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Does an Optimal Voluntary Approach Flexibly and Efficiently Control Emissions from Heterogeneous Firms?

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Does an Optimal Voluntary Approach Flexibly and Efficiently Control Emissions from Heterogeneous Firms?*

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Abstract

We theoretically examine voluntary policies' potential for controlling emissions from a large number of heterogeneous firms under an asymmetric information case as well as the flexibility of the optimal voluntary policy. The potential of flexible voluntary policies depends on the type of heterogeneity, distribution of firms and possibility of failure in introduction of inflexible mandatory regulation. If heterogeneity of emission abatement costs results from firms' technology level and there are few low technology firms, there is likely to be a flexible voluntary policy that generates higher social welfare than the inflexible mandatory regulation. However, an inflexible voluntary policy is the best among the feasible voluntary policies if there are more higher abatement cost firms than lower abatement cost firms. Moreover, no voluntary policy can generate higher social welfare than inflexible mandatory regulation if heterogeneity of emission abatement costs results from firms' emission size, there are many small emission size firms, some medium emission size firms and a few large emission size firms, and the introduction of the inflexible mandatory regulation can fail.

Keywords: Voluntary Approarch, Heterogeneous firms with private information

JEL Classification: Q58, D78, D82.

1 Introduction

A number of regulatory instruments have been considered by economists in the context of a benevolent social planner's problem. An omnipotent government could impose flexible regulations on each firm respectively, but in reality, any reasonable regulations must be suboptimal. An example is a mandatory policy, imposed directly by the central or local government, usually accompanied by criminal punishment. Although such a policy tends to be inflexible due to the limitation in law-making, such a policy has an advantage in enforceability. Such a policy is prevalent in many policy

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issues i.e. financial regulation, custom regulation, etc. Usually, public economists are interested in mandatory policy; optimal tax theory is one of the successful contributions they have made. Another example is voluntary policy, imposed by an industry group or other voluntary group, without any prima facie enforcement. Although such a policy is frequently used for environmental issues, it is also adopted in the cinema industry, fishery industry, etc.

Among all the policy issues, environmental issues are by far the most controversial in terms of the comparative merits and demerits of voluntary policy and mandatory regulation. Voluntary approaches, polluters' commitment to improve environmental performance beyond legal requirement, have gained popularity as new environmental policy tools since the early 1990s. The reason for the increasing popularity of voluntary approaches is that they are recognized as flexible and cost-effective policy tools. In contrast with voluntary approaches, traditional command and control approaches have been criticized due to their inflexibility and cost-ineffectiveness. Moreover, the introduction of economic instruments like pollution taxes and emissions trading has been politically difficult though the instruments are cost-effective. As a result, policymakers have encouraged polluting firms to make voluntary commitments.

What drives the adoption of voluntary approaches or the pursuit of flexible and cost-effective (and politically feasible) environmental policy tools is informational asymmetry about the emissions abatement cost of a large number of firms. If there were a small number of firms that are supposed to be subject to environmental policy, the costs of acquiring information on their abatement costs would not be so high and therefore, the introduction of mandatory regulations appropriate to the firms' abatement costs would neither be so difficult nor costly. Lacking only informational asymmetry, regulators can set a mandatory policy at least as socially desirable as firms do. Because polluting firms, knowing about their emissions abatement costs more than policymakers, can choose the lowest cost option of emissions abatement, it is possible that voluntary emissions abatement policies generate higher social welfare than mandatory regulations. However, it is also possible that the opposite result happens because, if they can choose how much they abate, the amount of their emissions abatement might be too low even though they can abate their emissions by the lowest cost option.

This paper examines voluntary policies' potential for the emissions abatement of a large number of heterogeneous firms under an asymmetric information case and the flexibility of the optimal voluntary policy. We develop a game theoretic model of emissions abatement policy played by a regulator and polluting firms with private information on their emissions abatement costs. Due to the inflexibility of mandatory regulation, all firms are subject to uniform emission standards under the mandatory regulation, whereas high abatement cost firms might be able to emit more than low abatement cost firms do under voluntary policy if the firms submit credible evidence (signal) that they are of high abatement cost to the regulator. A feasible set of voluntary emissions abatement policies is characterized by firms' commitment to voluntarily control of their emissions level with the requirement to submit credible evidence. We derive and characterize the optimal voluntary policy that generates the highest social welfare in the feasible set of voluntary policies and also compare the optimal voluntary policy with the mandatory policy in terms of social welfare.

The characteristics and performance of the optimal voluntary policy depend on the type of heterogeneity. When firms' emission size is heterogeneous, an inflexible voluntary policy (a kind of uniform emission standard) is optimal among the feasible voluntary policies unless the distribution of emission size is strongly biased toward a small size or medium size. The optimal inflexible voluntary policy generates strictly higher social welfare than a uniform mandatory standard only if the introduction of the mandatory standard might fail. Moreover, the optimal voluntary policy is flexible (different emission standard for firms of different emission size) but generates lower social welfare than the mandatory standard does if the distribution of firms' emission size sharply decreases for all emission size.

On the other hand, when firms' emission abatement technology is heterogeneous, a flexible voluntary policy always generates strictly higher social welfare than the mandatory standard does if the flexible policy is optimal. In addition to the performance of the flexible voluntary policy, relative to a case of heterogeneous emission size, it is likely that optimal voluntary policy is flexible. The difference in performance of voluntary policy under a heterogeneous emission size case and heterogeneous abatement technology case has roots in the difference of the best inflexible voluntary policy's performance relative to the mandatory standard. While the best inflexible voluntary policy always generates weakly higher social welfare than the mandatory standard does under the heterogeneous abatement technology cases, the policy does not always do so under the heterogeneous emission size case.

Although many researchers have studied voluntary approaches to environmental protection as the popularity of voluntary approaches increases, we are unaware of any theoretical study that addresses the potential and flexibility of voluntary policies targeted at a large number of firms under an asymmetric information case. Most researchers have focused on the case of a single polluter (e.g. Segerson and Miceli, 1998; Hansen, 1999; Segerson and Miceli, 1999; Glachant, 2007; and Fleckinger and Glachant, 2011) or mutiple polluters without informational asymmetry (Lutz et al., 2000; Maxwell et al., 2000; Manzini and Mariotti, 2003; Dawson and Segerson, 2008; and Brau and Carraro, 2011). Only Lyon and Maxwell (2003) examined a voluntary program for multiple polluters with private information on their technology. However, the voluntary program in Lyon and Maxwell's model is assumed to provide all participating firms with a uniform subsidy. Thus, Lyon and Maxwell (2003) did not examine how voluntary approaches can flexibly address the emissions abatement of a large number of firms under an asymmetric information case.

Technically, our model has aspects of both signaling and monopolistic screening. Firms' behavior (particularly, cost verification activity) in our model can be interpreted as that of informed agents in the signaling game that was first investigated by Spence (1973) and especially, as that of informed agents with continuous types whose strategy's property is characterized by Mailath (1987) and Mailath and von Thadden (2013). However, the regulator in our model, an uninformed agent, offers a voluntary emission control scheme to firms taking into account firms' behavior like monopolistic screening. Therefore, our model can be also interpreted as a monopolistic screening model where an agent's reservation utility depends on its type, like Jullien's (2000). In contrast to monopolistic screening models, monetary transfer between the regulator and firms is impossible in our model[1] and alternatively, firms might send a costly signal to reveal their type.

The remainder of this paper is organized as follows. In the next section, we describe the model environment. Section 3 presents the main results. Section 4 discusses what leads to the variance in the results of different types of heterogeneity and examines our main results in a more generalized setting. Section 5 concludes this paper.

2 Setup

We consider a two-stage policy game played by a regulator and heterogeneous firms in an industry to examine voluntary policies' potential for emissions abatement of heterogeneous firms with private information. To achieve this aim, we consider a case where the benevolent regulator has strong negotiation power under the voluntary policy making process. Specifically, we focus on a voluntary agreement between the regulator and the industry (a group of the firms) in a case where the regulator makes a take-it-or-leave-it offer to the industry under the voluntary policy making process.

In the first stage, the regulator makes the take-it-or-leave-it offer of voluntary agreement to the industry and then, the industry decides whether to accept the offer. The industry accepts the offer when all firms prefer the voluntary policy to the mandatory policy. If the industry accepts the offer, the firms abate their emissions by following the agreement. If the industry rejects the offer, then, in the second stage, the regulator tries to introduce a mandatory policy for the industry with a probability of failure of $1 - p$ ($0 < p \leq 1$). By introducing a possibility that the regulator fails to introduce the mandatory policy, we incorporate political difficulties in the mandatory policy-making process[2] as simply as possible to focus on firm heterogeneity. Thus, due to the risk of having no mandatory regulation, the regulator has an incentive to introduce the voluntary policy or to make the offer of the voluntary policy to the industry. Once the mandatory standard is introduced, all firms must follow it. The timing of the game is summarized as follows:

1. (Voluntary policy (VP)) The regulator makes the take-it-or-leave-it offer to the industry. If the industry accepts the offer, then firms abate their emissions by following the agreement.

2. (Mandatory policy (MP)) If the industry refuses the offer, then the regulator tries to introduce a mandatory policy for the industry. The mandatory policy is adopted with probability p and all firms comply. Otherwise, firms do not abate their emissions at all.

[1] We assume no monetary transfer between the regulator and firms judging from the characteristics of voluntary commitment to emission reduction.

[2] Fleckinger and Glachant (2011) pointed out that the "assumption that the adoption of legislation is subject to uncertainty is both realistic and common in papers dealing with voluntary abatement" (p.43). The simplest way to incorporate the uncertainty of adopting the legislation into a model is to assume that p is purely exogenous like Segerson and Miceli (1998). We follow Segerson and Miceli's method. See p.43 of Fleckinger and Glachant (2011) for other ways to incorporate the uncertainty of adopting the legislation in the literature.

Firm i emits a pollutant at e_i before the introduction of environmental policy (e_i can be interpreted as a natural emission level or emission size) and it's emission abatement cost is given by $\frac{1}{2}c_i a_i^2$, where a_i and c_i are the actual emission abatement level and the slope of marginal abatement cost (MAC) of firm i. Although we examine the heterogeneity of both e_i, and c_i, we first focus on the heterogeneity of emission size, e_i, and let $c_i = c$ for all i (and then we consider the heterogeneity of the slope of MAC, c_i). We assume that e_i is distributed over $[e_{\min}, e_{\max}]$ ($e_{\max} > e_{\min} \geq 0$) with continuously differentiable probability density $f(e_i)$[3] and that the regulator knows $f(\cdot)$ but does not know the emission size of any given firm. Thus, we implicitly assume that the regulator can observe the aggregate emission level but cannot observe the emission level of individual firms. On the other hand, environmental damage due to firms' emissions is assumed to be $\delta \int_{e_{\min}}^{e_{\max}} (e_i - a_i) f(e_i) de_i$.

