Model Based Estimation of Sovereign Default

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Feb. 4, 2020@IMFOAP-PRI Workshop

Motivation

- There is a large literature on sovereign default that has developed building on Eaton and Gersovitz (1981).
- While the existing papers quantitatively analyze different aspects of default using calibrated parameter values, our paper is the first attempt to estimate this model.
- We apply a structural estimation method for discrete choice dynamic programming models to the sovereign default model of Arellano (2008).
    - For the estimation, we use default, bond spread, and output paths from the data, not targeting any data moments.
    - We let the default data speak for itself without putting any restrictions other than matching the default behavior observed in the data.

What we ask

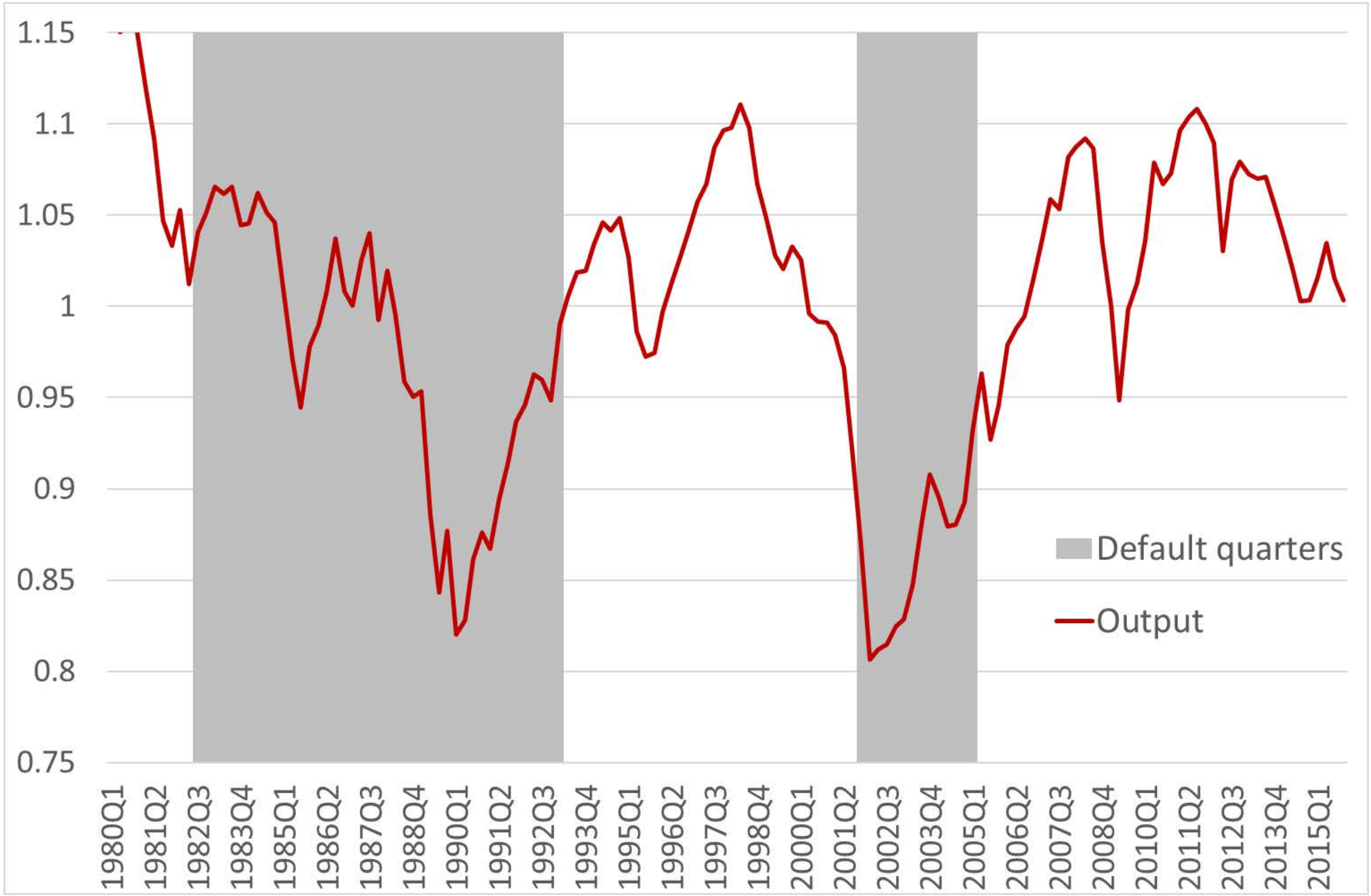
- What are the values of the "estimated" model parameters?
- How do they compare with the calibrated values from the literature?
- Can the actual timing of defaults be predicted by a canonical stochastic general equilibrium sovereign default model?
- How does the model's default prediction with the estimated parameters differ from those with Arellano's (2008) values?

