tax deduction, tax credit, or other tax expenditure schemes for private contributions in order to alleviate issues of under-provision.

Although tax expenditure is a powerful tool to enhance charitable contributions and its effects have been studied thus far (e.g. Clotfelter, 1985; Randolph, 1994; Auten et al., 2002; Bakija and Heim, 2011), the greatest caveat of tax expenditure is that only government can implement such a mechanism. Even government may face difficulty implementing (or modifying) the tax expenditure mechanism because of political resistance.

Therefore, private fundraisers and sometimes even government implement other kinds of instruments which do not rely on the tax mechanism that requires government's coercive power. A “thank you” letter is a typical (and less costly) example that private institutions can provide. Earmarked donations are frequently used to enhance private contributions (e.g. Li et al. 2011). Not only private institutions (e.g. DonorsChoose.org) but also governments accept earmarked donations; around 80% of local governments in Japan accept earmarked donations under the hometown tax payment system (Konishi, 2015) and the US government has accepted earmarked donations (the federal government of the USA has accepted earmarked donations for reducing the national debt since 1961. (Slemrod, 2003)). Some organizations publish the top donor's name (e.g. Bureau of Investigative Journalism). These measures are examples where fundraisers reflect the large donors' preference and modify the characteristics of the public goods. Another good example is engraving the top donor's name on a school building as well as applying it to a business school program (e.g. Ross School of Business in the University of Michigan). Further examples are introduced in a later section.

Considering these examples, one may recall a model where a certain good is a mixture of a pure public good and a private good. Such kind of good is called an *impure public good* and has already been studied (Andreoni, 1989; Andreoni, 1990; Cornes and Sandler, 1994). "Thank you" letters can be understood using the impure public good model. However, if fundraisers modify the characteristics of a public good in response to large donors' preferences, such public good cannot be analyzed under the impure public good model because one's donation may have both positive and negative externality.

Tiebout (1956) suggested in his classic paper that people "vote with their feet." People reveal their preference by choosing where to live. A similar phenomenon can be considered on private provision on public goods. People reveal their preferences by choosing where to donate or how to donate. Earmarked donations to local municipalities or universities also reveal donors' preferences. Therefore, it is natural to imagine that fundraisers, such as government, local government, universities and so on, reflect the revealed preferences of the donors. This reflection may take a variety of forms, e.g. engraving the donor's name, building a bridge closer to the donor's residence, or facilitating academic research that the donor prefers. The key aspect is that one's donation to a public good makes the whole public good more favorable to the donor, but may make the whole public good possibly less favorable to others[2].

A similar problem is analyzed by Morgan (2000) that analyzes the effect of a fixed prize raffle. His model exploits the structure of a lottery that the lottery revenue minus the amount of prize is used for providing public goods and that the winning probability is proportional to one's donation. In his model, one's donation has a positive externality because it increases the amount of public goods, but has a negative externality because it reduces other people's probability of winning the prize. He theoretically proves that the issue of under-provision is alleviated under this lottery mechanism. Our model is different from but shares ideas with Morgan (2000). Therefore, in this paper, we first generalize Morgan's (2000) model and show that our model is a special case of this generalized Morgan's (2000) model. Then, we show that the under-provision issue is alleviated under our model as is in Morgan's (2000) model.

[2] In this paper, public good is non-rival and non-excludable ex-post (after collecting individuals' donations). However, "public good" in this paper can be rival ex-ante, because one's donation may make the "public good" less favorable to others.

1.2 Explanation of crowding-in in our model

In this paper, we show that government direct subsidy to public goods may not only crowd-out private contributions but also sometimes crowd-in private contributions. This is something different from the model by Morgan (2000), where only the crowding-out effect is shown. The novel finding in this paper is that we provide a new model that explains both crowding-in and crowding-out in the same framework.

A simple model of pure public good provides a dismal prediction that a government direct subsidy for public goods completely crowds out private provision on public goods (Warr, 1982; Roberts, 1984; Bergstrom et al., 1986). If this prediction were true, government direct subsidy would generally not make sense from the viewpoint of fundraising. However, in reality, the situation is not so depressing; the crowding-out effect is generally incomplete (Andreoni, 1993; Payne, 1998; Andreoni and Payne, 2003). To explain this deviation, many scholars assume that each donor not only benefits from the public good but also acquires “warm-glow” utility from his/her act of giving (Becker, 1974; Cornes and Sandler, 1984; Steinberg, 1987; Andreoni, 1989; Andreoni, 1990). Some scholars explain this phenomenon by assuming that people have an incentive to signal their wealth (Glazer and Konrad, 1996; Harbaugh, 1998a; Harbaugh, 1998b; Blumkin and Sadka, 2007). However, in a few cases, government provision may even “crowd-in” private provision of public goods (Rose-Ackerman, 1981; Sugden, 1982; Segal and Weisbrod, 1998; Khanna and Sandler, 2000; Okten and Weisbrod, 2000; Payne, 2001; Andreoni et al., 2014), which cannot solely be derived from the “warm-glow” explanation. Crowding-in may be explained by asymmetric information where a donated grant reveals the quality of the public goods (Romano and Yildirim 2001; Vesterlund, 2003; Potters et al. 2005; Andreoni, 2006). Although this explanation is predominant, there exist other explanations. For example, in a case where a public good comes into effect only when the total contribution exceeds a certain threshold, a large amount of government direct contribution will act as seed money; it may enhance private contribution because government contribution increases the probability that the public good comes into effect (Andreoni, 1998).

Our model provides totally different explanations to understand the crowding-in phenomenon. If fundraisers reflect large donors' preference on public goods, the influence of one's donation is leveraged by the government direct subsidy. If this crowding-in effect dominates the crowding-out effect, which government direct subsidy has by nature, a government subsidy crowds-in private contributions.

1.3 Construction of this paper

Our paper is constructed as follows. In the next section, we explain Morgan's (2000) fixed prize raffle model and its generalization. This generalization connotes our model in Section 3. In Section 3, we will see our basic model without warm-glow and then we will incorporate the warm-glow effect in the model. The model is also extended to multiple individuals and heterogeneity is considered briefly. Examples are provided in Section 4. In the final section, we conclude.

2. Morgan's (2000) model and its generalization

2.1 Explanation of Morgan's (2000) model

Our model formalized in Section 3 shares the basic idea with Morgan's (2000) fixed prize raffle model. Therefore, before introducing our model, we explain the Morgan's (2000) model and generalize it in this section. Note that our model in the following section is a special case of the generalized Morgan's (2000) model established in this section.

Morgan asks in his paper why lotteries are frequently used for the fundraising purpose in spite of the criticism of being inequitable and inefficient compared to tax instruments. He argues that this comparison is not fair because lotteries are often held when tax instruments are not feasible. He then argues that lottery is a more effective instrument for the fundraising purpose than just soliciting voluntary donations.

It is well-known that, when individuals voluntarily contribute to public goods and government does not provide any subsidy, the public goods result in under-provision (Bergstrom et al., 1986). The reason is that each individual does not internalize the positive externality her contribution creates. Morgan (2000) finds that lottery alleviates this problem; if a lottery prize is prefixed, one's contribution increases his expected prize and *decreases* others' expected prizes. Therefore, the lottery mechanism connotes a negative externality mechanism which partially cancels out the positive externality one's contribution creates.

The similarity between Morgan (2000) and our model is that one's contribution to public good not only has positive externality but also have negative externality, and the latter (maybe partially) cancels out the former.

In order to describe Morgan's (2000) mechanism formally, let us assume that there are N people, indexed $1, 2, \cdots, N$. Individual i has positive initial endowment w_i. Each individual divides its initial endowment into private consumption and purchase of lottery. The amount of lottery tickets that individual i purchases is g_i. The prize of the lottery, P, is prefixed and the winning probability of the prize is proportional to the amount of lottery tickets she purchases. Therefore, her expected prize is $\frac{g_i P}{\sum_{j=1}^{N} g_j}$. If she wins the prize, she uses the prize for her private consumption. Assuming a quasi-linear utility function and risk neutrality, her expected utility is expressed as

$$U_i = w_i - g_i + \frac{g_i P}{\sum_{j=1}^{N} g_j} + h_i(G) \qquad (1)$$

where G is the total amount of public goods and $h_i(G)$ is her utility from public goods. Individuals experience positive but diminishing marginal utility from public goods, hence $h_i'(\cdot) > 0$ and $h_i''(\cdot) < 0$ are assumed. We assume that the utility from public goods is non-negative, i.e. $h_i(G) \geq 0 \; \forall G \geq 0$. The total amount of public goods is the sum of individuals' contributions net of the prize, formally

$$G = \sum_{j=1}^{N} g_j - P. \qquad (2)$$

Combining individual utility (1) and the public good provision (2), any individual maximizes the following expected utility:

$$U_i = w_i - g_i + \frac{g_i P}{\sum_{j=1}^{N} g_j} + h_i\left(\sum_{j=1}^{N} g_j - P\right). \qquad (3)$$

Note that, when the prize P converges to zero, this model converges to a voluntary contribution model. Upon this limit, purchase of lottery tickets is identical to voluntary contribution to public goods.