The regulator's objective is to minimize social cost, the sum of damage and aggregate emissions abatement cost given by

$$\delta \int_{e_{\min}}^{e_{\max}} (e_i - a_i) f(e_i) de_i + \int_{e_{\min}}^{e_{\max}} \frac{1}{2} c a_i^2 f(e_i) de_i. \tag{1}$$

We assume that the mandatory policy is uniform emission standard (imposing upper-bound of emission e_{MP}, formally $a(e_i) = \min\{e_i - e_{MP}, 0\}$ for all i). This assumption seems reasonable because the traditional "command-and-control" approach has been criticized as being inflexible and cost-ineffective whereas the voluntary approach is considered to be flexible. It is also possible that the mandatory policy is an economic incentive approach such as emission tax or an emission trading scheme rather than the traditional "command-and-control" approach. Although the economic incentive approach is cost-effective in contrast with the "command-and-control" approach, the introduction of the "effective" economic incentive approach has been much more politically fraught and difficult than that of the "command-and-control" approach due to opposition from manufacturing sectors and industry associations. This is because the economic incentive approach imposes tax payments or permit purchases as well as emissions abatement cost on polluting industries or firms. As a result of the opposition to the economic incentive approach, the feasible mandatory policy instruments generating the highest social welfare might be a uniform emission standard. We consider such a case.

The mandatory uniform standard (MP) is

$$e_{MP} \in \underset{e_{\min} \leq e' \leq e_{\max}}{\arg\min} \; p \left\{ \int_{e'}^{e_{\max}} \left[\delta e' + \frac{1}{2} c (e - e')^2 \right] f(e) de + \delta \int_{e_{\min}}^{e'} e f(e) de \right\} + (1-p)\delta \int_{e_{\min}}^{e_{\max}} e f(e) de.$$

Unlike the mandatory policy, emission levels of individual firms can be different under voluntary policy. For example, under the voluntary policy, it is possible that firms of large emission size emit more than those of small emission size do, whereas both kinds of firms emit the same amount, e_{MP} (or less), under the mandatory policy. However, firms have to convince the regulator that their emission size is large if they want to emit more since the regulator does not know the emission sizes

[3]We assume that $e_{\min}$ and $e_{\max}$ have the following properties; $f(e_{\min}) > 0$ or if $f(e_{\min}) = 0$, $\exists \lambda > 0$, $\forall \theta \in (0, \lambda)$, $f(e_{\min} + \theta) > 0$; and $f(e_{\max}) > 0$ or if $f(e_{\max}) = 0$, $\exists \lambda > 0$, $\forall \theta \in (0, \lambda)$, $f(e_{\max} + \theta) > 0$.

of individual firms. To convince the regulator of their emission sizes or send it signals about their emission sizes, firms adopt costly certificated environmental management systems (EMSs) such as ISO14001, issue detailed environmental reports (with a third party certification) or do both, for instance. Let firm's signaling cost be such that its emission size is e_i be $s(e_i)(\geq 0)$. The signaling is assumed to generate no social benefit or to just waste resources.

At the stage of voluntary policy negotiation, the regulator offers the industry a voluntary abatement rule a that depends on signaling cost $(a(e_j) = a(s(e_j))$) and is semi-continuous. Then, the total cost of firm i that convinces the regulator that its emission size is e_j is $\frac{1}{2}c(e_i - e_j + a(e_j))^2 + s(e_j)$ and social cost under the voluntary policy is

$$\int_{e_{\min}}^{e_{\max}} \left[\delta(e_j - a(e_j)) + \frac{1}{2}c(e_i - e_j + a(e_j))^2 + s(e_j)\right] f(e_i)de_i.$$

By the Revelation Principle, we can focus on a direct mechanism to analyze what happens in equilibrium. Therefore, we can write the regulator's problem under the voluntary policy as

$$\min_g \int_{e_{\min}}^{e_{\max}} \left[\delta(e_i - a(e_i)) + \frac{1}{2}c[a(e_i)]^2 + s(e_i)\right] f(e_i)de_i \tag{2}$$

subject to

$$\text{(IC)} \quad e_i = \arg\min_{e'_i}[\frac{1}{2}c(e_i - e'_i + a(e'_i))^2 I_{[e_i > e'_i - a(e'_i)]} + s(e'_i)] \quad \forall i \tag{3}$$

$$\text{(PC)} \quad \frac{1}{2}c(a(e_i))^2 + s(e_i) \leq p\frac{1}{2}c(e_i - e_{MP})^2 I_{[e_i > e_{MP}]} \quad \forall i \tag{4}$$

where IC is the incentive compatibility condition, PC is the participation condition and I_B is an indicator function that is equal to 1 if B is true and 0 otherwise. The participation condition is that firms' cost under the voluntary policy is smaller than their expected cost under the mandatory policy (taking into account the possibility that the regulator fails to introduce the mandatory policy). Otherwise, the industry rejects the voluntary policy rule proposed by the regulator.

3 Results

3.1 Heterogeneity in e

Before we show general results, let us consider a simple case when e_i is distributed uniformly to understand what happens under the MP and the implication of IC and PC. In this case, the mandatory uniform emission standard is as follows.

Example 1. If the distribution of e_i is uniform under the assumption of $e_{\min} + \frac{2\delta}{c} \leq e_{\max}$, then the MP satisfies $e_{MP} = e_{\max} - \frac{2\delta}{c}$.

Proof. the MP must satisfy $e_{MP} = \arg\max_{e_{MP}} \int_{e_{\min}}^{e_{\max}} \{-\frac{c}{2}\{\max(0, e - e_{MP})\}^2 + \delta \max(0, e - e_{MP})\}de$. A simple calculation thus results in $e_{MP} = e_{\min} - \frac{2\delta}{c}$. ◻

Thus, under the MP, firm i emits e_{MP} (or abates its emissions by $(e_i - e_{MP})$) if $e_i \geq e_{MP}$; and it emits e_i (or does not abate its emissions at all) otherwise. From this fact and PC, firms whose emission size is smaller than e_{MP} ($e_i < e_{MP}$) have no incentive to abate their emissions under the VP or to participate in VP.

We can rewrite the IC and PC conditions. As we show in the Appendix A.1, the IC condition can be simplified to $c(1-a'(e_i))a(e_i) = s'(e_i)$ and $a'(e_i) \leq 1$ for all i when $s(\cdot)$ and $a(\cdot)$ are differentiable. These conditions can be written in a non-differentiable form, such as

$$s(e_i) = c\{A(e_i) - \frac{(a(e_i))^2}{2}\} \ \forall i \tag{5}$$

and $a'(e_i) \leq 1 \ \forall i$ where $A'(\cdot) = a(\cdot)$. Note that the IC condition implies that $s(\cdot)$ is weakly monotonically increasing derived from $c(1 - a'(e_i))a(e_i) = s'(e_i)$ and $a'(e_i) \leq 1$. Substituting (5) for the PC condition, the PC condition is simplified to:

$$A(e_i) \leq \frac{p(e_i - e_{MP})^2}{2} \ \forall i. \tag{6}$$

Then, the following theorem that characterizes the optimal VP holds.

Theorem 1. *If $f'(e) > -\frac{cf(e)}{\delta}$ for all e, then the optimal VP is uniform emission standard e_{VP} for all e (or emission abatement rule $a(e) = \max\{0, e - e_{VP}\}$ for all e) and $s(e) = 0$ holds for all e. If $p < 1$ and there exists e such that $f'(e) < -\frac{cf(e)}{\delta}$, then the optimal VP is not a uniform emission standard and there exists e' such that $s(e') > 0$.*

Proof. See Appendix A.2. □

This theorem states the following: if probability distribution $f(\cdot)$ is weakly monotonically increasing like a uniform distribution, or decreasing but flatter than $\exp(-\frac{ce}{\delta})$ everywhere, then, the signaling cost is too expensive and, as a result, the optimal VP is a uniform emission standard like the MP. Sending a costly signal and adopting a flexible emission standard (relative to the MP) is justified only if $f(\cdot)$ is decreasing sharply from the social welfare point of view.

What does a sharp decrease in probability density $f(\cdot)$ imply? If $f(\cdot)$ sharply decreases around e_i, $f(\cdot)$ must be relatively high just below e_i. Thus, focusing on the small region around e_i, there are likely to be many firms of small emission size if f sharply decreases. In such a case, the regulator sets the mandatory uniform standard to mainly control emissions for firms of small emission size i.e. the mandatory standard is too strict for firms of large emission size. Therefore, it is better for large emission size firms to emit more than they do under the mandatory standard even though they have to send a costly signal.

If there is a large number of large emission size firms ($f(\cdot)$ is not sharply decreasing), the regulator sets the mandatory uniform standard to mainly control emissions for firms of large emission size. Such a standard is generally too lax for low emission level firms although it might be a little strict for high emission level firms. The low emission level firms have no more incentive to voluntarily agree to commit themselves to abate their emissions than they do under the mandatory standard. In addition, the regulator does not want to relax emission control policy for firms of large emission size relative to the mandatory standard. As a result, costly signaling and flexible policy adoption is not optimal.

Then, the next question is whether social welfare under optimal VP is greater then social welfare under the MP. The result is actually as follows.

Theorem 2. *If $f'(e) > -\frac{cf(e)}{\delta}$ holds for all e, then social welfare under the optimal VP is weakly greater than that under the MP. If $p < 1$, the inequality is strict. On the other hand, if $f'(e) \leq -\frac{cf(e)}{\delta}$ holds for all e, then social welfare under optimal VP is weakly lower than that under the MP.*

Proof. See Appendix A.3. □

The implication of this theorem is fairly understandable. Whether the VP is more desirable than the MP depends on the distribution of e. If $f(e)$ is weakly increasing everywhere or decreasing but relatively flat, the VP is desirable. However, in such a case, the VP is not flexible at all because the first theorem ensures that the optimal VP is $a(e) = \max\{0, e - e_{VP}\}$ and does not distinguish large emission size firms with small emission size firms. If $f(\cdot)$ is sharply decreasing everywhere, the MP does better than the optimal VP. In such a case, the regulator should reject any VP and should try to adopt the MP anytime even though the introduction of the MP might fail. Additionally, we can compare the optimal VP and the MP when $p = 1$. When $p = 1$, VP can implement $a(e) = \max\{0, e - e_{VP}\}$ which satisfies $e_{VP} = e_{MP}$. In such a case, social welfare under this VP is the same as that under the MP. Therefore, even if $f'(e) \leq -\frac{cf(e)}{\delta}$ holds for all e, if $p = 1$, social welfare under the optimal VP must be exactly the same as that under the MP.