What we do

- Estimate Arellano (2008, AER) using
  - Data for Argentina: output, default, bond spread
  - Sample period: 1993Q2-2015Q4
  - A maximum simulated likelihood method
  - Allow variation in output cost: flat vs. quadratic
- Simulate the model with the estimated parameters given the output data series
- Analyze the default predictions of the model
- Analyze model-implied variables and business cycle properties

What we find

- With Arellano’s (2008) parameter values,
  - The model can account for the 2001 default but falsely predicts a default in 2009 and predicts the 1982 default with a lengthy delay
  - The probability that the model replicates the whole observed default sequence (including default duration) is zero
- With the estimated values,
  - The model correctly predicts no default in 2009, while other default predictions remain the same as those with Arellano’s (2008) values
  - Business cycle properties are in line with the literature
- The estimated output cost is much higher than the calibrated value.
- Using bond spread information in estimation helps to achieve the global maximum in the maximum likelihood estimation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1980Q1
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1983Q4
1985Q1
1986Q2
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1988Q4
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1992Q3
1993Q4
1995Q1
1996Q2
1997Q3
1998Q4
2000Q1
2001Q2
2002Q3
2003Q4
2005Q1
2006Q2
2007Q3
2008Q4
2010Q1
2011Q2
2012Q3
2013Q4
2015Q1
Default quarters
Output

Related literature

Sovereign debt crisis models

- Eaton and Gersovitz (1981, REStu),
- Arellano (2008, AER)
- Aguiar and Gopinath (2006, JIE)
- Mendoza and Yue (2012, QJE)
- Chatterjee and Eyigungor (2012, AER)
- Hatchondo and Martinez (2009, JIE)
- Hachondo, Martinez and Sapriza (2010, RED)
- Asonuma and Trebesch (2016, JEEA)
- Yue (2010, JIE)
- Na et al. (2017, AER)

Discrete choice model estimation

- Train (2009, a textbook), Keane and Wolpin (2009, RED)

Arellano (2008, AER): The state variables

- 3 state variables: (d, B, y)
  - The default variable: d
    - $d_t = 1$ (if the country defaults in period t) or 0 (if it repays)
      The behavioral process: $d_t = d(d_{t-1}, B_t, y_t)$
  - The debt variable: (minus B)
    - -B_t: the country's debt obligation in period t
      The saving policy function: $B(B_{t-1}, y_{t-1})$
  - The output variable: y
    - lny_t=ρ lny_{t-1}+ ε_t

Arellano (2008): Seq. of decisions

- In period t, a country faces debt obligation $-B_t$
- It then observes output, y_t
- If the country had repaid in the previous period, then it would be able to choose to repay or default in period t
  - If the country chooses to repay, then it also decides how much it borrows in that period $-B_{t+1}$
  - If it chooses to default, it can write off its debt obligations at the expense of losing a fraction of output and being excluded from world financial markets for a stochastic number of period

Arellano (2008): The behavioral process: $d(d_{t-1}, B_t, y_t)$

- $d_t = d(d_{t-1} = 0, B_t, y_t)$

$$d_t = \begin{cases} 1, & \text{if } V^D(y_t) > V^R(B_t, y_t) \\ 0, & \text{otherwise} \end{cases},$$

- $d_t = d(d_{t-1} = 1, B_t, y_t)$

$$d_t = \begin{cases} 1, & \text{w/ prob. } 1 - \lambda \\ 0, & \text{otherwise} \end{cases},$$

Arellano (2008): The resource constraints

$$c_t \quad = y_t - q(B_{t+1}, y_t)B_{t+1} + B_t, \qquad \text{under repayment,}$$
$$c_t \quad = h(y_t), \qquad \text{under default,}$$

$$\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t$$

$$h(y_t) = \overline{y} \text{ if } y_t > \overline{y} \text{ and } h(y_t) = y_t \text{ if } y_t \leq \overline{y}$$

Arellano (2008): The value functions

- $V^D(y) = u\big(h(y)\big) + \beta E[\lambda V^R(0, y') + (1-\lambda) V^D(y')],$