Under this setting, Morgan (2000) shows that the total amount of public goods, G is a function of prize P (see Proposition 2 in Morgan, 2000), namely $G(P)$.

Let G^* be the Pareto optimal amount of public goods, namely $\sum_{j=1}^{N} h_j'(G^*) = 1$ (Samuelson rule[3]). Then, Morgan (2000) shows the following proposition that increasing prize P (1) increases the total amount of public goods $G(P)$, (2) is always welfare improving, and (3) pushes the total amount of public goods to converge to the optimal amount.

Proposition 1

The total amount of public goods is a strictly increasing function of prize P and it converges to optimal amount G^* when prize P increases; formally,

$$G'(P) > 0, \quad (4)$$

and

$$\lim_{P\to\infty} G(P) = G^*. \quad (5)$$

Proof:

See Theorem 1 and Theorem 2 in Morgan (2000).

Morgan (2000) briefly investigates the effect of government direct subsidy on public goods and concludes that government direct subsidy incompletely crowds out private contributions. Formally, when equation (2) is substituted by $G = \sum_{j=1}^{N} g_j - P + g_G$ where g_G is government direct subsidy, $-1 < \frac{\partial \sum_{j=1}^{N} g_j}{\partial g_G} < 0$ is obtained (See Proposition 3 in Morgan, 2000). This point is further analyzed in the following subsection.

---

[3] In Samuelson (1954), he showed that sum of marginal rate of substitutions is equal to marginal rate of transformation between the public good and private good if the amount of public good is optimal.

2.2 Generalization of Morgan’s (2000) model

Although Morgan (2000) only analyzes the lottery example in his paper, his idea is fairly easily generalized. There are three aspects to be extended. First, in Morgan’s (2000) model, the expected prize one gets is proportional to one’s contribution, but this weighting can be generalized; the weight is generalized to be a function of one’s contribution divided by total contributions. Second, in Morgan’s (2000) model, the prize is solely financed from private contributions, but we can consider that not only the source of the public goods (private contributions) but also the fruits from the public goods can be distributed to contributors. Consider a company collecting investment. It does not distribute the invested money itself to investors but it distributes the profit made from the investment to investors. We can consider such mechanism in fundraising; not only the private contributions but also the fruits from public goods can be the prize for contributors. Third, we incorporate government direct subsidy in the model explicitly. Even with these generalizations, Morgan’s (2000) basic idea is valid that negative externality created from the prize is a key feature to alleviate the under-provision problem. With these generalizations, individual expected utility is as follows:

$$U_i = w_i - g_i + \varphi\left(\frac{g_i}{\sum_{j=1}^N g_j}\right)\left(P + l\left(\sum_{j=1}^N g_j - P + g_G\right)\right)$$
$$+ \left(k_i\left(\sum_{j=1}^N g_j - P + g_G\right) - \frac{l\left(\sum_{j=1}^N g_j - P + g_G\right)}{N}\right), \quad (6)$$

where one’s contribution has a weakly negative externality effect (i.e. $\varphi'(\cdot) \geq 0$), the prize is always non-negative for anyone, and the total prize is not affected by who contributed for the public goods. ($\varphi(\cdot)$ is a weighting function, i.e.

$\sum_{i=1}^{N}\varphi\left(\frac{g_i}{\sum_{j=1}^{N}g_j}\right)\equiv 1$ and $\varphi(\cdot)\geq 0$). g_G can be considered as government's direct subsidy. The prize consists of two parts, fixed amount P and variable amount $l\left(\sum_{j=1}^{N}g_j-P+g_G\right)$, where Morgan (2000) only considers the former part. It is assumed that the public good effect is non-negative ($k_i\left(\sum_{j=1}^{N}g_j-P+g_G\right)\geq\frac{l\left(\sum_{j=1}^{N}g_j-P+g_G\right)}{N}\forall i$). We also assume that the fixed amount of prize P should be non-negative. The variable amount of prize $l\left(\sum_{j=1}^{N}g_j-P+g_G\right)$ is assumed to be non-negative and its marginal utility is assumed to be non-negative and weakly diminishing as usual. We assume that the variable amount of prize $l\left(\sum_{j=1}^{N}g_j-P+g_G\right)$ is not so progressively increasing; formally we assume that $\frac{P+l(\bar{G}-P+g_G)}{\bar{G}}$ is always a weakly decreasing function of $\bar{G}$ for mathematical convenience. Morgan's (2000) fixed prize raffle model is a special case of this generalization, where $\varphi\left(\frac{g_i}{\sum_{j=1}^{N}g_j}\right)=\frac{g_i}{\sum_{j=1}^{N}g_j}$, $k_i(\cdot)=h_i(\cdot)$, and $l(\cdot)=0$ hold and government direct subsidy is disregarded.

The novel finding in Morgan (2000) is that, even if the prize is solely financed from private contributions ($l(\cdot)=0$), the increment of prize P increases not only the total contributions $\sum_{j=1}^{N}g_j$ but also the amount of public goods $\sum_{j=1}^{N}g_j-P$. We can generalize his results as follows. Primarily, under this generalized framework, the size of the public goods $\sum_{j=1}^{N}g_j-P+g_G$ is an increasing function of the fixed amount of prize P if and only if the total contributions $\sum_{j=1}^{N}g_j$ exceeds the size of prize $P+l\left(\sum_{j=1}^{N}g_j-P+g_G\right)$. Morgan's (2000) result is a special case of this; in his model, total contributions $\sum_{j=1}^{N}g_j$ always dominate the size of prize P. Moreover, we can derive the necessary and sufficient conditions that the increase of fixed amount of prize P and the increase of government direct subsidy g_G enhance total contributions $\sum_{j=1}^{N} g_j$. The novel finding here is that, in any case, either fixed amount of prize P or government direct subsidy g_G (possibly both) must enhance total contributions $\sum_{j=1}^{N} g_j$. Therefore, we can adopt at least one measure to enhance private contributions. Finally, increase of the slope of the weighting function $\varphi(\cdot)$ as well as the increment of the variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ always increase the total contribution $\sum_{j=1}^{N} g_j$ and thus increase the size of the public goods $\sum_{j=1}^{N} g_j - P + g_G$.

Proposition 2

(a) If prize $P + l(\cdot)$ is zero ($P = l(\cdot) = 0$), public goods are under-provided. For later statements in this Proposition, we assume an interior solution and assume $N \geq 3$ or symmetric contributions in $N = 2$.

(b) The size of public goods $\sum_{j=1}^{N} g_j - P + g_G$ is increasing in the fixed amount of prize P if and only if the total contributions $\sum_{j=1}^{N} g_j$ exceeds the size of the prize $P + l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$. Morgan's (2000) result is a special case of this statement because total contributions always dominate the size of the prize in his model.

(c) The effects of the fixed amount of prize P and government direct subsidy g_G to private contributions are as follows:

(c-1) Total contribution $\sum_{j=1}^{N} g_j$ is increasing in fixed amount of prize P if and only if $(N-1)\varphi'\left(\frac{1}{N}\right) - (N-1)\varphi'\left(\frac{1}{N}\right) l'\left(\sum_{j=1}^{N} g_j - P + g_G\right) - \sum_{j=1}^{N} g_j \sum_{i=1}^{N} k_i''\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ is positive. Therefore, if the variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ is disregarded as in Morgan (2000), fixed amount of prize P always enhances private contributions[4].

(c-2) Total contribution $\sum_{j=1}^{N} g_j$ is increasing in government direct subsidy g_G if and only if $(N-1)\varphi'\left(\frac{1}{N}\right) l'\left(\sum_{j=1}^{N} g_j - P + g_G\right) + \sum_{j=1}^{N} g_j \sum_{i=1}^{N} k_i''\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ is positive. Therefore, the slope of variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ improves the effect of government direct subsidy g_G, the effect of which is totally the opposite of that in (c-1).

(c-3) At least either fixed amount of prize P or government direct subsidy g_G enhances private contributions.

(d) Increase in $l(\cdot)$ (i.e. by multiplying a fixed number that is more than one) results in an increase of total contribution $\sum_{j=1}^{N} g_j$. The increase of the slope of the weighting function $\varphi(\cdot)$ also results in an increase of total contribution $\sum_{j=1}^{N} g_j$.

(e) Public goods are underprovided if and only if $\varphi'\left(\frac{1}{N}\right)\left(P + l\left(\sum_{j=1}^{N} g_j - P + g_G\right)\right) < \sum_{j=1}^{N} g_j$.[5] Statement (a) is a special case of this.

Proof:

See Appendix I.1

As explained in Proposition 2 (b)-(c), at least either the fixed amount of prize or

---

[4] It is natural that a variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$ discourages the effect of fixed amount of prize P, because increase of fixed amount of prize P decreases the size of variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$.