The PC condition (4) plays a key role in generating Theorem 1 and 2. Firms' emission level under the VP that satisfies (4) with equality when signaling cost is zero is increasing with their emission size if $p < 1$. This emission level of small emission size firms does not satisfy (4) for large emission size firms. Therefore, if the regulator wants to set an emission standard under the VP for small emission size firms as strictly as possible because the ratio of small emission size firms to the whole industry is very high, the regulator has to differentiate small- and large- emission size firms or the emission standards for large- and small- emission size firms must be different. Then, the regulator has to require firms to pay for verification that their emission size is high and as a result of high verification cost, the standard for large emission size firms is lenient. Otherwise, large emission size firms have an incentive to pretend to be of large emission size. Due to too high cost signal and a lenient emission standard for firms of large emission size, the optimal VP generates lower social welfare than the MP does (Theorem 2). Moreover, because of the PC condition, it is better to set a uniform emission standard for all firms unless the distribution of firms' emission sizes is strongly biased toward small or medium emission size (Theorem 1). This is also because due to the PC condition, the regulator has to accept high signaling cost and a lenient emission standard for large emission size firms if it sets different emission standards for firms of different emission sizes.

The main implications of the theorems are as follows: (1) if there are many small emission size firms, some medium emission size firms and a very few large emission size firms, like the Pareto distribution, and the introduction of a uniform MP can fail ($p < 1$), the optimal VP is flexible but it generates lower social welfare than the uniform MP does; (2) if the distribution of firms' emission size (e) is not strongly biased toward small or medium size, the optimal VP generates higher social welfare than the uniform MP does but the optimal VP is a kind of uniform emission standard and the VP's superiority over the MP is due to the possibility that the introduction of MP fails. Thus, our results imply that flexibility is unlikely to be an advantage of the VP over the uniform MP. However, note that it is possible that a flexible VP is optimal and generates higher social welfare than the MP does if most firms have medium emission size, like a normal distribution with small variance.

In addition to Theorem 2, Theorem 1 has the limitation that the latter half of the theorem is valid only when there exists a certain kind of risk in introducing mandatory policy i.e. $p < 1$. Therefore, we do not know whether there is an occasion where a flexible VP is optimal among every VP when $p = 1$. Such a flexible VP must generate higher social welfare than the mandatory standard when $p = 1$ since the flexible VP generates higher social welfare than an infexible VP that generates the same emissions abatement and social welfare as the mandatory standard does. Actually, when $p = 1$, we can see the following theorem hold.

Theorem 3. *Suppose that there is no risk in implementing mandatory policy (i.e. $p = 1$). Then, the inflexible VP ($a(e) = e - e_{MP}$ for all $e \in [e_{MP}, e_{max}]$) is optimal if*

$$\int_{e_{th}}^{e_{\max}} \{c(1 - F(e)) - \delta f(e)\} de \leq 0 \text{ for all } e_{th} \in [e_{MP} + \frac{\delta}{c}, e_{max}] \tag{7}$$

Otherwise, there exists a flexible VP ($0 < a(e') < e' - e_{MP}$ for some $e' \in [e_{MP} + \frac{\delta}{c}, e_{max}]$) that generates higher social welfare than the inflexible VP and the mandatory standard.

Proof. See Appendix A.4. □

Please note that $a(e) = \delta/c$ is optimal if the regulator knows the emission size of individual firms. Therefore, the mandatory standard is too lenient for firms with emission size $e < e_{MP} + \frac{\delta}{c}$ and the regulator wants these firms to voluntarily commit to abating their emissions more than they do under the mandatory standard. However, due to the PC condition, it is impossible for the VP to be more stringent than the mandatory standard. As a result, emissions abatement of firms with emission size $e < e_{MP} + \frac{\delta}{c}$ under the optimal VP is the same as it is under the mandatory standard. Thus, the regulator's problem becomes how much it should reduce emission abatement for firms with emission size $e > e_{MP} + \frac{\delta}{c}$ from $e - e_{MP}$ to minimize the social cost. This is why the distribution of emission size below $e_{MP} + \frac{\delta}{c}$ does not affect whether the optimal VP is flexible or inflexible.

$\delta f(e')$ is marginal social benefits (or decrease in environmental damage) when only firms with emission size e' would increase their emissions abatement, whereas $c(1 - F(e'))$ is the aggregate marginal costs of firms with emission size e' that include an increase in abatement cost and signaling cost due to an increase in emissions abatement. However, when the regulator slightly decreases the emission abatement of firms with emission size e' from $(e' - e_{MP})$, it must also decrease the emissions abatement of firms whose emission size is greater than e' by (at least) the same amount as that of firms with emission size e' because of the IC condition or $a'(e) \leq 1$ for all $e \in [e_{MP}, e_{\max}]$. Therefore, (7) implies that the marginal benefits of "decrease" in emission abatement of firms with emission size e from $(e - e_{MP})$, $\int_e^{e_{\max}} c(1 - F(e'))de'$, are smaller than their marginal costs, $\int_e^{e_{\max}} \delta f(e')de'$, for all e. If so, the inflexible VP is optimal.

We can easily see that (7) is likely to hold if δ is great enough relative to c. How about the distribution of emission size, $f(e)$? What types of the distribution of firms' emission size satisfy (7)? Of course, (7) holds if $c(1 - F(e)) - \delta f(e) < 0$ for all $e \in [e_{MP} + \frac{\delta}{c}, e_{\max}]$ or $f'(e) > -\frac{cf(e)}{\delta}$ for all $e \in [e_{MP} + \frac{\delta}{c}, e_{\max}]$. (7) may hold even if $c(1 - F(e)) - \delta f(e)$ is positive in some range. Figure 1 shows an example that (7) holds even if $c(1 - F(e)) - \delta f(e)$ is positive in some range (figure 1(a)) and another example that (7) does not hold (figure 2). Note that $c(1 - F(e)) - \delta f(e)$ is non-positive at the limit of $e \to e_{\max}$ because $1 - F(e) \to 0$ but $f(e) \geq 0$ as $e \to e_{\max}$. As we can see, (7) holds if the area below X-axis and on the right side of e_{th} (area A) is greater than the area above X-axis and on the right side of e_{th} (area B) like figure 1(a), for any e_{th}. However, A is smaller than B in figure 2 and therefore, (7) does not hold.

Conversely, when does (7) not hold or is there an inflexible VP that generates higher social welfare than the mandatory standard? (7) is less likely to hold if $1 - F(e)$ is large relative to $f(e)$, in particular, for almost all medium- and large- sized e. Taking into account that $1 - F(e)$ is strictly decreasing (because $f(e) > 0 \; \forall e$), $1 - F(e)$ is large relative to $f(e)$ if $f(e)$ sharply decreases. Therefore, (7) does not hold when $f(e)$ sharply decreases for all e or for almost all medium- and large- emission size. This implies that there exists a flexible VP such that generates higher social welfare than the best inflexible VP and the mandatory standard do if most firms have medium emission size, like a normal distribution with small variance, or if there are many small emission size firms, some medium emission size firms and a very few large emission size firms, like the Pareto distribution.

3.2 Heterogeneity in c

We will see briefly the case where c is heterogeneous between firms but e is homogeneous. Let c_i be distributed in $[c_{\min}, c_{\max}]$ with probability distribution $f(\cdot)$. $f(\cdot)$ satisfies the same property we assumed in heterogeneity in e. MP and VP are defined as the same as in the case where there is heterogeneity in e. In order to understand our setting, let us show an example about the MP when c_i is distributed uniformly.

Example 2. *Let us define $a_{MP} \equiv e - e_{MP}$. If the distribution of c_i is uniform, then $a_{MP} = \frac{\delta}{c^{ave}}$ holds where $c^{ave} = \int_{c_{\min}}^{c_{\max}} cf(c)dc$.*

Proof. the MP must satisfy $a_{MP} = \arg\max_{a_{MP}} \int_{c_{\min}}^{c_{\max}} \{-\frac{c}{2}(a_{MP})^2 + \delta a_{MP}\}dc$. Simple calculation thus results in $a_{MP} = \frac{\delta}{c^{ave}}$. $\square$

Then, as we derived in the case where there is heterogeneity in e, we derive IC and PC conditions in the case where there is heterogeneity in c. On one hand, the PC condition is described as $\frac{ca(c)^2}{2} + s(c) \leq \frac{pca_{MP}^2}{2}$. On the other hand, IC condition is described as $\arg\min_{c'}\{\frac{ca(c')^2}{2} + s(c')\} = c$. The IC condition is simplified to $s'(c) = -ca(c)a'(c)$ and $a'(c) \leq 0$ if $s(\cdot)$ and $a(\cdot)$ are differentiable, or $s(c) = -\frac{ca(c)^2}{2} + \int \frac{a(c)^2}{2}dc + \zeta$ regardless of the differentiability of $s(\cdot)$ and $a(\cdot)$ where ζ is a constant. With these settings, we can see the following theorem and corollary.

Theorem 4. *Let $c_{MP} = \frac{\delta}{\sqrt{p}a_{MP}}$ where a_{MP} is the optimal mandatory abatement. $\forall c; a(c) = \sqrt{p}a_{MP}$ holds and signaling cost $s(c)$ is always zero if $f(\cdot)$ satisfies*

$$c_{MP} + k \geq \frac{\int_k^{c_{max}} cf(c)dc}{1 - F(k)} \quad for\ any\ k \in [c_{MP}, c_{\max}]. \tag{8}$$

Otherwise, there exists a flexible VP that generates higher social welfare than the inflexible VP. Social welfare under the optimal VP is always weakly greater than that under the MP and the former is strictly greater than the latter if and only if either $p < 1$ holds or $a(c) = \sqrt{p}a_{MP}$ does not hold for all c as the optimal VP.