- $V^R(B, y) = \max_{B'} u(y - q(B', y)B' + B) + \beta E[\max\{V^D(y'), V^R(B', y')\}],$
  $= u(y - q(B(B,y), y)B(B,y) + B) + \beta E[\max\{V^D(y'), V^R(B(B,y), y')\}]$

Arellano (2008): Bond pricing

- Lenders are assumed to be risk neutral.

$$q(B(B,y),y) = \frac{1 - \delta(B(B,y),y)}{1+r},$$

Arellano (2008): Default probability

- $\Pr(d_t = 1 | d_{t-1} = 0, B_{t-1}, y_{t-1}) = \delta(B(B_{t-1}, y_{t-1}), y_{t-1})$,
- $\Pr(d_t = 0 | d_{t-1} = 0, B_{t-1}, y_{t-1}) = 1 - \delta(B(B_{t-1}, y_{t-1}), y_{t-1})$,
- $\Pr(d_t = 1 | d_{t-1} = 1, B_{t-1}, y_{t-1}) = 1 - \lambda$,
- $\Pr(d_t = 0 | d_{t-1} = 1, B_{t-1}, y_{t-1}) = \lambda$,

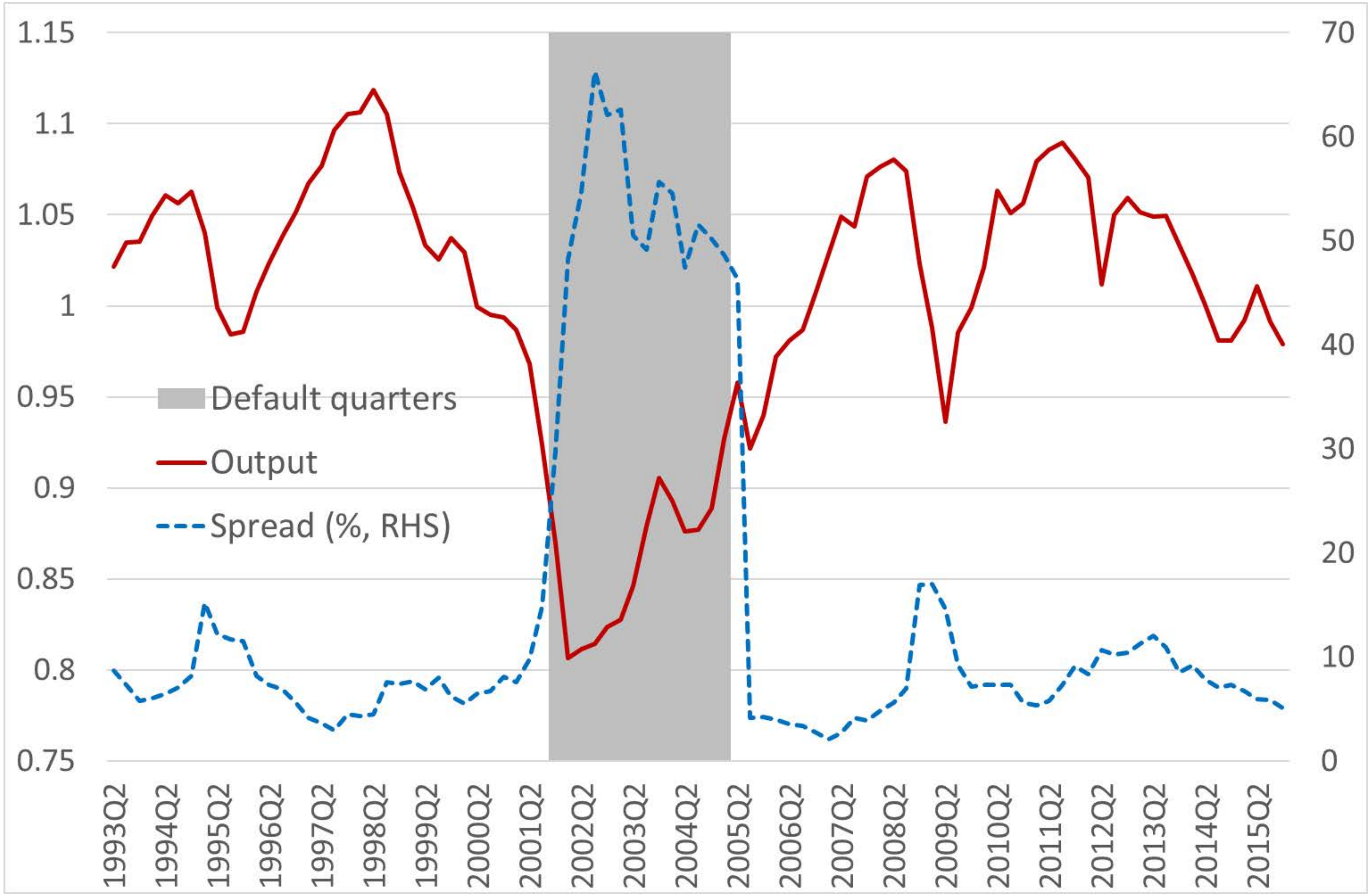
where

$\delta(B_t, y_{t-1}) = Pr(y_t \in I(B_t))$, with $I(B_t) = \{y_t \in \mathcal{Y}: V^D(y_t) > V^R(B_t, y_t)\}$

Data

- Data on default:
  - Asonuma and Trebesch (2016) covers 1978/1/1-2010/12/1
  - BoC-BOE database

  Default quarters: 1982Q3-1993Q1 and 2001Q4-2005Q1
- Data on real output: 1980Q1-2015Q4
  - The National Institute of Statistics and Censuses (INDEC) of Argentina.
  - We remove a linear trend from the log of the real GDP at constant prices, and then use the detrended component as ln(y).
- Data on bond spread: 1983Q3-2013Q4
  - Neumeyer and Perri (2005)
  - EMBI Global Argentina (stripped spread)



1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
70
60
50
40
30
20
10
0
Default quarters
Output
Spread (%, RHS)
1993Q2
1994Q2
1995Q2
1996Q2
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1999Q2
2000Q2
2001Q2
2002Q2
2003Q2
2004Q2
2005Q2
2006Q2
2007Q2
2008Q2
2009Q2
2010Q2
2011Q2
2012Q2
2013Q2
2014Q2
2015Q2

Estimation strategy

- The state-space representation is nonlinear and non-Gaussian. We use a simulated likelihood method.