[5] If $N \geq 3$, the weighting function $\varphi(\cdot)$ is a linear function (Appendix I.2). Therefore, when $N \geq 3$, $\varphi'\left(\frac{1}{N}\right)$ can be substituted by $\varphi'(\cdot)$ in this statement.

government direct subsidy enhances private contributions. Morgan (2000) finds a case where the former works very well (The former not only enhances private contribution but also increases the size of the public goods.) but the latter not. My model in the following section finds a case where the latter works well.

The abovementioned model is too general to obtain a meaningful result. Therefore, in order to answer our problem, we will see another special case of this generalized model. In the following section, we will focus on a case where the prize is solely financed from the fruits from public goods and public goods only appear as the prize ($k_i\left(\sum_{j=1}^{N} g_j - R + g_G\right) = \frac{l\left(\sum_{j=1}^{N} g_j - P + g_G\right)}{N}$ and $P = 0$).

3. Model

3.1 Model setting

In this section, we hereafter see that generalized Morgan's (2000) model formalized in the previous section can be applied to our issue.

For the sake of simplicity, firstly, we assume that there are two people, A and B, privately providing public goods. Let the contribution from A be g_A where that from B be g_B. Government is also able to provide public goods directly. Let the government's direct contribution be g_G. The total amount of public goods is assumed to be a simple summation of these provisions, $G = g_A + g_B + g_G$. We assume a quasi-linear utility function depending on one's consumption and public goods for simplicity.

Our main assumption in this model is that one's donation on public goods makes the public goods preferable to the donor. This assumption is an application of

Morgan's (2000) idea that one's donation has an aspect of negative externality. This idea is clearly imagined when a bridge is built where two donors live in separated places. If one donates for the bridge, the bridge will be made closer to the donor and thus farther from the other. This characteristics can be described by a symmetrical function $f(\cdot)$. Imagine that $f(g_A/(g_A+g_B))$ is the accessibility of the bridge from A and $f(g_B/(g_A+g_B))$ is that from B. Accessibility is negatively correlated with the distance from the residence to the bridge. In such a case, it is natural to assume that $f'(\cdot) \geq 0$. We normalize that $1 \geq f(\cdot) \geq 0$ is satisfied and assume symmetricity on $f(\cdot)$, i.e. $f(x) + f(1-x) = 1 \forall x \in [0,1]$. The reason why we assume symmetricity is that we would like to hold the total utility from public goods constant irrespective of who provided the public goods in order to simplify the discussion hereafter. We then assume that individual utility is

$$u_i(c_i, g_i, g_{-i}, G) = c_i + f\left(\frac{g_i}{g_i+g_{-i}}\right) h(G) \qquad (7)$$

where c_i is i's consumption, g_i is i's contribution, g_{-i} is the other person's contribution, and $h(G)$ is the total utility derived from the public goods. Note that $g_i + g_{-i}$ does not include government contribution where $G = g_i + g_{-i} + g_G$ does. We assume that total utility derived from the public goods is weakly positive and strictly increasing with respect to the size of the public goods, and its marginal utility is diminishing; formally $h(\cdot) \geq 0$, $h'(\cdot) > 0$ and $h''(\cdot) < 0$, as usual. The individual budget constraint is

$$w_i = c_i + g_i \qquad (8)$$

where w_i is i's initial endowment. In order to ensure an inner solution, we assume a sufficient initial endowment so that private consumption always takes a strictly positive value.

Note that total utility derived from public goods in Bentham's sense is $h(G)$, independent from who provided the public goods. It is also worth noting that this model converges to the standard model if we assume $f(\cdot)$ to be constant, i.e. $f(x) = 1/2 \forall x$.

It is easy to see that our problem is another special case of the generalized Morgan's (2000) model. If we substitute $k_i\left(\sum_{j=1}^{N} g_j - R + g_G\right) = \frac{l\left(\sum_{j=1}^{N} g_j - P + g_G\right)}{N}$, $P = 0$, $l(\cdot) = h(\cdot)$, $\varphi(\cdot) = f(\cdot)$ and $N = 2$ in the generalized Morgan's (2000) model and we can obtain our model.

The well-known fact is that, under the standard model, private donations alone result in under-provision of public goods and government provision completely crowds out private donations (Warr, 1982; Roberts, 1984; Bergstrom et al., 1986).

Proposition 3

If the model in equation (7)-(8) converges to the standard model: $f(x) = 1/2 \forall x$,

(a) Private provision $g_A + g_B$ alone results in under-provision of public goods.

(b) Government provision g_G completely crowds out private provision $g_A + g_B$ if $g_A + g_B > 0$. Formally, $\frac{\mathrm{d}(g_A + g_B)}{d g_G} = -1$ if $g_A + g_B > 0$.

Proof:

See Appendix I.3

However, if one's donation makes the public goods more preferable to the donor, the story is not that simple. Under the quasi-linear utility function, the initial endowment only affects private consumption and does not affect private provision. Therefore, there is a reasonable ground to assume symmetric contributions. Then, as stated in the following proposition, private provision will be enhanced and government provision may result in partial crowding-out or even crowding-in.

Proposition 4

Suppose that an inner solution is guaranteed and consider symmetric contributions: $g \equiv g_A = g_B$. Then,

(a) Private provision $g_A + g_B$ is always weakly larger than that under the standard model: $f(x) = 1/2 \forall x$. The inequality is strict if and only if $f'\left(\frac{g_A}{g_A+g_B}\right) > 0$.

(b) Government provision g_G incompletely crowds out private contribution if $0 < f'\left(\frac{1}{2}\right) < -\frac{2gh''(G)}{h'(G)}$ and $g_A + g_B > 0$ hold. Government provision even enhances private contribution if $f'\left(\frac{1}{2}\right) > -\frac{2gh''(G)}{h'(G)}$ holds.

Proof: See Appendix I.4

The intuition of these results is rather simple. With respect to (a), the intuition is as follows: if $f'\left(\frac{g_A}{g_A+g_B}\right) > 0$ holds, there is an additional incentive for A to donate because her donation makes the public goods more preferable for herself (e.g. the bridge will be built closer to her house.).

With respect to (b), we should note that there are two ambivalent incentives for individuals. An individual has an incentive to free-ride as we saw in Proposition 1. However, if $f'\left(\frac{1}{2}\right)$ is large enough, an individual has an additional incentive to donate so as to make the public goods more preferable for him/herself.

Then, let us consider a case where government provides a unit amount to the public goods. It decreases an individual's incentive to donate because it decreases the marginal utility in the public goods ($-\frac{1}{2}h''(G)$). However, it increases an individual's incentive to donate because the effect of one's donation is leveraged by the government provision ($\frac{f'\left(\frac{1}{2}\right)h'(G)}{4g}$). Government provision may result in partial crowding-in or crowding-out due to the comparison of the two ambivalent elements.

3.2 Optimality condition

Perhaps the optimality condition is of interest. The Samuelson rule states that the optimal amount of public goods is given by $h'(G) = 1$. Under a symmetric assumption, the first order condition is given as $f'\left(\frac{1}{2}\right)h(G) + 2gh'(G) = 4g$. Therefore, at the optimal, $f'\left(\frac{1}{2}\right)h(G) = 2g$ with $h'(G) = 1$ and $G \geq 2g$ should be satisfied. Without government subsidy, this condition converges to $f'\left(\frac{1}{2}\right)h(G) = G$ with $h'(G) = 1$ and therefore the optimal slope of f is given by $f'\left(\frac{1}{2}\right) = \frac{h'^{-1}(1)}{h(h'^{-1}(1))}$.

3.3 Sufficient condition for symmetric solution

The abovementioned propositions hold for symmetric contribution. The necessary and sufficient condition that a symmetric contribution is in Nash equilibrium is described in the following proposition.

In addition, although our consideration is only restricted in symmetric contribution, there may exist asymmetric equilibrium. Therefore, it would be useful to show the application limit of the abovementioned results. We provide a sufficient condition that there exists no asymmetric Nash equilibrium in the following proposition. Under this condition, we may restrict our consideration in symmetric contribution.

Proposition 5

(a) Symmetric contribution $g \equiv g_A = g_B$ is in Nash equilibrium if and only if $2f\left(\frac{g'}{g+g'}\right)h(g+g'+g_G)+2(g-g') \leq h(2g+g_G)\forall g' \geq 0$ is satisfied.

(b) There exists no asymmetric Nash equilibrium if the following system has no roots in $(x, \alpha) \in [0, \infty) \times (\frac{1}{2}, 1]$

$$\begin{cases} f'(\alpha) = \frac{\left(2-h'(x+g_G)\right)x}{h(x+g_G)}, & (9) \\ \frac{f(\alpha)-1/2}{\alpha-1/2} = \frac{2}{h'(x+g_G)} - 1. & (10) \end{cases}$$

Proof: See Appendix I.5

An example that the condition in Proposition 5 is satisfied is as follows.

Example 1.

Suppose $f(\alpha) = 2\left(\alpha - \frac{1}{2}\right)^3 + \frac{1}{2}\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}$ and $h(G) = \sqrt{G}$. Then, $(g_A, g_B) = \left(\frac{1}{8}, \frac{1}{8}\right)$ is the only Nash equilibrium where government contribution $g_G = 0$.