Proof. See Appendix A.5. $\square$

Corollary 1. *The optimal VP is inflexible if $f(\cdot)$ is weakly monotonically increasing.*

Proof. If $f(\cdot)$ is weakly monotonically increasing, $c_{MP} + k \geq 2c_{MP} = \frac{2c^{ave}}{\sqrt{p}} \geq \frac{c_{max}}{\sqrt{p}} \geq c_{\max} \geq \int_k^{c_{\max}} cf(c)dc/[1 - F(k)]$ for any $k \in [c_{MP}, c_{\max}]$. Thus, (8) holds if $f(\cdot)$ is weakly increasing. $\square$

Note that the RHS of (8) is an conditional expected value of c given that c is greater than k ($E[c|c > k]$). k can be any technology inefficiency level such that the best inflexible VP is optimal ($k = c_{MP}$) or too repressive ($k > c_{MP}$). Therefore, Theorem 4 claims that the inflexible VP is optimal if, for any technology inefficiency level (k) as abovementioned, the expected value of technology inefficiency level which is higher than k is lower than ($c_{MP} + k$).

Inequality(8) evaluates the impact of a slight decrease in $a(k)$ on social cost for $k \in [c_{MP}, c_{\max}]$ when $a(c) = \sqrt{p}a_{MP}$ for all c. If the regulator reduces $a(k)$ by ϵ, due to IC condition or $a'(c) \leq 0$ for all c, it must also reduce the emission abatement of firms with a higher technology inefficiency level than k ($a(c)$ for all $c > k$) by ϵ. Because the portion of firms with a higher technology inefficiency level than k is $\int_k^{c_{\max}} f(c)dc = 1 - F(k)$, the product of the RHS of (8), $[1 - F(k)]$ and $\sqrt{p}a_{MP}$ is a marginal benefit from decrease in $a(k)$ from $\sqrt{p}a_{MP}$(decrease in emissions abatement cost, $\sqrt{p}a_{MP} \int_k^{c_{max}} cf(c)dc$), whereas the product of the LHS of (8), $[1 - F(k)]$ and $\sqrt{p}a_{MP}$ is a marginal cost from decrease in $a(k)$ from $\sqrt{p}a_{MP}$ (the sum of increase in environmental damage, $\delta[1 - F(k)]$[4], and signaling cost, $k\sqrt{p}a_{MP}[1 - F(k)]$). Thus, Theorem 4 implies that the regulator should not

[4]Please note that $c_{MP} = \delta/(\sqrt{p}a_{MP})$.

decrease emissions abatement from $\sqrt{p}a_{MP}$ if the marginal benefit from decrease in $a(k)$ is smaller than the marginal cost.

If c_{MP} is large, (8) is likely to hold. When is c_{MP} large? From the proof of Corollary 1, we can see that c_{MP} is large and (8) holds if $f(\cdot)$ is weakly monotonically increasing. Moreover, from this fact, we can geuss that c_{MP} is large or (8) is likely to hold if the distribution of firms is biased toward a high inffeciency level (high c). This is intuitive because the main target of the mandatory standard consists of firms with a high inffeciency level in such a case. The probability that the mandatory policy is adopted, p, also affects c_{MP}. c_{MP} is large or (8) is likely to hold due to the PC condition if p is small or the introduction of the mandatory standard is likely to fail. Finally, (8) holds if the difference in technology inefficiency level is small or in particular, $c_{\min}$ is greater than $c_{\max}/2$. This indicates that the efficiency loss of the inflexible VP is small relative to signaling cost if the difference in technology inefficiency level is small.

If c_{MP} is small, (8) is not likely to hold or there is likely to exist a flexible VP that is optimal among feasible VPs. c_{MP} is small if the distribution of firms is biased toward a low inefficiency level (low c), the mandatory policy is very likely to be adopted, and the difference in technology inefficiency level is large. In Appendix A.8, we give an example of a distribution such that signaling cost is non-zero under the optimal VP. Figure 2 also gives such an example. The distribution of figure 2 is biased toward a low inefficiency level, and therefore, c_{MP} is near to $c_{\min}$ if the mandatory policy is very likely to be adopted. Because some firms have very inefficient technology (very high c), for lower-middle k, the difference in k and $E[c|c > k]$ is large. Therefore, there exists k such that $c_{MP} + k < E[c|c > k]$ or the optimal VP is flexible.

In contrast to the heterogeneity of emission size, the optimal VP always generates at least as high social welfare as the MP does. This is because there always exists an inflexible VP that generates weakly higher social welfare than the MP. Such a VP is $g(c) = \sqrt{p}a_{MP}$, which satisfies the PC condition with equality when signaling cost is zero for all c (the same expected abatement cost as the MP) and that generates weakly greater expected aggregate emission abatement and smaller expected damage than the MP does. If there exists a flexible VP that generates strictly higher social welfare than $a(c) = \sqrt{p}a_{MP}$ does, the flexible VP must generate strictly higher social welfare than the MP too. This is a mechanism that generates the results of Theorem 4 on social welfare.

4 Discussion

In this section, we discuss what makes the results of the heterogeneous emission size case and the heterogeneous abatement technology case different and thus consider the implication of our results. Then, we check whether negative results of the VP under the heterogeneous emission size case are still valid even if environmental damage is not linear.

We show that if the optimal VP is flexible, the optimal VP always generates weakly higher social welfare than the MP does under the heterogeneous abatement technology case, whereas the optimal and flexible VP does not always generate weakly higher social welfare than the MP does under the heterogeneous emission size case. The difference in the results under these two cases has roots in the difference in the best inflexible VP's performance relative to the MP. The best inflexible VP under the heterogeneous abatement technology case always generates weakly higher social welfare than the MP does. However, whether the best inflexible VP generates higher social welfare than the MP does under the heterogeneous emission size case depends on the distribution of emission size. If the distribution weakly increases or only slightly decreases for all emission sizes, the best inflexible VP is better than the MP from the social welfare point of view, whereas the best inflexible VP is worse than the MP if the distribution sharply decreases for all emission sizes.

The best inflexible VP is determined by the PC condition. On the one hand, under the heterogeneous abatement technology case, the best inflexible VP is the abatement level that satisfies the PC condition with equality for all abatement technology level. This inflexible VP generates weakly higher social welfare than the MP does. Therefore, the best flexible VP generates higher social welfare than the MP does if there exists a flexible VP that generates higher social welfare than the optimal inflexible VP does. On the other hand, under the heterogeneous emission size case, the best inflexible VP is the abatement level that satisfies the PC condition with equality for firms of the highest emission size. Under this abatement level, the PC condition for firms other than the highest emission size firms is not binding if the introduction of the MP fails at some positive probability. Therefore, the best inflexible VP is too lenient for low emission size firms and is worse than the MP if the ratio of low emission size firms is very high (or if the distribution sharply decreases for all emission sizes).

As a result of the too lenient best inflexible VP, the regulator chooses a flexible VP to control emissions from low emission size firms. While the VP is imposed on low emission size firms, which do not have an incentive to pretend to be a high emission size firm, the regulator has to require firms of high emission size to pay for verification that their emission size is high. Due to high verification cost, the standard for high emission size firms is lenient so that the PC condition for high emission size firms is satisfied. Because signaling cost is high and emissions abatement by high emission size firms is small, the optimal flexible VP generates lower social welfare than the MP does, although the optimal flexible VP is better than the best inflexible VP.

Quadratic damage

Previously, environmental damage was considered to be linear: $\delta \int (e_i - a_i) f(e) de$; that is, environmental damage due to one firm's emission has no relation to that due to other firms' emission. This assumption is fairly plausible, but we can consider another assumption, i.e. environmental damage due to one firm's emission is positively related to that due to other firms' emission. Formally, such environmental damage can be written as $\frac{\delta}{2}\{\int (e_i - a_i) f(e) de\}^2$. In order to understand this setting, we provide an example about it.

Example 3. If e is uniformly distributed, the MP satisfies $e_{MP} = e_{max} - \frac{-c+\sqrt{c^2+4\delta^2(e_{max}^2-e_{min}^2)}}{2\delta}$ if $2\delta e_{min} < c$.

Proof. See the Appendix A.9. □

Previously, we show that signaling cost is always zero under optimal VP if probability distribution $f(\cdot)$ is weakly monotonically increasing, or decreasing but fairly flat. A similar phenomenon can be seen where environmental damage due to one firm's emission is positively related to that due to other firms' emission. For simplicity, we just show an example where probability distribution $f(\cdot)$ is weakly increasing (including uniform distribution).

Proposition 1. *Suppose the probability distribution of e is weakly increasing. Then, signaling cost is always zero under optimal VP. In this case, social welfare under optimal VP is weakly greater than that under MP and strictly greater than that under the MP when $p < 1$.*

Proof. See the Appendix A.7. □

Since environmental damage does not change the behavior of firms and only affects the government's behavior, the proof of Example 4 is almost the same as what we see when environmental damage is linear.

Is optimal VP always inflexible? Actually not. Let us restrict our discussion to where there is no risk in implementing mandatory policy, i.e. $p = 1$, where $f(e)$ is a non-zero function, and where $f(e)$ is not decreasing exponentially at any region. Then, we can see that optimal VP is inflexible VP if and only if $c\int_{e_{th}}^{e_{\max}}(1-F(e))de \leq \delta\{\int_{e_{min}}^{e_{MP}} ef(e)de + e_{MP}\int_{e_{MP}}^{e_{max}} f(e)de\}\int_{e_{th}}^{e_{\max}} f(e)de \ for \ all \ e_{th} \in [e_{MP}, e_{max}]$. Therefore, if c is large enough or δ is small enough, it is possible that optimal VP turns out to be flexible.

Theorem 5. *Suppose that there is no risk in implementing mandatory policy (i.e. $p = 1$). Then, the optimal VP is inflexible ($a(e) = e - e_{MP}$ for all $e \in [e_{MP}, e_{max}]$) if*

$$c\int_{e_{th}}^{e_{\max}}(1-F(e))de \leq \delta\{\int_{e_{min}}^{e_{MP}} ef(e)de + e_{MP}\int_{e_{MP}}^{e_{max}} f(e)de\}\int_{e_{th}}^{e_{\max}} f(e)de \ for \ all \ e_{th} \in [e_{MP}, e_{max}] \quad (9)$$

Otherwise, there exists a flexible VP ($0 < a(e') < e' - e_{MP}$ for some $e' \in [e_{MP}, e_{max}]$) that generates higher social welfare than the inflexible VP and the mandatory standard.

Proof. See the Appendix A.6. □

Like (7), (9) implies that marginal benefits of "decrease" in the emission abatement of firms with emission size e from $(e - e_{MP})$ (the RHS of (9)) are smaller than their marginal costs (the LHS of (9)) for all e. Although the marginal benefits are the same as those under a linear damage function case, the marginal costs are different from those under the linear damage function case. Increase in marginal damage when a firm reduces its emission abatement is given by $\delta\{\int_{e_{min}}^{e_{MP}} ef(e)de + e_{MP}\int_{e_{MP}}^{e_{max}} f(e)de\}$ and if the regulator slightly reduces $a(e)$, the regulator must reduce emissions abatement of firms whose emission size is larger than e, $\int_{e}^{e_{\max}} f(e')de'$, due to the IC condition. Thus, the LHS of (9) is marginal cost due to decreased emission abatement of firms with emission size e from $(e - e_{MP})$. Comparing (7) and (9), in the quadratic damage function case, the optimal

VP tends to be inflexible unless the slope of the marginal abatement cost curve is very steep relative to the linear damage function case.

5 Conclusion

This paper examines voluntary policies' potential for the emissions abatement of a large number of firms under an asymmetric information case and flexibility of the optimal voluntary policy by developing a game theoretic model of emissions abatement policy. In our model, all firms are subject to a uniform emission standard under the mandatory regulation, whereas firms' emission level under voluntary policy can depend on their abatement cost if they submit credible evidence on their abatement cost to a regulator. We derive and characterize the most socially desirable voluntary policy in the feasible set of voluntary policies that is characterized by firms' response regarding their commitment to voluntarily control on their emissions level with the requirement of credible evidence submission. We also compare social welfare under the optimal voluntary policy and the mandatory standard.

The characteristics and performance of the optimal voluntary policy depend on the type of heterogeneity. When firms' emission size is heterogeneous, an inflexible voluntary policy (a kind of uniform emission standard) is optimal among feasible voluntary policies unless the distribution of emission size is strongly biased toward small or medium size. The optimal inflexible voluntary policy generates strictly higher social welfare than a uniform mandatory standard only if the introduction of the mandatory standard might fail. Moreover, the optimal voluntary policy is flexible (different emission standard for firms of different emission size) but generates lower social welfare than the mandatory standard does if the distribution of firms' emission size sharply decreases for all emission sizes.

On the other hand, when firms' emission abatement technology is heterogeneous, a flexible voluntary policy always generates strictly higher social welfare than the mandatory standard does if such a voluntary policy is optimal. In addition to the performance of the flexible voluntary policy, relative to a case of heterogeneous emission size, it is likely that optimal voluntary policy is flexible. The difference in the performance of voluntary policy under heterogeneous emission size case and heterogeneous abatement technology case has roots in the difference of the best inflexible voluntary policy's performance relative to the mandatory standard. While the best inflexible voluntary policy always generates weakly higher social welfare than the mandatory standard does under the heterogeneous abatement technology case, it does not always hold under the heterogeneous emission size case.

In our model, we implicitly assumed that the abilities to carry out verification of abatement costs are homogeneous among firms although emissions abatement cost are heterogeneous among firms. However, one way to extend our model is to introduce the heterogeneity of abatement technology among firms. Given that most voluntary approaches cannot enforce a firm's commitment, it would be interesting to consider the case in which voluntary policies are not enforceable. Exploring these types of extensions remains an endeavor for future research.

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A Appendix

A.1 Proof of "IC condition $\Leftrightarrow$ (5) and $a'(e_i) \leq 1$ for all i"

Proof of $\Rightarrow$

It is obvious that (5) holds for all $e \in [e_{MP}, e_{\max}]$ if the IC condition holds. Therefore, what we have to show is that $a'(e_i) \leq 1$ for all $e \in [e_{MP}, e_{\max}]$ if the IC condition holds. From the IC condition, $\forall e,\ e' \in [e_{MP}, e_{\max}]$,

$$\frac{1}{2}c(a(e))^2 + s(e) \leq \frac{1}{2}c(e-(e'-a(e')))^2 + s(e') \tag{10}$$
$$\frac{1}{2}c(a(e'))^2 + s(e') \leq \frac{1}{2}c(e'-(e-a(e)))^2 + s(e). \tag{11}$$

Without loss of generality, we can assume $e > e'$. Therefore, combining 10 and 11,

$$\begin{aligned}\frac{1}{2}c(e-(e'-a(e')))^2 - \frac{1}{2}c(a(e'))^2 \geq \frac{1}{2}c(a(e))^2 - \frac{1}{2}c(e'-(e-a(e)))^2 \qquad (12)\\ (e-e')(e-e'+2a(e')) \geq (e-e')(e'-e+2a(e))\\ e-e' \geq a(e)-a(e')\\ \int_{e'}^{e} 1dt \geq \int_{e'}^{e} a'(t)dt\end{aligned}$$

Therefore, $a'(e) \leq 1$ for all $e \in [e_{MP}, e_{\max}]$.

Proof of $\Leftarrow$

Suppose that the IC condition does not hold. Then, there exist e' and e'' (we can assume $e' > e''$without loss of generality) such that

$$\frac{1}{2}c(e'-(e''-a(e'')))^2 + s(e'') < \frac{1}{2}c(a(e'))^2 + s(e').$$

Therefore,

$$\frac{1}{2}c(e'-(e''-a(e'')))^2+s(e'')-[\frac{1}{2}c(a(e''))^2+s(e'')] < \frac{1}{2}c(a(e'))^2+s(e')-[\frac{1}{2}c(a(e''))^2+s(e'')].$$
$$\frac{1}{2}(e'-e''+a(e''))^2-\frac{1}{2}(e''-e''+a(e''))^2 < A(e')-A(e'')$$
$$\int_{e''}^{e'}[e-e''+a(e'')]de < \int_{e''}^{e'}a(e)de.$$
$$\int_{e''}^{e'}[e-e'']de < \int_{e''}^{e'}[a(e)-a(e'')]de.$$

However, $\int_{e''}^{e'}[e-e'']de \geq \int_{e''}^{e'}[a(e)-a(e'')]de$ must hold because $a'(e) \leq 1$ implies that $e-e'' \geq a(e)-a(e'')$ for all $e \in [e'', e']$.

A.2 Proof of Theorem 1

We first define "manipulation" here. Consider $\tilde{e}$, which satisfies $e_{\min} < \tilde{e} < e_{\max}$. Suppose that VP is not a uniform emission standard. Then, there must be an open region where $a'(\cdot) < 1$. Let $\tilde{e}$ be taken arbitrary from the open region. Then, because of the IC condition and the fact that $s(\cdot)$ is weakly monotonically increasing, $s(\tilde{e}) > 0$ must hold. Let ϵ and ℓ be sufficiently small positive numbers and η be a positive value much smaller than $\ell\epsilon$. If $a(\cdot)$ is not continuous at $\tilde{e}$, slightly change $\tilde{e}$ so that $a(\cdot)$ is continuous at $\tilde{e}$ and $s(\tilde{e}) > 0$ still holds. Equation (5) ensures that $s(\cdot)$ is also continuous at $\tilde{e}$. Then, we call the following transformation from $a(\cdot)$ to $\hat{a}(\cdot)$ "manipulation".

$$\hat{a}(e) = \begin{cases} a(e) - \frac{(e-\tilde{e}+\epsilon)\eta}{\ell\epsilon} & \text{where} \quad \tilde{e}-\epsilon \leq e < \tilde{e}-\epsilon+\ell\epsilon, \\ a(e) - \eta & \text{where} \quad \tilde{e}-\epsilon+\ell\epsilon \leq e < \tilde{e}-\ell\epsilon, \\ a(e) + \frac{(e-\tilde{e})\eta}{\ell\epsilon} & \text{where} \quad \tilde{e}-\ell\epsilon \leq e < \tilde{e}+\ell\epsilon, \\ a(e) + \eta & \text{where} \quad \tilde{e}+\ell\epsilon \leq e < \tilde{e}+\epsilon-\ell\epsilon, \\ a(e) + \frac{(\tilde{e}+\epsilon-e)\eta}{\ell\epsilon} & \text{where} \quad \tilde{e}+\epsilon-\ell\epsilon \leq e < \tilde{e}+\epsilon, \\ a(e) & \text{otherwise.} \end{cases}$$

This manipulation does not violate PC because this manipulation just weakly decreases $A(\cdot)$ and thus weakly eases the PC (6). Since $s(\tilde{e}) > 0$, this manipulation does not violate the non-negativity of signaling cost. Since $a'(\tilde{e}) < 1$, this manipulation does not violate the condition $a' \leq 1$ as far as $\frac{\eta}{\ell\epsilon}$ is sufficiently small. Let $L = f(\tilde{e})$ and $k = f'(\tilde{e})$. Let us consider the manipulation effect for social welfare. Let Δ be an operator to describe the difference between post-manipulation and pre-manipulation. Let SW be social welfare. Then,

$$\begin{aligned}\Delta SW &= \int \{-c\Delta A(e) + \delta \Delta a(e)\} f(e) de \\ &= cL\epsilon^2\eta + \delta k \epsilon^2 \eta + O(\epsilon^3\eta) \\ &= (\delta k + cL)\epsilon^2\eta + O(\epsilon^3\eta)\end{aligned}$$

We can see that manipulation increases SW if $k > -\frac{cL}{\delta}$. This condition is equivalent to $f'(e) > -\frac{cf(e)}{\delta}$. Therefore, if $\forall e; \quad f'(e) > -\frac{cf(e)}{\delta}$ holds, manipulation increases SW. Berge's Theorem ensures the existence of optimal VP which ensures $a(\cdot)$ to be semi-continuous[5]. Thus, if $a(\cdot)$ and $s(\cdot)$ ensure optimal VP, signaling cost must be zero for all e. Therefore, if $\forall e; \quad f'(e) > -\frac{cf(e)}{\delta}$ holds, $\forall e; s(e) = 0$ holds. Combining this finding and (5), $A(e) = \frac{(a(e))^2}{2}$ holds. Solving this equation, either $a(e) = 0$ or $a(e) = e - e_0$ holds and $a(e) = \max\{0, e - e_{VP}\}$ is derived.