Proof: See Appendix I.6

This example is the case where government contribution neither crowds out nor crowds in private contribution. If you would like to see a case where symmetric equilibrium is the only Nash equilibrium and government contribution crowds in private contribution, slightly modify the abovementioned example to $h(G) = G^{\gamma}$ where $\gamma = 0.51$ and you can see that $(g_A, g_B) = \left(\frac{1}{2}\left(\frac{1}{4} + \frac{\gamma}{2}\right)^{\frac{1}{1-\gamma}}, \frac{1}{2}\left(\frac{1}{4} + \frac{\gamma}{2}\right)^{\frac{1}{1-\gamma}}\right)$ is the only Nash equilibrium where government contribution crowds in private contribution. If you choose $\gamma = 0.49$, it will be the only Nash equilibrium where government contribution partially crowds out private contribution.

3.4 Model with warm-glow

The abovementioned model does not incorporate warm-glow effect. However, as Diamond (2006) investigated, warm-glow effect is not disregarded in private donation literature. Then, the natural question would be how our results would change if we would incorporate it into our model. If we incorporate warm-glow effect, which was formalized by Andreoni (1990), government provision only partially crowds out private provisions in the standard model (i.e. $f(x) = 1/2 \forall x$). However, the striking result is that Proposition 2 is still valid even if we incorporate warm-glow effect in the utility function. This statement is formally explained in the following proposition.

To be precise, let us assume that the individual utility function is:

$$u_i(c_i, g_i, g_{-i}, G) = c_i + k(g_i) + f\left(\frac{g_i}{g_i + g_{-i}}\right) h(G), \qquad (11)$$

where $k(g_i)$ reflects a warm-glow effect with the standard assumption $k(\cdot) > 0$, $k'(\cdot) > 0$ and $k''(\cdot) < 0$. Note that our previous model described in equation (7)-(8) is a special case of equation (11) with $k(\cdot) = 0$. It is known that taking into account such an effect improves the description of people's behavior (Andreoni, 1989; Andreoni, 1990). Taking into account warm-glow effect inevitably enhances private donation (Appendix I.7). Moreover, we can see that Proposition 2 is still valid even if warm-glow effect is incorporated.

Proposition 6

Let us consider a case where warm-glow effect is incorporated into our model. Suppose that an inner solution is guaranteed and consider symmetric contributions: $g \equiv g_A = g_B$. Then,

(a) Private provision $g_A + g_B$ is always weakly larger than that under the standard model: $f(x) = 1/2 \forall x$. The inequality is strict if and only if $f'\left(\frac{g_A}{g_A+g_B}\right) > 0$.

(b) Government provision g_G incompletely crowds out private contribution if $0 < f'\left(\frac{1}{2}\right) < -\frac{2gh''(G)}{h'(G)}$ and $g_A + g_B > 0$ hold. Government provision even enhances private contribution if $f'\left(\frac{1}{2}\right) > -\frac{2gh''(G)}{h'(G)}$ holds.

Proof: See Appendix I.7

The reason why warm-glow does not change the main result of Proposition 4 is rather straightforward. It is natural that Proposition 4 (a) still holds with warm-glow effect because this statement is a qualitative statement, rather than a quantitative statement. The reason why Proposition 4 (b) still holds with warm-glow effect is that the mechanism behind this statement is not affected by the warm-glow effect. Proposition 4 (b) compares the two effects, government provision dis-incentivizing private provision and government provision incentivizing private provision. The former effect comes from the decreasing marginal utility of public goods, which is irrelevant to warm-glow effect. The latter comes from the leverage effect of government provision, which is also irrelevant to warm-glow effect. Therefore, warm-glow effect does not affect the condition whether government provision crowds out private provision or crowds it in.

3.5 Multiple individuals

The abovementioned model with two individuals is extended to multiple individuals fairly easily. Suppose there are n individuals indexed by $1,2,\cdots,n$. Let contribution from individual i be g_i, consumption of individual i be c_i, and government's direct contribution be g_G. Let g_{-i} be total contributions from individuals other than individual i, i.e. $g_{-i} = \sum_{j=1,j\neq i}^{n} g_j$. The total amount of public goods is assumed to be a simple summation of all contributions, $G = \sum_{i=1}^{n} g_i + g_G$. Individual utility is assumed to be

$$u_i(c_i, g_i, g_{-i}, G) = c_i + f\left(\frac{g_i}{g_i + g_{-i}}\right) h(G) \qquad (12)$$

where $h(G)$, the total utility derived from the public goods, satisfies increasing and diminishing marginal utility, i.e. $h'(\cdot) > 0$ and $h''(\cdot) < 0$, and the non-negative and weakly increasing function $f: [0,1] \to [0,1]$ as a weighting function ($f\left(\frac{g_1}{\sum_{i=1}^{n} g_i}\right) + f\left(\frac{g_2}{\sum_{i=1}^{n} g_i}\right) + \cdots + f\left(\frac{g_n}{\sum_{i=1}^{n} g_i}\right) = 1$ holds, i.e. formally

$\forall(\alpha_1, \alpha_2, \cdots, \alpha_n) \in [0,1]^n; \sum_{i=1}^n \alpha_i = 1 \Rightarrow \sum_{i=1}^n f(\alpha_i) = 1$). If $n = 2$, this model converges to the model in equation (7)-(8). In this extension, it should be noted that weighting function f is restricted to a linear function, specifically $f(\alpha) = \beta\left(\alpha - \frac{1}{n}\right) + \frac{1}{n}$ with $\beta \in [0,1]$, if $n \geq 3$ holds (See lemma and Appendix I.8). Also, it should be noted that this is also a special case of a generalized Morgan's (2000) model with $k_i\left(\sum_{j=1}^N g_j - R + g_G\right) = \frac{l\left(\sum_{j=1}^N g_j - P + g_G\right)}{N}$, $P = 0$, $l(\cdot) = h(\cdot)$, and $\varphi(\cdot) = f(\cdot)$. Similar to the simple model with two individuals, if we focus on the symmetric Nash equilibrium, Proposition 3 and Proposition 4 are extended in the following way.

Proposition 7

(a) If weighting function f converges to a constant function i.e. $f(x) = 1/n \forall x$, private provision $\sum_{j=1}^n g_j$ alone results in under-provision of public goods and government provision g_G completely crowds out private provision $\sum_{j=1}^n g_j$ if $\sum_{j=1}^n g_j > 0$ holds. Formally, $\frac{\mathrm{d}\left(\sum_{j=1}^n g_j\right)}{d g_G} = -1$ is satisfied if $\sum_{j=1}^n g_j > 0$ holds.

(b) Focusing on symmetric contribution, private provision $\sum_{j=1}^n g_j$ is always weakly larger than that under the abovementioned case (a): $f(x) = 1/n \forall x$. The inequality is strict if and only if $f'\left(\frac{g_i}{\sum_{j=1}^n g_j}\right) > 0$ holds.

(c) Focusing on symmetric contribution, government provision g_G incompletely crowds out private contribution if $0 < f'\left(\frac{1}{n}\right) < -\frac{ngh''(G)}{(n-1)h'(G)}$ and $\sum_{j=1}^n g_j > 0$ hold. Government provision even enhances private contribution if $f'\left(\frac{1}{n}\right) > -\frac{ngh''(G)}{(n-1)h'(G)}$ holds.

Proof: See Appendix I.9

In order to clarify what is stated in the abovementioned proposition, we herein provide an example of multiple individuals. With this example, it is observed that crowding-in is not a rare case. Rather, under this example, when the number of people n gets larger, it may become more probable that government contribution crowds-in private contribution.

Example 2.

Suppose there are n identical individuals. Suppose $f(\alpha) = \beta\left(\alpha - \frac{1}{n}\right) + \frac{1}{n}$ with $0 < \beta < \frac{1}{2}$ and $h(G) = \sqrt{G}$.

Without government contribution, the symmetric contribution equilibrium is $g_i = g = \frac{(2\beta(n-1)+1)^2}{4n^3}\forall i$. Therefore, $h'(G) = \frac{n}{2\beta(n-1)+1} > 1$ holds, implying that public goods are under-provided in this case. Small government contribution partially crowds out private contribution if $\beta < \frac{1}{2(n-1)}$ and crowds in if $\beta > \frac{1}{2(n-1)}$.

Proof: Simple calculation results in $g = \frac{(2\beta(n-1)+1)^2}{4n^3}$, $h'(G) = \frac{n}{2\beta(n-1)+1}$ and $h''(G) = -\frac{2n^3}{(2\beta(n-1)+1)^3}$. Proposition 7 (c) results in the conclusion.

3.6 Heterogeneity

It is natural to ask how heterogeneity can be incorporated into our model. Many scholars considered heterogeneity in wage rate (e.g. Mirrlees, 1971; Blumkin and Sadka, 2007) or in initial endowment (e.g. Bergstrom et al., 1986). However, due to the characteristics of the additively separable quasi-linear utility function, such heterogeneity does not affect anyone's choice of private contribution, unless one or more people's consumption takes the lower bound zero. Instead of considering the heterogeneity in initial endowment, we may slightly modify our model and consider heterogeneity. The example is shown as follows.