What happens if $\exists e$ such that $f'(e) < -\frac{cf(e)}{\delta}$? Because of the continuity of f, we can assume without loss of generality that such e is less than $e_{\max}$ and more than $e_{\min}$. Suppose signaling cost is always zero. In such a case, since $a(e) = \max\{0, e - e_{VP}\}$ holds, the participation condition is not binding in $e < e_{\max}$ if $p < 1$. Then, inverse-manipulation increases social welfare where inverse-manipulation is the transformation from $a(\cdot)$ to $\breve{a}(\cdot)$ defined as follows:

$$\breve{a}(e) = \begin{cases} a(e) + \frac{(e-\tilde{e}+\epsilon)\eta}{\ell\epsilon} & \text{where} \quad \tilde{e} - \epsilon \le e < \tilde{e} - \epsilon + \ell\epsilon, \\ a(e) + \eta & \text{where} \quad \tilde{e} - \epsilon + \ell\epsilon \le e < \tilde{e} - \ell\epsilon, \\ a(e) - \frac{(e-\tilde{e})\eta}{\ell\epsilon} & \text{where} \quad \tilde{e} - \ell\epsilon \le e < \tilde{e} + \ell\epsilon, \\ a(e) - \eta & \text{where} \quad \tilde{e} + \ell\epsilon \le e < \tilde{e} + \epsilon - \ell\epsilon, \\ a(e) - \frac{(\tilde{e}+\epsilon-e)\eta}{\ell\epsilon} & \text{where} \quad \tilde{e} + \epsilon - \ell\epsilon \le e < \tilde{e} + \epsilon, \\ a(e) & \text{otherwise.} \end{cases}$$

This inverse-manipulation tightens the PC, but it does not violate the PC because the PC is not binding in this case. Let Δ' be an operator to describe the difference between post-inverse-

[5] Let $\Gamma(e)$ be the correspondence of optimal abatement which satisfies both IC and PC. Berge's Theorem ensures that such $\Gamma(\cdot)$ is upper-semicontinuous (Berge, 1959). Then, we can take semi-continuous function $a(\cdot)$, which satisfies $a(e) \in \Gamma(e)$ for all e (see proof of this statement as follows). Therefore, Berge's Theorem ensures the existence of optimal VP, which ensures $a(\cdot)$ to be semi-continuous.

Proof. Let $a^*(e)$ be defined as $a^*(e) = \max\{x \in \Gamma(e)\}$. The existence of the maximum value is ensured by the compactness of $\Gamma(e)$. Then, $a^*(e)$ is semi-continuous in $[e_{\min}, e_{\max}]$ except for at most finite points (otherwise, there exists a condensed point of points which are neither right-continuous nor left continuous, which results in the violation of upper-semicontinuity). Let e_i^* $(1 \le i \le N)$ be the points at which $a^*(\cdot)$ is neither right continuous nor left continuous. If such point does not exist, set $N = 0$. Then, let $a^{**}(e)$ be defined as $a^{**}(x) = \lim_{e \to x-0} a^*(e)$ where there exists i such that $x = e_i^*$ and $a^{**}(x) = a^*(x)$ otherwise. Because of the upper-semicontinuity, $a^{**}(e) \in \Gamma(e)$ holds for all e. Also, $a^{**}(\cdot)$ is semi-continuous for all e. Therefore, we can take semi-continuous function $a(\cdot)$, which satisties $a(e) \in \Gamma(e)$ for all e. $\square$

manipulation and pre-inverse-manipulation. Then,

$$\Delta' SW = \int \{-c\Delta' A(e) + \delta\Delta' a(e)\} f(e) de = -(\delta k + cL)\epsilon^2\eta + O(\epsilon^3\eta) > 0.$$

Berge's Theorem ensures the existence of optimal VP, which ensures $a(\cdot)$ to be semi-continuous. Thus, if $a(\cdot)$ and $s(\cdot)$ ensure optimal VP and if $\exists e$ such that $f'(e) < -\frac{cf(e)}{\delta}$, $\exists e'$ such that $s(e') > 0$.

A.3 Proof of Theorem 2

Social welfare under the MP, written as SWMP, is written as follows:

$$SWMP = p \int_{e_{MP}}^{e_{\max}} \{-\frac{c(e - e_{MP})^2}{2} - \delta(e - e_{MP})\} f(e) de - \int_{e_{\min}}^{e_{\max}} \delta e f(e) de.$$

Then, let us define $\xi(e)$ as follows:

$$\xi(e) = \begin{cases} p\frac{(e-e_{MP})^2}{2} - A(e) & \text{when } e \geq e_{MP} \\ 0 & \text{otherwise.} \end{cases}$$

Because of PC, $\xi(e) \geq 0$ always holds.

Social welfare under a VP, written as SWVP, is calculated as follows:

$$\begin{aligned} SWVP &= \int_{e_{MP}}^{e_{\max}} \{-\frac{ca(e)^2}{2} - s(e) - \delta(e - a(e))\} f(e) de - \int_{e_{\min}}^{e_{MP}} \delta e f(e) de \\ &= \int_{e_{MP}}^{e_{\max}} \{-cA(e) + \delta a(e)\} f(e) de - \int_{e_{\min}}^{e_{\max}} \delta e f(e) de \\ &= \int_{e_{MP}}^{e_{\max}} \{-p\frac{c(e - e_{MP})^2}{2} + c\xi(e) - p\delta(e - e_{MP}) - \delta\xi'(e)\} f(e) de - \int_{e_{\min}}^{e_{\max}} \delta e f(e) de \\ &= SWMP + \int \{c\xi(e) - \delta\xi'(e)\} f(e) de \\ &= SWMP + \int c\xi(e) f(e) de - [\delta\xi(e) f(e)]_{e_{MP}}^{e_{\max}} + \int \delta\xi(e) f'(e) de \\ &= SWMP + \int \{cf(e) + \delta f'(e)\}\xi(e) de - \delta\xi(e_{\max}) f(e_{\max}) \end{aligned}$$

where the second equality holds due to IC condition (3). If $\forall e; f'(e) > -\frac{cf(e)}{\delta}$ holds, then consider a VP in which PC is binding at $e = e_{\max}$ (i.e. $a(e) = \max\{0, e - e_{VP}\}$ where $e_{VP} = e_{\max} - \sqrt{p}(e_{\max} - e_{MP})$). Inequality $cf(e) + \delta f'(e) > 0$ holds by assumption and $\xi(e_{\max}) = 0$ holds because PC is binding at $e = e_{\max}$. Therefore, this VP satisfies the inequality $SWVP \geq SWMP$. Therefore, social welfare under optimal VP must be weakly greater than that under the MP. If $p < 1$, $\exists e$ such that $\xi(e) > 0$ under the VP we selected. This VP thus satisfies $SWVP > SWMP$. Therefore, social welfare under optimal VP must be strictly greater than that under the MP if $p < 1$.

If $\forall e; f'(e) \leq -\frac{cf(e)}{\delta}$ holds, $cf(e) + \delta f'(e) \leq 0$ holds by assumption. Since $\delta\xi(e_{max})f(e_{max}) \geq 0$

holds, social welfare under any VP is weakly lower than that under the MP. Therefore, social welfare under optimal VP is weakly lower than that under the MP.

A.4 Proof of Theorem 3

At first, we can see $a(e) = e - e_{MP}$ must hold for $e \le e_{MP} + \frac{\delta}{c}$ because of the PC and optimality condition. Therefore, we can focus our attention on how the regulator should set $a(e)$ for $e \in [e_{MP} + \frac{\delta}{c}, e_{max}]$. The social cost function can be written as $\int_{e_{MP}+\frac{\delta}{c}}^{e_{max}} \{c(1-F(e)) - \delta f(e)\} a(e) de$ disregarding any term irrelevant to the functional form of $a(e)$. Let us define the following infinitedesimal function for sufficiently small positive value ϵ:

$$\ell_{e_{th}}(e) = \begin{cases} 0 & \text{when } e \le e_{th} \\ e_{th} - e & \text{when } e_{th} < e \le e_{th} + \epsilon \\ -\epsilon & \text{when } e_{th} + \epsilon < e. \end{cases}$$

For any abatement function $\bar{a}(\cdot)$ which is infinitesimally different from $a(e) = e - e_{MP}$, the set of the above infinitesimal functions constitutes the basis for $\bar{a} - a$, i.e. $\bar{a} - a$ can be expressed as the superposition of the above infinitesimal functions. Therefore, the necessary and sufficient condition that $a(e) = e - e_{MP}$ is the best VP is that the social cost under abatement function $a + \ell_{e_{th}}$ is always (weakly) greater than that under abatement function a for all $e_{th} \in [e_{MP} + \frac{\delta}{c}, e_{\max}]$. Taking the difference between them, this condition can be calculated as $\int_{e_{th}}^{e_{\max}} \{c(1 - F(e)) - \delta f(e)\} de \le 0$ for all $e_{th} \in [e_{MP} + \frac{\delta}{c}, e_{\max}]$. If this condition does not hold, then there exists a flexible VP such that $0 < a(e') < e' - e_{MP})$ for some $e' \in [e_{MP} + \frac{\delta}{c}, e_{\max}]$ and generates smaller social costs than the inflexible VP and the mandatory standard.

A.5 Proof of Theorem 4

At first, we can see that $a(c) = \sqrt{p} a_{MP}$ must hold for $c \le \frac{\delta}{\sqrt{p} a_{MP}} = c_{MP}$ because of the PC and optimality condition. Let us assume $c_{\max} > \frac{\delta}{\sqrt{p} a_{MP}}$ hereinafter (otherwise, the theorem is already solved.). Therefore, the PC is always binding for $c \le \frac{\delta}{\sqrt{p} a_{MP}}$.

Since signaling cost is zero in $c \le \frac{\delta}{\sqrt{p} a_{MP}}$, signaling cost is calculated as $s(c) = -\frac{ca(c)^2}{2} + \int_{c_{MP}}^{c} \frac{a(c')^2}{2} dc' + \frac{\sqrt{p}\delta a_{MP}}{2}$ where $c \ge c_{MP}$. Firm's cost is $\frac{1}{2} \int_{c_{MP}}^{c} a(c')^2 dc' + \frac{\sqrt{p}\delta a_{MP}}{2}$ if $c \ge c_{MP}$ and $\frac{pca_{MP}^2}{2}$ otherwise. Taking also into account the environmental damage, total social cost is $\frac{1}{2} \int_{c_{MP}}^{c_{max}} \{\int_{c_{MP}}^{c} a(c')^2 dc'\} f(c) dc + \frac{\sqrt{p}\delta a_{MP}}{2} \int_{c_{MP}}^{c_{max}} f(c) dc + \int_{c_{min}}^{c_{MP}} \frac{pca_{MP}^2}{2} f(c) dc - \delta \int_{c_{min}}^{c_{MP}} \sqrt{p} a_{MP} f(c) dc - \delta \int_{c_{MP}}^{c_{max}} a(c) f(c) dc$.

Therefore, by simple calculation, total social cost is calculated as $\frac{pa_{MP}^2}{2} \int_{c_{min}}^{c_{MP}} cf(c) dc + \frac{\sqrt{p}\delta a_{MP}}{2} (1 - 3F(c_{MP})) + \frac{1}{2} \int_{c_{MP}}^{c_{max}} \{\int_{c_{MP}}^{c} a(c')^2 dc'\} f(c) dc - \delta \int_{c_{MP}}^{c_{max}} a(c) f(c) dc$. Therefore, our interest is whether $a(c) = \sqrt{p} a_{MP} \forall c$ minimizes $\frac{1}{2} \int_{c_{MP}}^{c_{max}} \{\int_{c_{MP}}^{c} a(c')^2 dc'\} f(c) dc - \delta \int_{c_{MP}}^{c_{max}} a(c) f(c) dc$.

Let us define the following infinitedesimal function for sufficiently small positive value ϵ where

$k \in [c_{MP}, c_{max}]$:

$$\ell_k(c) = \begin{cases} 0 & \text{when } c \leq k \\ k - c & \text{when } k < c \leq k + \epsilon \\ -\epsilon & \text{when } k + \epsilon > c. \end{cases}$$

For any abatement function $\bar{a}(\cdot)$ which is infinitesimally different from $a(c) = \sqrt{p} a_{MP} \forall c$, the set of the above infinitesimal functions constitutes the basis for $\bar{a} - a$, i.e. $\bar{a} - a$ can be expressed as the superposition of the above infinitesimal functions. Therefore, the necessary and sufficient condition that $a(c) = \sqrt{p} a_{MP} \forall c$ is the best VP is that the $\frac{1}{2} \int_{c_{MP}}^{c_{max}} \{\int_{c_{MP}}^{c} a(c')^2 dc'\} f(c) dc - \delta \int_{c_{MP}}^{c_{max}} a(c) f(c) dc$ under abatement function $a + \ell_k$ is always (weakly) greater than that under abatement function a for all $k \in [c_{MP}, c_{max}]$. Taking the difference between them, this condition can be calculated as

$$\begin{aligned} \frac{1}{2} \int_{c_{MP}}^{c_{max}} \{-2 \min\{0, c - k\} \epsilon \sqrt{p} a_{MP}\} f(c) dc + \delta \epsilon \int_{k}^{c_{max}} f(c) dc \geq 0 \\ \delta \epsilon \int_{k}^{c_{max}} f(c) dc \geq \int_{k}^{c_{max}} (c - k) \epsilon \sqrt{p} a_{MP} f(c) dc \\ c_{MP} \int_{k}^{c_{max}} f(c) dc \geq \int_{k}^{c_{max}} (c - k) f(c) dc \end{aligned}$$

Therefore, the necessary and sufficient condition that $a(c) = \sqrt{p} a_{MP} \forall c$ is optimal VP is $c_{MP} \int_{k}^{c_{max}} f(c) dc \geq \int_{k}^{c_{max}} (c - k) f(c) dc$ for all $k \in [c_{MP}, c_{max}]$.

Social welfare achieved under VP of $\forall c; a(c) = \sqrt{p} a_{MP}$ is always weakly greater than that under the MP because expected emission abatement cost under both policies are the same where expected environmental damage under the MP is weakly greater than that under optimal VP because $\sqrt{p} \geq p$ holds. Since the inequality of $\sqrt{p} \geq p$ is the strict inequality if and only if $p < 1$, social welfare achieved under VP of $\forall c; a(c) = \sqrt{p} a_{MP}$ is always strictly greater than that under the MP if and only if $p < 1$ holds. If $\forall c; a(c) = \sqrt{p} a_{MP}$ does not hold under optimal VP, social welfare under optimal VP is strictly greater than that under the MP regardless of the value of p.

A.6 Proof of Theorem 5

Assume that $a(e) = e - e_{MP}$ for all $e \in [e_{MP}, e_{\max}]$ is the best VP. Let us define the following infinitedesimal function for sufficiently small positive value ϵ:

$$\ell_{e_{th}}(e) = \begin{cases} 0 & \text{when } e \leq e_{th} \\ e_{th} - e & \text{when } e_{th} < e \leq e_{th} + \epsilon \\ -\epsilon & \text{when } e_{th} + \epsilon < e. \end{cases}$$

For any abatement function $\bar{a}(\cdot)$ which is infinitesimally different from $a(e) = e - e_{MP}$, the set of the above infinitesimal functions constitutes the basis for $\bar{a} - a$, i.e. $\bar{a} - a$ can be expressed as the superposition of the above infinitesimal functions. Therefore, the necessary and sufficient condition that $a(e) = e - e_{MP}$ is the best VP is that the social cost under abatement function

$a + \ell_{e_{th}}$ is always (weakly) greater than that under abatement function a for all $e_{th} \in [e_{MP}, e_{max}]$. (Note that, via simple calculation, the social cost function is written as $\int_{e_{MP}}^{e_{max}} c(1 - F(e))a(e)de + \frac{\delta}{2}\{\int_{e_{min}}^{e_{max}} ef(e)de - \int_{e_{MP}}^{e_{max}} a(e)f(e)de\}^2$.) Taking the difference between them, this condition can be calculated as $\frac{\int_{e_{th}}^{e_{\max}}(1-F(e))de}{\int_{e_{th}}^{e_{\max}} f(e)de} \leq \frac{\delta}{c}\{\int_{e_{min}}^{e_{MP}} ef(e)de + e_{MP}\int_{e_{MP}}^{e_{max}} f(e)de\}$. If this condition does not hold, then there exists a flexible VP such that $0 < a(e') < e' - e_{MP})$ for some $e' \in [e_{MP}, e_{\max}]$ and generates smaller social costs than the inflexible VP and the mandatory standard.

A.7 Proof of Proposition 1

The IC condition is the same as when environmental damage is linear. Suppose $\exists\tilde{e}$ such that $s(\tilde{e}) > 0$. Then, apply the same manipulation as we have defined where environmental damage is linear. This manipulation just weakly eases PC. Since $s(\tilde{e}) > 0$ and (as we discussed previously) $s(\cdot)$ is continuous around $\tilde{e}$, this manipulation does not violate the non-negativity of $s(\cdot)$. Since this manipulation weakly decreases the environmental damage due to the fact that $f(\cdot)$ is weakly increasing, this manipulation just strictly increases social welfare. The existence of optimal VP is ensured by Berge's Theorem and manipulation must not be conducted under optimal VP. Therefore, under optimal VP, $\forall e; s(e) = 0$.

In the case above, social cost under optimal MP is

$$p\left[\frac{1}{2}c\int(\max\{0, e - e_{MP}\})^2 f(e)de + \frac{1}{2}\delta\left\{\int \min\{e, e_{MP}\}f(e)de\right\}^2\right] + (1-p)\frac{1}{2}\delta[\int ef(e)de]^2$$

where social cost under VP is

$$\frac{1}{2}c\int(\max\{0, e - e_{VP}\})^2 f(e)de + \frac{1}{2}\delta[\int \min\{e, e_{VP}\}f(e)de]^2.$$

Let us consider a VP that satisfies $e_{max} - e_{VP} = \sqrt{p}(e_{max} - e_{MP})$ and we will see that the social cost under such a VP is strictly less than that under MP when $p < 1$, which implies that social welfare under the optimal VP is strictly greater than that under MP when $p < 1$.

First, let us define κ to satisfy $\kappa = e_{max} - e_{MP}$. κ must be positive. Also, let us define

$$l(p) = \frac{1}{2}c\int(\max\{0, e - e_{max} + \sqrt{p}\kappa\})^2 f(e)de + \frac{1}{2}\delta[\int \min\{e, e_{max} - \sqrt{p}\kappa\}f(e)de]^2.$$

Social cost under this VP is $l(p)$ where social cost under MP is $pl(1)+(1-p)l(0)$. Then, the sufficient condition that social cost under this VP is strictly less than that under MP when $p \in (0, 1)$ is that $l(\cdot)$ is a convex function. By simple calculation, due to the fact that $f(\cdot)$ is weakly increasing, $l''(\cdot) > 0$. Therefore, social welfare under the optimal VP is strictly greater than that under MP when $p < 1$. When $p = 1$, social welfare under the optimal VP is at least weakly greater than that under MP.

A.8 Example where signaling cost is non-zero under VP

We will see an example with a discrete case, but we can modify this example in a continuous case fairly easily. Let there be 2 firms with $c = c_0$ and a firm with $c = 4c_0$. In this case, $c^{ave} = 2c_0$. Let $p = 1$ hold. If there is no signaling cost under optimal VP, $a = \frac{\delta}{2c_0}$. By simple calculation, the social welfare under this program is $\frac{3\delta^2}{4c_0}$. Besides, we can consider a VP where $a_0 = \frac{\delta}{2c_0}$ (PC binding) for firms with $c = c_0$, $a_1 = \frac{\delta}{4c_0}$ (PC not binding) for firms with $c = 4c_0$ and signaling cost for any firm to signal that $c = 4c_0$ to be $\frac{c_0\{a_0^2 - a_1^2\}}{2} = \frac{3\delta^2}{32c_0}$. The social welfare under this program is $\frac{25\delta^2}{32c_0}$. Therefore, social welfare under the latter program is greater than the social welfare under the former program. Therefore, signaling cost under optimal VP is not always zero in this case.

A continuous example can be made as follows. Let's consider the following flexible VP;

$$a(c) = \begin{cases} \sqrt{p}\delta/c_{ave} & if\ c < c_{ave}/\sqrt{p} \\ \delta/c & if\ c \geq c_{ave}/\sqrt{p}. \end{cases}$$

From F.O.C. for maximization of social welfare under the mandatory policy, $a_{MP} = \delta/c_{ave}$. When $a_{MP} = \delta/c_{ave}$,

$$SWMP = \int (-ca_{MP}^2/2 + \delta a_{MP}) f(c)dc = \delta^2/(2c_{ave})$$

$$Expected\ SWMP = p\delta^2/(2c_{ave}) = \frac{1}{2}p\frac{\delta^2}{c_{ave}}$$

Therefore, the PC condition is $a(c) \leq \sqrt{p}\delta/c_{ave}$. Social welfare under the inflexible VP is $SWinflexVP = \int(-ca^2/2 + \delta a)f(c)dc = -p\delta^2/(2c_{ave}) + \sqrt{p}\delta^2/c_{ave} = \delta^2/c_{ave}(-p/2 + \sqrt{p})$. Under the flexible VP, signaling cost is $\arg\max_{c'} ca(c')^2/2 + s(c') = c$. From F.O.C, $ca(c)a'(c) + s'(c) = 0$. Because $a(c) = \delta/c$ and $a'(c) = -\delta/c^2$ for $c \geq c_{ave}/\sqrt{p}$, $ca(c)a'(c) = -\delta^2/c^2$ and we obtain $s'(c) = \delta^2/c^2$ from the F.O.C.. Therefore, $s(c) = -\delta^2/c + \delta^2/(c_{ave}/\sqrt{p}) = \delta^2(-1/c + \sqrt{p}/c_{ave})$.