Let us assume that there are two groups of people, group A and group B. The former consists of N_A people and the latter N_B. Consider a case where people in group A live in Town A and people in group B live in Town B. They are arguing where to build a bridge. The bridge will be built closer to Town A if total private provision from group A exceeds that from group B and vice versa. In such a case, heterogeneity is taken into account with regard to the number of people in each group. Note that, in such a case, private contribution from a person in group A has both positive and negative externality for people in group B, but it has only positive externality for other people in group A.

Formally, let the total private provision from group A be G_A where that from group B is G_B. We then assume that individual i's utility is

$$u_i\left(c_i, G_{\varphi(i)}, G_{-\varphi(i)}, G\right) = c_i + f\left(\frac{G_{\varphi(i)}}{G_{\varphi(i)}+G_{-\varphi(i)}}\right) h(G) \qquad (13)$$

where c_i is i's consumption, $\varphi(i) = \mathrm{A}$ if individual i belongs to group A and $\varphi(i) = \mathrm{B}$ otherwise. Let $-\varphi(i)$ denote A if $\varphi(i) = \mathrm{B}$ and $-\varphi(i)$ denote B otherwise. Assume G is the total amount of public goods, i.e. $G = G_{\varphi(i)} + G_{-\varphi(i)} + g_G$ where g_G is government's contribution. This model converges to our basic model in section 3.1 if we assume $N_A = N_B = 1$.

Under this model, the first order condition results in the following equation:

$$f'\left(\frac{G_{\varphi(i)}}{G_{\varphi(i)}+G_{-\varphi(i)}}\right)\frac{G_{-\varphi(i)}}{\left(G_{\varphi(i)}+G_{-\varphi(i)}\right)^2}h(G)+f\left(\frac{G_{\varphi(i)}}{G_{\varphi(i)}+G_{-\varphi(i)}}\right)h'(G)=1. \qquad (14)$$

This first order condition is completely parallel to the first order condition in our basic model in section 3.1 (i.e. see equation (4-1) of Appendix I.4) and both G_A and G_B do not depend on the number of people in each group, i.e. increase in the number of a group results in a decrease of private contribution per capita so that total private provision from the group remains constant. Therefore, this model is dealt with similar to our basic model in section 3.1. Note that an increase of a person's private contribution in group A is totally cancelled out by the decrease of other people's contribution in the same group.

4. Examples

There are several examples to which our model may be applied. Our model is easily understood by the bridge metaphor where a bridge made by private contributions constructed closer to large donor (and further from small donors), providing real world examples would be useful. Although we briefly look at such examples in the first section, we will look at such examples deeply.

(1) A typical example to which our model may be applicable is an earmarked donation because it conveys the donor's preference to fundraisers. As is predicted by our model, allowing earmarking donations significantly increases private contributions. For example, Li et al. (2011) report that providing the opportunity to earmark one's donation more than doubles the contribution per capita and the likelihood to give to government organizations. Although there are pros and cons for earmarking, evidence exists that this opportunity enhances

private contributions.

(2) Another example is international public goods provided by the International Monetary Fund (IMF). The IMF's financial resources are provided by quota subscriptions from member countries. Quota subscription not only determines the amount member countries have to provide to the IMF, but also determines the voting power of member countries. Therefore, quota subscription has an aspect to contribute for international public goods, and a large contributor has negotiation power to determine the IMF's policy. This voting share is almost proportional to the quota. Although the quota is determined by the countries' economic statistics such as GDP, openness, economic variability and international reserves, it can be modified by negotiation and consent of an 85% majority of the total voting power. Now that one's contribution increases its voting power compared to other countries, our model predicts that this mechanism greatly enhances the incentive to contribute. Anecdotal evidence to support this prediction is that many countries have incentive to contribute more rather than less; for example, Buira (2005, p.287) writes that "*with the exception of Honduras in 1948, no country has ever requested a reduction in its quota.*"

(3) The other example is the non-price competitive auction adopted by JGB auction.

Let me first explain how the auction works. The auction is held twice during the day; the first bidding is held at noon, and this is a price competitive auction. In this first bidding, Primary Dealers (PDs) are obliged to bid for an adequate amount and purchase at least a specified share of the planned total issuance amount. The result of this auction is published at 12:45.

Then, the second bidding is closed at 14:30, and it is a non-price competitive auction. The PDs are eligible to participate in this second bidding, and they are able to purchase JGBs at their average successful bidding price at the first bidding.

Now, each PD can make profit without any risk if the market value of JGBs at 14:30 is greater than the average successful bidding price at the first bidding. The total amount of issuance in a non-price competitive auction (2nd bidding) is predefined and the maximum amount that each PD can buy is proportional to the successful bid in the JGB price competitive auction (1st bidding).

This auction system, which is a combination of a price-competitive and a non-price competitive auction, has the following features.

--- the more one purchases JGBs in the first bidding, the more one can purchase JGBs in the second bidding.

--- the more one purchases JGBs in the first bidding, the less other PDs can purchase JGBs in the second bidding.

Based on these features, the government can enhance the incentive for PDs to purchase more JGBs at the first bidding. My model can explain this.

(4) Finally, our model is applicable to explain the phenomenon reported by Brooks (2000) that a low level of public subsidy for concerts crowds-in private donations where a high level of it crowds-out private donations. Brooks (2000) does not provide any theoretical model to explain this phenomenon. Consider a case where the utility function of public goods $h(G)$ is quadratic, say $\alpha G - \beta G^2$ where $G \in \left[0, \frac{\alpha}{2\beta}\right]$. Let there be n identical people contributing for concerts and let $g(g_G)$ be donation per capita where government direct subsidy is g_G. Since

government budget is sufficiently large, people ignore any government budget problem. Assume $f'\left(\frac{1}{n}\right) > -\frac{ng(0)h''\left(ng(0)\right)}{(n-1)h'\left(ng(0)\right)} = \frac{2\beta ng(0)}{(n-1)(\alpha-2\beta ng(0))}$. In such a case, a low level of public subsidy crowds in private donation (See Proposition 7). However, when crowd-in occurs, $-\frac{ng(g_G)h''(ng(g_G)+g_G)}{(n-1)h'(ng(g_G)+g_G)} = \frac{2\beta ng(g_G)}{(n-1)\{\alpha-2\beta\left(ng(g_G)+g(g_G)\right)\}}$ is an increasing function of g_G. Therefore, if $f'\left(\frac{1}{n}\right)$ is not large enough, there exists a certain amount of government direct contribution $\overline{g_G}$ such that government contribution neither crowds in nor crowds out private contributions ($f'\left(\frac{1}{n}\right) = -\frac{ng(\overline{g_G})h''(ng(\overline{g_G})+\overline{g_G})}{(n-1)h'(ng(\overline{g_G})+\overline{g_G})}$). Let S be the set of these numbers. Let $\overline{g_G}'$ be the smallest number of S. Since S is closed and lower-bounded, the smallest number exists in S. Then, if government subsidy is smaller than $\overline{g_G}'$, government subsidy crowds in private donation, but (slightly) larger government subsidy crowds out private donation.

This explanation is dependent on the assumption of the utility function of public goods $h(G)$, but clearly interprets the coexistence of crowding-in and crowding-out. In this example, marginal utility for public goods is constantly decreasing. Therefore, the disincentive for private donation caused by government subsidy is always constant. However, the incentive, leverage effect, is decreasing. That is why a low level of government subsidy stimulates private donation where a high level discourages private donation.

5. Conclusion

Although theorists have already come up with several ideas to achieve the first-best public good provision, fundraisers as well as governments tend to rely on simpler measures to enhance private contributions. Among the measures that do not rely on government's coercive power, in order to incentivize individuals and corporate companies to contribute, fundraisers sometimes reflect major donors' preferences on public goods *as if donation has an aspect of voting*. There are many examples where public goods become preferable to donors: engraving a large donor's name, earmarked donation to local municipalities, giving special privileges to large donors when getting access to the public goods, etc. Although related ideas such as impure public goods, warm-glow, prestige etc. have been modeled so far, this idea has not been formalized. We provide a rigorous theoretical framework to reflect our idea, point out the similarity between our model and the fixed prize raffle model by Morgan (2000), and prove that private contribution is enhanced under our framework.

Also, it is known that government direct subsidy, which generally causes partial crowding-out, sometimes results in crowding-in of private contributions. For example, Payne (2001) points out that government subsidy for research universities increments private contributions to the same universities. Our model provides a novel explanation to understand crowding-out and crowding-in under the same framework and finds the conditions where crowding-in/crowding-out occur. In our model, government direct subsidy incentivizes private contribution because the influence on private contribution is leveraged by government subsidy. As is well-known, government direct subsidy causes crowding-out by its nature. If the former effect dominates the latter, crowding-in occurs. This explanation is totally different from preexisting explanations such as asymmetric information or seed money, which explain the crowding-in phenomenon.