Now, we can calculate the difference between social welfare under flexible VP and under the best inflexible VP, $a = \sqrt{p}\delta/c_{ave}$. It is given by

$$\begin{aligned} SWflexVP - SWinflexVP &= \int_{c \geq \frac{\sqrt{p}}{c_{ave}}} [\frac{c}{2}(\frac{p\delta^2}{c_{ave}^2} - \frac{\delta^2}{c^2}) - \delta(\frac{\sqrt{p}\delta}{c_{ave}} - \frac{\delta}{c}) - \delta^2(-\frac{1}{c} + \frac{\sqrt{p}}{c_{ave}})]f(c)dc \\ &= \int [\frac{pc\delta^2}{2c_{ave}^2} - \frac{\delta^2}{2c} - \frac{\sqrt{p}\delta^2}{c_{ave}} + \frac{\delta^2}{c} + \frac{\delta^2}{c} - \frac{\sqrt{p}\delta^2}{c_{ave}}]f(c)dc \\ &= (\delta^2/2)\int [pc/c_{ave}^2 + 3/c - 4\sqrt{p}/c_{ave}]f(c)dc \\ &= (\delta^2/2)\int c[1/c - \sqrt{p}/c_{ave}][3/c - \sqrt{p}/c_{ave}]f(c)dc \end{aligned}$$

If this difference is positive, then flexible VP generates strictly higher social welfare than inflexible

VP. If we consider $f(c) \propto c^{-\alpha}$ and let $c = c_{ave}x/\sqrt{p}$, then

$$(\delta^2/2)\int_{c \geq c_{ave}/\sqrt{p}} c[1/c - \sqrt{p}/c_{ave}][3/c - \sqrt{p}/c_{ave}]f(c)dc = C\int_{x\geq 1} x[1/x-1][3/x-1]f(x)dx$$

where C is a positive constant. By letting $f(x) = kx^{-\alpha}$ where k is a positive constant,

$$\begin{aligned}
C\int_{x\geq 1} x[1/x-1][3/x-1]f(x)dx &= Ck\int_{x\geq 1} x(1/x-1)(3/x-1)x^{-\alpha}dx \\
&= Ck\int_{x\geq 1}(3x^{-\alpha-1} - 4x^{-\alpha} + x^{-\alpha+1})dx \\
&= Ck[-3x^{-\alpha}/\alpha + 4x^{-\alpha+1}/(\alpha-1) - x^{-\alpha+2}/(\alpha-2)]_1^{\infty} \\
&= Ck[3/\alpha - 4/(\alpha-1) + 1/(\alpha-2)] \\
&= Ck[3\alpha^2 - 9\alpha + 6 - 4\alpha^2 + 8\alpha + \alpha^2 - \alpha]/[\alpha(\alpha-1)(\alpha-2)] \\
&= Ck[6-2\alpha]/[\alpha(\alpha-1)(\alpha-2)] > 0
\end{aligned}$$

if $2 < \alpha < 3$. Thus, the flexible VP is more socially desirable than the best inflexible VP if $f(x) \propto c^{-\alpha}$ and $2 < \alpha < 3$.

A.9 Proof of Example 3

The MP requires

$$e_{MP} = \arg\min_{e_{MP}}[\int\{\frac{c}{2}(\max\{0, e - e_{MP}\})^2\}de + \frac{\delta}{2}\{\int\{\min[e, e_{MP}]\}de\}^2]$$

as an inner solution. The right hand is calculated as follows:

$$\begin{aligned}
& \int\{\frac{c}{2}(\min\{0, e - e_{MP}\})^2\}de + \frac{\delta}{2}\{\int\{\min[e, e_{MP}]\}de\}^2 \\
= \quad & \frac{c(e_{max} - e_{MP})^3}{6} + \frac{\delta}{2}(\frac{(e_{MP} - e_{min})(e_{MP} + e_{min})}{2} + e_{MP}(e_{max} - e_{MP}))^2 \\
= \quad & \frac{c(e_{max} - e_{MP})^3}{6} + \frac{\delta}{2}(\frac{e_{MP}^2 - e_{min}^2}{2} + e_{MP}(e_{max} - e_{MP}))^2 \\
= \quad & \frac{c(e_{max} - e_{MP})^3}{6} + \frac{\delta}{2}(-\frac{e_{MP}^2}{2} - \frac{e_{min}^2}{2} + e_{MP}e_{max})^2
\end{aligned}$$

Taking the derivative of e_{MP},

$$\begin{aligned}
0 &= -\frac{c(e_{max}-e_{MP})^2}{2} + \delta(-\frac{e_{MP}^2}{2} - \frac{e_{min}^2}{2} + e_{MP}e_{max})(-e_{MP}+e_{max}) \\
&= -\frac{cM^2}{2} + \delta(-\frac{M^2}{2} + \frac{e_{max}^2}{2} - \frac{e_{min}^2}{2})M \\
&= \left(-\frac{cM}{2} + \delta(-\frac{M^2}{2} + \frac{e_{max}^2}{2} - \frac{e_{min}^2}{2})\right) M \\
&= \left(-\frac{\delta}{2}M^2 - \frac{c}{2}M + \delta(\frac{e_{max}^2}{2} - \frac{e_{min}^2}{2})\right) M \\
&= \frac{M}{2}\{-\delta M^2 - cM + \delta(e_{max}^2 - e_{min}^2)\}
\end{aligned}$$

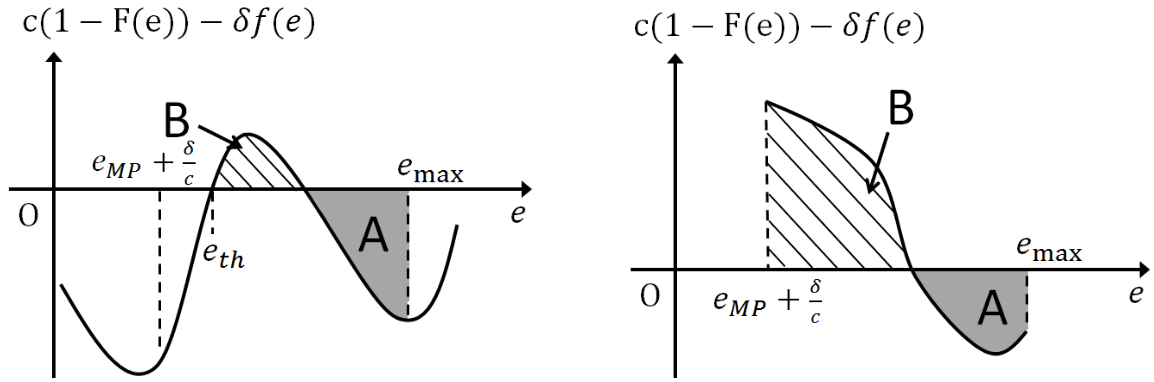
Note that $M \equiv e_{max} - e_{MP}$ here. Solving this quadratic equation with $M > 0$,

$$\begin{aligned}
0 &= -\delta M^2 - cM + \delta(e_{max}^2 - e_{min}^2) \\
0 &= \delta M^2 + cM - \delta(e_{max}^2 - e_{min}^2) \\
M &= \frac{-c \pm \sqrt{c^2 + 4\delta^2(e_{max}^2 - e_{min}^2)}}{2\delta}
\end{aligned}$$

Since $M > 0$ holds,

$$\begin{aligned}
M &= \frac{-c + \sqrt{c^2 + 4\delta^2(e_{max}^2 - e_{min}^2)}}{2\delta} \\
e_{MP} &= e_{max} - \frac{-c + \sqrt{c^2 + 4\delta^2(e_{max}^2 - e_{min}^2)}}{2\delta}.
\end{aligned}$$

It is obvious to see that $e_{MP} < e_{max}$. Also, the assumption $2\delta e_{min} < c$ ensures $e_{MP} > e_{min}$. Therefore, $e_{MP} = e_{max} - \frac{-c+\sqrt{c^2+4\delta^2(e_{max}^2-e_{min}^2)}}{2\delta}$ gives the the MP.



(a) Eq. (7) holds

(b) Eq. (7) does not hold

Figure 1: Examples that whether $c(1-F(e)) - \delta f(e)$ is positive or negative depends on e

In figures 1(a) and 2, $\int_{e_{th}}^{e_{\max}} [c(1-F(e)) - \delta f(e)] de = B - A$.

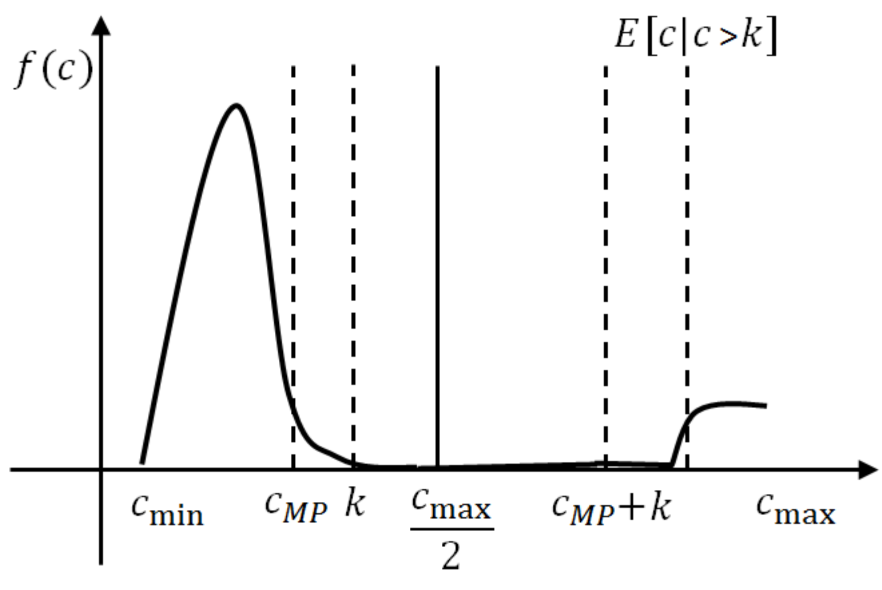


Figure 2: A case where the inflexible VP is not optimal ((8) does not hold)