The analysis in this paper is largely restricted to symmetric contribution. Further analysis should be conducted when the Nash equilibrium is asymmetric.

Appendices

Appendix I.1 Proof of Proposition 2

(a) When prize is zero, this model converges to a standard private provision problem. Therefore, public goods are under-provided. See Bergstrom et al (1986).

(b) When $N \geq 3$, it is proven that the weighting function $\varphi(\cdot)$ is a linear function from the following lemma.

Lemma

If $N \geq 3$, the weighting function $\varphi(\cdot)$ is a linear function.

Proof: See Appendix I.2

Let the slope of weighting function $\varphi(\cdot)$ be $\beta \geq 0$ (When $N = 2$, $\beta = \varphi'\left(\frac{1}{2}\right)$).

Then, an individual's first order condition is

$$\beta\left(P + l\left(\sum_{j=1}^{N} g_j - P + g_G\right)\right)\frac{\sum_{j=1,j\neq i}^{N} g_j}{\left(\sum_{j=1}^{N} g_j\right)^2} + \left(\varphi\left(\frac{g_i}{\sum_{j=1}^{N} g_j}\right) - \frac{1}{N}\right) l'\left(\sum_{j=1}^{N} g_j - P + g_G\right)$$

$$+ k_i'\left(\sum_{j=1}^{N} g_j - P + g_G\right) = 1. \qquad \text{(1-1)}$$

Taking the sum of this equality, the following equation is derived:

$$\beta(N-1)\frac{P + l\left(\sum_{j=1}^{N} g_j - P + g_G\right)}{\sum_{j=1}^{N} g_j} + \sum_{i=1}^{N} k_i'\left(\sum_{j=1}^{N} g_j - P + g_G\right) = N. \qquad \text{(1-2)}$$

The second order condition is

$$N - \sum_{i=1}^{N} k_i'\left(\sum_{j=1}^{N} g_j - P + g_G\right) - (N-1)\beta l'\left(\sum_{j=1}^{N} g_j - P + g_G\right) -$$

$$\frac{\sum_{j=1}^{N} g_j \sum_{i=1}^{N} k_i''\left(\sum_{j=1}^{N} g_j - P + g_G\right)}{2} \geq 0 \qquad (1\text{-}3)$$

Taking the total difference in equation (1-1), the effect of the increase of fixed amount of prize P to total contributions $\bar{G} = \sum_{j=1}^{N} g_j$ is derived as follows.

$$\frac{\partial \bar{G}}{\partial P} = \frac{(N-1)\beta\left(1 - l'(\bar{G} - P + g_G)\right) - \bar{G}\sum_{i=1}^{N} k_i''(\bar{G} - P + g_G)}{N - (N-1)\beta l'(\bar{G} - P + g_G) - \sum_{i=1}^{N} k_i'(\bar{G} - P + g_G) - \bar{G}\sum_{i=1}^{N} k_i''(\bar{G} - P + g_G)} \qquad (1\text{-}4)$$

Note that the denominator of the right-hand side of equation (1-4) is always positive due the second order condition (1-3).

The right-hand side of equation (1-4) is more than one if and only if $\bar{G} > P + l(\bar{G} - P + g_G)$.

(c) The right-hand side of equation (1-4) is positive if and only if the numerator of it is positive.

Also, taking the total difference in equation (1-1), the effect of the increase of government direct subsidy g_G to total contributions $\bar{G} = \sum_{j=1}^{N} g_j$ is derived as follows.

$$\frac{\partial \bar{G}}{\partial g_G} = \frac{(N-1)\beta l'(\bar{G} - P + g_G) + \bar{G}\sum_{i=1}^{N} k_i''(\bar{G} - P + g_G)}{N - (N-1)\beta l'(\bar{G} - P + g_G) - \sum_{i=1}^{N} k_i'(\bar{G} - P + g_G) - \bar{G}\sum_{i=1}^{N} k_i''(\bar{G} - P + g_G)} \qquad (1\text{-}5)$$

The right-hand side of equation (1-5) is positive if and only if the numerator of it is positive.

Since the numerator of the right-hand side of (1-4) plus the numerator of the right-hand side of (1-5) is always positive ($(N-1)\beta$), either fixed amount of prize P or government direct subsidy g_G enhances private contributions and it may occur that both measures enhance private contributions simultaneously.

(d) The left-hand side of equation (1-2) is increasing in β (the slope of the weighting function $\varphi(\cdot)$) and in the variable amount of prize $l\left(\sum_{j=1}^{N} g_j - P + g_G\right)$. This increase is to be compensated by the increase of total contributions $\sum_{j=1}^{N} g_j$. Note that both the first term and the second term in the left-hand side of equation (1-2) are decreasing in total contributions $\sum_{j=1}^{N} g_j$.

(e) The Samuelson condition states that public goods are underprovided if and only if $\sum_{i=1}^{N} k_i'\left(\sum_{j=1}^{N} g_j - P + g_G\right) > 1$. Then, our statement is easily derived. Note that public goods are always underprovided in the original Morgan's (2000) model because $\varphi'(\cdot) = \beta = 1$, $l(\cdot) = 0$, $g_G = 0$ and $\sum_{j=1}^{N} g_j > P$.

Appendix I.2 Proof of Lemma

If $n \geq 3$, note that; $\forall x, y(1 - y \geq x \geq y \geq 0)$; $2\varphi(x) + \varphi(1 - 2x) = \varphi(x + y) + \varphi(x - y) + \varphi(1 - 2x) \Rightarrow \varphi(x) = \frac{\varphi(x+y)+\varphi(x-y)}{2}$. Therefore, $\varphi(\cdot)$ is locally point-symmetric everywhere (if we assume C^2 in $\varphi(\cdot)$, it is equivalent to $\varphi''(\cdot) = 0$ everywhere.). Therefore, taking into account that $\varphi(\cdot)$ is a (weakly) increasing function, $\varphi(\cdot)$ must be linear.

Appendix I.3 Proof of Proposition 3

(a) This is a direct consequence of Proposition 2 (a). If you want to see direct proof, see the following.

The optimal level of public goods satisfies the Samuelson condition: $h'(g_A + g_B) = 1$. However, under the standard model, the first order condition satisfies

$$f'\left(\frac{g_A}{g_A+g_B}\right)\frac{g_B}{(g_A+g_B)^2}h(g_A + g_B) + f\left(\frac{g_A}{g_A+g_B}\right)h'(g_A + g_B) =$$

$$f'\left(\frac{g_B}{g_A+g_B}\right)\frac{g_A}{(g_A+g_B)^2}h(g_A + g_B) + f\left(\frac{g_B}{g_A+g_B}\right)h'(g_A + g_B) = 1 \Leftrightarrow h'(g_A + g_B) = 2 \quad .$$

Therefore, private provision alone results in under-provision of public goods.

(b) The first order condition of private optimization yields $h'(G) = 2$. Therefore, the total amount of public goods $G = g_A + g_B + g_G$ is always constant.

Appendix I.4 Proof of Proposition 4

(a) The first order condition satisfies

$$f'\left(\frac{g_A}{g_A+g_B}\right)\frac{g_B}{(g_A+g_B)^2}h(g_A+g_B)+f\left(\frac{g_A}{g_A+g_B}\right)h'(g_A+g_B)$$

$$=f'\left(\frac{g_B}{g_A+g_B}\right)\frac{g_A}{(g_A+g_B)^2}h(g_A+g_B)+f\left(\frac{g_B}{g_A+g_B}\right)h'(g_A+g_B)=1. \qquad \text{(4-1)}$$

Since $f'\left(\frac{g_A}{g_A+g_B}\right)=f'\left(\frac{g_B}{g_A+g_B}\right)$ and $f\left(\frac{g_A}{g_A+g_B}\right)+f\left(\frac{g_B}{g_A+g_B}\right)=1$,

$$h'(g_A+g_B)=2-f'\left(\frac{g_A}{g_A+g_B}\right)\frac{h(g_A+g_B)}{g_A+g_B}\leq 2. \qquad \text{(4-2)}$$

Therefore, the private provision g_A+g_B is always larger than that under the standard model and this inequality is strict if and only if $f'\left(\frac{g_A}{g_A+g_B}\right)>0$.

(b) Under the assumption of an inner solution and symmetric contributions[6], the first order condition for individual's optimization is

$$f'\left(\frac{1}{2}\right)h(G)+2gh'(G)=4g \qquad \text{(4-3)}$$

and the second order condition[7] is

$$2g\{f'\left(\frac{1}{2}\right)h'(G)+gh''(G)\}\leq f'(\tfrac{1}{2})h(G). \qquad \text{(4-4)}$$

Combining the first order condition (4-3) and the second order condition (4-4), we can derive a simple form of the second order condition:

$$-2+h'(G)+f'\left(\frac{1}{2}\right)h'(G)+gh''(G)\leq 0. \qquad \text{(4-5)}$$

Taking the total differential of the first order condition (4-3), we can derive the derivative

$$\frac{d(g_A+g_B)}{dg_G}=\frac{d(2g)}{dg_G}=-\frac{f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)}{-2+h'(G)+f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)}. \qquad \text{(4-6)}$$

The second order condition (4-5) guarantees that the denominator of the

[6] Note that $f\left(\frac{1}{2}\right)=\frac{1}{2}$ always holds.

[7] Note that $f''\left(\frac{1}{2}\right)=0$ because $f(x)$ is point symmetric around $x=\frac{1}{2}$.

right-hand side of derivative condition (4-6) is always negative. Therefore, the derivative condition of how government provision affects private provision depends on the sign of the numerator of the right-hand side of derivative condition (4-6). If the numerator is positive ($f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)>0$), government provision increases private contribution. If the numerator is negative ($f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)<0$), since the denominator is smaller than the numerator[8], government provision crowds out private contribution and this crowding-out is incomplete if and only if $f'\left(\frac{1}{2}\right)>0$.

Appendix I.5 Proof of Proposition 5

With regard to the statement (a), the definition of Nash equilibrium $f\left(\frac{g'}{g+g'}\right)h(g+g'+g_G)-g'\leq\frac{1}{2}h(2g+g_G)-g\forall g'\geq 0$ directly results in the conclusion. Note that the first order condition of statement (a) is $f'\left(\frac{1}{2}\right)h(2\mathrm{g}+g_G)+2gh'(2g+g_G)=4g$ and the second order condition is $2g\left\{f'\left(\frac{1}{2}\right)h'(2g+g_G)+gh''(2g+g_G)\right\}<f'\left(\frac{1}{2}\right)h(2g+g_G)$.

With regard to the statement (b), let $\alpha=\frac{g_A}{g_A+g_B}$ and $x=g_A+g_B$ where $g_A\neq g_B$. Assume $\alpha>\frac{1}{2}$ without loss of generality. Then, the first order condition that (g_A,g_B) is Nash equilibrium is $f'(\alpha)=\frac{\left(2-h'(x+g_G)\right)x}{h(x+g_G)}$ and $\frac{f(\alpha)-1/2}{\alpha-1/2}=\frac{2}{h'(x+g_G)}-1$. There is no asymmetric Nash equilibrium if the system has no roots.

[8] Note that $h'(G)\leq 2$ from the first order condition (1-1).

Appendix I.6 Proof of Example 1

It is easy to confirm that $(g_A, g_B) = \left(\frac{1}{8}, \frac{1}{8}\right)$ is the only symmetric Nash equilibrium. In order to find out that there is no asymmetric Nash equilibrium, it should be noted that the right-hand side of equation (3), the left-hand side of equation (3), the right-hand side of equation (4), and the left-hand side of equation (4) are all increasing functions where $\alpha > \frac{1}{2}$ is satisfied. Therefore, the infimum value that x takes in equation (3) is $x = \frac{1}{4}$ because $\lim_{\alpha \to \frac{1}{2}+0} f'(\alpha) = \lim_{x \to \frac{1}{4}+0} \frac{\left(2-h'(x+g_G)\right)x}{h(x+g_G)}$ is satisfied. The maximum value that x takes in equation (4) is also $x = \frac{1}{4}$ because $\left[\frac{f(\alpha)-1/2}{\alpha-1/2}\right]_{\alpha=1} = \left[\frac{2}{h\prime(x+g_G)} - 1\right]_{x=\frac{1}{4}}$ is satisfied. Thus, the system, (3) and (4), has no roots, which proves that there is no asymmetric Nash equilibrium.

Appendix I.7 Proof of Proposition 6

First, we will see that incorporating warm-glow effect inevitably enhances private donations.

The first order condition without warm-glow effect is

$$f'\left(\frac{1}{2}\right)\frac{h(2\bar{\bar{g}}+g_G)}{4\bar{\bar{g}}} + \frac{h'(2\bar{\bar{g}}+g_G)}{2} = 1 \qquad \text{(7-1)}$$

where it is

$$k'(g^*) + f'\left(\frac{1}{2}\right)\frac{h(2g^*+g_G)}{4g^*} + \frac{h'(2g^*+g_G)}{2} = 1 \qquad \text{(7-2)}$$

with warm-glow effect. Then, it is obvious to see that private contribution with warm-glow is more than that without warm-glow i.e. $g^* \geq \bar{\bar{g}}$ because both

$\frac{h(2g+g_G)}{4g}$ and $h'(2g+g_G)$ are decreasing functions in g [9] where the inequality $k'(g^*) > 0$ always holds.

The following proof is an extension of Appendix I.4.

(a) The first order condition results in

$$k'(g) + f'\left(\frac{1}{2}\right)\frac{h(G)}{4g} + \frac{h'(G)}{2} = 1. \qquad (7\text{-}3)$$

Then, private contribution converges to $\bar{g}$ under the standard model ($f'(x) = \frac{1}{2}\forall x$) where

$$k'(\bar{g}) + \frac{h'(2\bar{g}+g_G)}{2} = 1 \qquad (7\text{-}4)$$

is satisfied. If we relax the restriction on $f(\cdot)$, private contribution converges to g^* where

$$k'(g^*) + f'\left(\frac{1}{2}\right)\frac{h(2g^*+g_G)}{4g^*} + \frac{h'(2g^*+g_G)}{2} = 1 \qquad (7\text{-}5)$$

is satisfied. Then, it is obvious to see that private contribution under this model is more than that under the standard model ($f'(x) = \frac{1}{2}\forall x$) i.e. $g^* \geq \bar{g}$ because both $k'(\cdot)$ and $h'(\cdot)$ are decreasing functions where inequality $f'\left(\frac{1}{2}\right)\frac{h(2g^*+g_G)}{4g^*} \geq 0$ is satisfied. The inequality $g^* \geq \bar{g}$ is strict if and only if $f'\left(\frac{1}{2}\right) > 0$ holds.

(b) Under the assumption of an inner solution and symmetric contributions, the first order condition for individual's optimization is

---

[9] $\frac{h(2g+g_G)}{4g}$ is decreasing in g. It is easily shown by taking the derivative $\frac{d\left(\frac{h(2g+g_G)}{4g}\right)}{dg} = \frac{8gh'(2g+g_G)-4h(2g+g_G)}{16g^2} \leq 0$. Note that $h'(x) \leq \frac{h(x)}{x}$ is always satisfied due to the assumption on $h(\cdot)$.

$$4gk'(g) + f'\left(\frac{1}{2}\right)h(G) + 2gh'(G) = 4g \qquad \text{(7-6)}$$

and the second order condition is

$$2g\{2gk''(g) + f'\left(\frac{1}{2}\right)h'(G) + gh''(G)\} \leq f'(\frac{1}{2})h(G). \qquad \text{(7-7)}$$

Combining the first order condition (7-6) and the second order condition (7-7), we can derive a simple form of the second order condition:

$$-2 + h'(G) + f'\left(\frac{1}{2}\right)h'(G) + gh''(G) + 2k'(g) + 2gk''(g) \leq 0. \qquad \text{(7-8)}$$

Taking the total differential of the first order condition (4-6), we can derive the derivative

$$\frac{d(g_A+g_B)}{dg_G} = \frac{d(2g)}{dg_G} = -\frac{f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)}{-2+h'(G)+f'\left(\frac{1}{2}\right)h'(G)+2gh''(G)+2k'(g)+2gk''(g)}. \qquad \text{(7-9)}$$

The second order condition (7-8) guarantees that the denominator of the right-hand side of derivative condition (7-9) is always negative. Therefore, the derivative condition of how government provision affects private provision depends on the sign of the numerator of the right-hand side of derivative condition (7-9). If the numerator is positive ($f'\left(\frac{1}{2}\right)h'(G) + 2gh''(G) > 0$), government provision increases private contribution. If the numerator is negative ($f'\left(\frac{1}{2}\right)h'(G) + 2gh''(G) < 0$), since the denominator is smaller than the numerator, government provision crowds out private contribution and this crowding-out is incomplete if and only if $f'\left(\frac{1}{2}\right) > 0$.

Appendix I.8 Characteristics of function f

Since f is weakly increasing and $n \geq 3$, the abovementioned lemma guarantees that it is a linear function in $[0,1]$. Remembering that $f\left(\frac{1}{n}\right)=\frac{1}{n}$, the linear function is described as $f(\alpha)=\beta\left(\alpha-\frac{1}{n}\right)+\frac{1}{n}$ with $\beta\in[0,1]$ where the upper bound of β is derived from the fact: $f(0)\geq 0$ and $f(1)\leq 1$.

Appendix I.9 Proof of Proposition 7

(a) The first order condition for individual i is

$$f'\left(\frac{g_i}{\sum_{j=1}^{n}g_j}\right)\frac{\sum_{j=1,j\neq i}^{n}g_j}{\left(\sum_{j=1}^{n}g_j\right)^2}h(G)+f\left(\frac{g_i}{\sum_{j=1}^{n}g_j}\right)h'(G)=1. \quad \text{(9-1)}$$

Taking the sum of (9-1) for $i=1\sim n$ and recalling that $\sum_{i=1}^{n}f\left(\frac{g_i}{\sum_{j=1}^{n}g_j}\right)=1$,

$$h(G)\sum_{i=1}^{n}f'\left(\frac{g_i}{\sum_{j=1}^{n}g_j}\right)\frac{\sum_{j=1,j\neq i}^{n}g_j}{\left(\sum_{j=1}^{n}g_j\right)^2}+h'(G)=n. \quad \text{(9-2)}$$

Since $f'(x)=0\forall x$ is satisfied, $h'(G)=n$ holds and this result implies under-provision as well as complete crowding-out of government contribution.

(b) Due to the first order condition and symmetric condition,

$$f'\left(\frac{1}{n}\right)=\frac{ng(n-h'(G))}{(n-1)h(G)} \quad \text{(9-3)}$$

is satisfied where $g_i=g\forall i$. If $f'\left(\frac{1}{n}\right)=0$ holds, $h'(G)=n$ is derived, which implies that the total amount of public goods is the same as in the previous case. If $f'\left(\frac{1}{n}\right)>0$ holds, $h'(G)<n$ holds, which implies that the total amount of public goods is more than the previous case (a).

(c) Due to the first order condition and symmetric condition,

$$\frac{d(ng)}{dg_G}=-\frac{(n-1)f'\left(\frac{1}{n}\right)h'(G)+ngh''(G)}{(n-1)f'\left(\frac{1}{n}\right)h'(G)-n+h'(G)+ngh''(G)} \quad \text{(9-4)}$$

holds. Equality (9-3) and $f'(\cdot)\geq 0$ imply the inequality $h'(G)\leq n$, which means that the numerator of the right-hand side of equation (9-4) is weakly larger than the denominator. Due to the second order condition $-2\left(n-h'(G)\right)+2(n-1)f'\left(\frac{1}{n}\right)h'(G)+ngh''(G)<0$, the denominator is always negative. Therefore, if the numerator is negative, there is partial crowding-out if $f'>0$. If the numerator is positive, government contribution results in crowding-in.

References

Andreoni, James, 1989. “Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence.” *Journal of Political Economy* 97 (6), 1447-1458.

Andreoni, James, 1990. “Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving.” *Economic Journal* 100 (401), 464-477.

Andreoni, James, 1993. “An Experimental Test of the Public-Goods Crowding-Out Hypothesis.” *American Economic Review* 83 (5), 1317-1327.

Andreoni, James, 1998. “Toward a Theory of Charitable Fund-Raising.” *Journal of Political Economy* 106 (6), 1186-1213

Andreoni, James, 2006 “Leadership Giving in Charitable Fund-Raising.” *Journal of Public Economic Theory* 8 (1), 1-22.

Andreoni, James, and A. Abigail Payne, 2003. “Do Government Grants to Private Charities Crowd Out Giving or Fund-raising?” *American Economic Review* 93 (3), 792-812.

Andreoni, James, A. Abigail Payne, and Sarah Smith, 2014. “Do grants to charities crowd out other income? Evidence from the UK.” *Journal of Public Economics* 114 (C), 75-86

Auten, Gerald, Holger Sieg, and Charles Clotfelter, 2002. “The Distribution of Charitable Giving, Income and Taxes: An Analysis of Panel Data.” *American Economic Review* 92 (1), 371-382.

Bakija, Jon, and Bradley Heim, 2011. “How Does Charitable Giving Respond to Incentives and Income? New Estimates from Panel Data.” *National Tax Journal* 64 (2), 615-650.

Becker, Gary S., 1974. “A Theory of Social Interactions.” *Journal of Political Economy*. 82 (6), 1063-1093.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian, 1986. “On the Private Provision of Public Goods.” *Journal of Public Economics* 29 (1), 25-49.

Blumkin, Tomer, and Efraim Sadka, 2007 “A Case for Taxing Charitable Donations.” *Journal of Public Economics* 91 (7-8), 1555–1564

Brooks, Author C., 2000. “Public Subsidies and Charitable Giving: Crowding out, Crowding in, or Both?.” *Journal of Policy Analysis and Management* 19 (3), 451-464.

Buira, Ariel, 2005. “Reforming the Governance of the IMF and the World Bank.” London: *Anthem Press*

Clotfelter, Charles T., 1985. “Federal Tax Policy and Charitable Giving.” Chicago: University of Chicago Press.

Cornes, Richard, and Todd Sandler, 1984. “Easy Riders, Joint Production, and Public Goods.” *Economic Journal*. 94 (3), 580-598.

Cornes, Richard and Todd Sandler, 1994. "The Comparative Static Properties of the Impure Public Good Model." *Journal of Public Economics*, 54(3), 403-421.

Diamond, Peter, 2006. “Optimal tax treatment of private contributions for public goods with and without warm glow preferences.” *Journal of Public Economics* 90 (4-5), 897–919

Glazer, Amihai, and Kai A. Konrad, 1996 “A Signaling Explanation for Charity.” *American Economic Review* 86 (4), 1019-1028

Groves, Theodore and John Ledyard, 1977. "Optimal Allocation of Public Goods: A Solution to the Free Rider Problem." *Econometrica* 45(4), 783-809.

Harbaugh, William T., 1998a “The Prestige Motive for Making Charitable Transfers.” *American Economic Review* 88 (2), 277-282

Harbaugh, William T., 1998b “What do donations buy? A model of philanthropy based on prestige and warm glow.” *Journal of Public Economics* 67 (2), 269-284

Khanna, Jyoti, and Todd Sandler, 2000. “Partners in giving: The crowding-in effects of UK government grants.” *European Economic Review* 44 (8), 1543-1556

Konishi, Sachio, 2015. “Hometown tax payment system supported by local economy vitalizing policy.” In *Chihou Zaimu* No. 733, 2-13, Gyosei, in Japanese

Li, Sherry Xin, Catherine Eckel, Philip J. Grossman, and Tara Larson Brown, 2011. "Do Earmarks Increase Giving to Government?"

http://cbees.utdallas.edu/papers/gtg2.pdf

Mirrlees, J. A., 1971. “An Exploration in the Theory of Optimum Income Taxation.” *Review of Economic Studies* 38 (2), 175-208

Morgan, John, 2000. “Financing Public Goods by Means of Lotteries.” *Review of Economic Studies* 67 (4), 761-784.

Payne, A. Abigail, 1998. “Does the government crowd-out private donations? New evidence from a sample of non-profit firms.” *Journal of Public Economics* 69 (3), 323-345.

Payne, A. Abigail, 2001. “Measuring the Effect of Federal Research Funding on Private Donations at Research Universities: Is Federal Research Funding More Than a Substitute for Private Donations?” *International Tax and Public Finance* 8 (5-6), 731-751.

Potters, Jan, Martin Sefton, and Lise Vesterlund, 2005. “After You - Endogenous Sequencing in Voluntary Contribution Games.” *Journal of Public Economics* 89 (8),1399-1419.

Randolph, William C., 1994. “Dynamic Income, Progressive Taxes, and the Timing of Charitable Contributions.” *Journal of Political Economy* 103 (4), 709-738.

Roberts, Russell D., 1984. ”A Positive Model of Private Charity and Public Transfers.” *Journal of Political Economy* 92 (1), 136-148.

Romano, Richard, and Huseyin Yildirim, 2001. “Why Charities Announce Donations: a Positive Perspective.” *Journal of Public Economics* 81 (3), 423-447.

Rose-Ackerman, Susan, 1981. “Do Government Grants to Charity Reduce Private Donations?” in M. White, ed., *Non-Profit Firms in a Three Sector Economy,* COUPE Papers in Public Economics #6, Urban Institute, Washington, 95-114.

Segal, Lewis M., and Burton A. Weisbrod, 1998. “Interdependence of Commercial and Donative Responses.” In Burton A. Weisbrod (ed.), *To Profit or Not to Profit*. Cambridge University Press.

Slemrod, Joel, 2003. “Trust in Public Finance.” In S. Cnossen and H.-W. Sinn (eds.), *Public Finance and Public Policy in the New Century*, MIT Press, 49-88.

Steinberg, Richard, 1987. “Voluntary Donations and Public Expenditures in a Federalist System.” *American Economic Review* 77 (1), 24-36.

Sugden, Robert, 1982. “On the Economics of Philanthropy.” *Economic Journal* 92, 341-350.

Tiebout, Charles M., 1956. “A Pure Theory of Local Expenditures.” *The Journal of Political Economy* 64 (5), 416-424

Vesterlund, Lise, 2003. “The Informational Value of Sequential Fund-raising.” *Journal of Public Economics* 87 (3-4), 627-657.

Walker, Mark, 1981. "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations.” *Econometrica* 49 (1), 65-71.

Warr, Peter G., 1982. “Pareto Optimal Redistribution and Private Charity.” *Journal of Public Economics* 19 (1), 131-138.