- We define $z_t = (1 - d_t)s_t + d_t$, where $s_t =$ bond spread and $d_t =$ default variable
- We allow i.i.d. measurement (forecast) errors to the default and spread variables
- Only 10 parameters with pre-specified ranges
  - σ (risk aversion), r (risk-free rate), β (discount factor), λ (reentry probability), ρ and η (coefficients in the output equation), $\overline{y}$ (output cost), and B_0 (initial asset level), a_1 and a_0 (measurement error parameters)
  - We fix σ *at 2, r* at 0.01, and set the initial debt at zero.
- Grids setting: Hatchondo et al. (2010)

  151 and 66 grids for debt and output in estimation; Once obtained parameter values, we increase grids to 751 and 131 respectively
- The sample period: 1993Q2:2015Q4
- We use output and “z” data series for estimation

The state-space representation

Measurement equations

- $\ln(y_t^o) = \ln(y_t)$,
- $\Pr(d_t^o = 0 \mid d_t = i) = a_i$ for $i = 1$ or 0
- $z_t^o = \begin{cases} z(B_t, y_t) + u_t, & if\, d_t^o = 0 \\ 1 & o/w \end{cases}$, Where $z_t^o = (1 - d_t^o)s_t^o + d_t^o$

State equations

- $\ln(y_t) = \rho \ln(y_{t-1}) + \varepsilon_t,$
- $d_t = d(d_{t-1}, B_t, y_t),$
- $z_t = (1 - d_t)s_t + d_t$ Where $s(B_t, y_t) = \frac{1}{q(B_t, y_t)} - (1 + r)$
- $B_t = \begin{cases} B(B_{t-1}, y_{t-1}) & if\ d_t = 0 \\ 0 & o/w \end{cases}$, $s(B_{t-1}, y_{t-1}) = \begin{cases} s(B_{t-1}, y_{t-1}) & if\ d_t = 0 \\ \infty & o/w \end{cases}$
- $\varepsilon_t \sim\ i.i.d.\ N(0, \eta), u_t \sim\ i.i.d.\ N(0, \eta^s),$

Let $Z^o \equiv \{z_t^o\}, D^o \equiv \{d_t^o\}, D \equiv \{d_t\}$ and $Y \equiv \{y_t\}$.

The joint distribution of Z^o, D^o and *Y* implied by the model is given by

$$P(Z^o, D^o, Y; \boldsymbol{\theta}) = \left[\int P(Z^o | D^o, S) P(D^o | D) P(Z, D | Y) dD \right] P(Y)$$

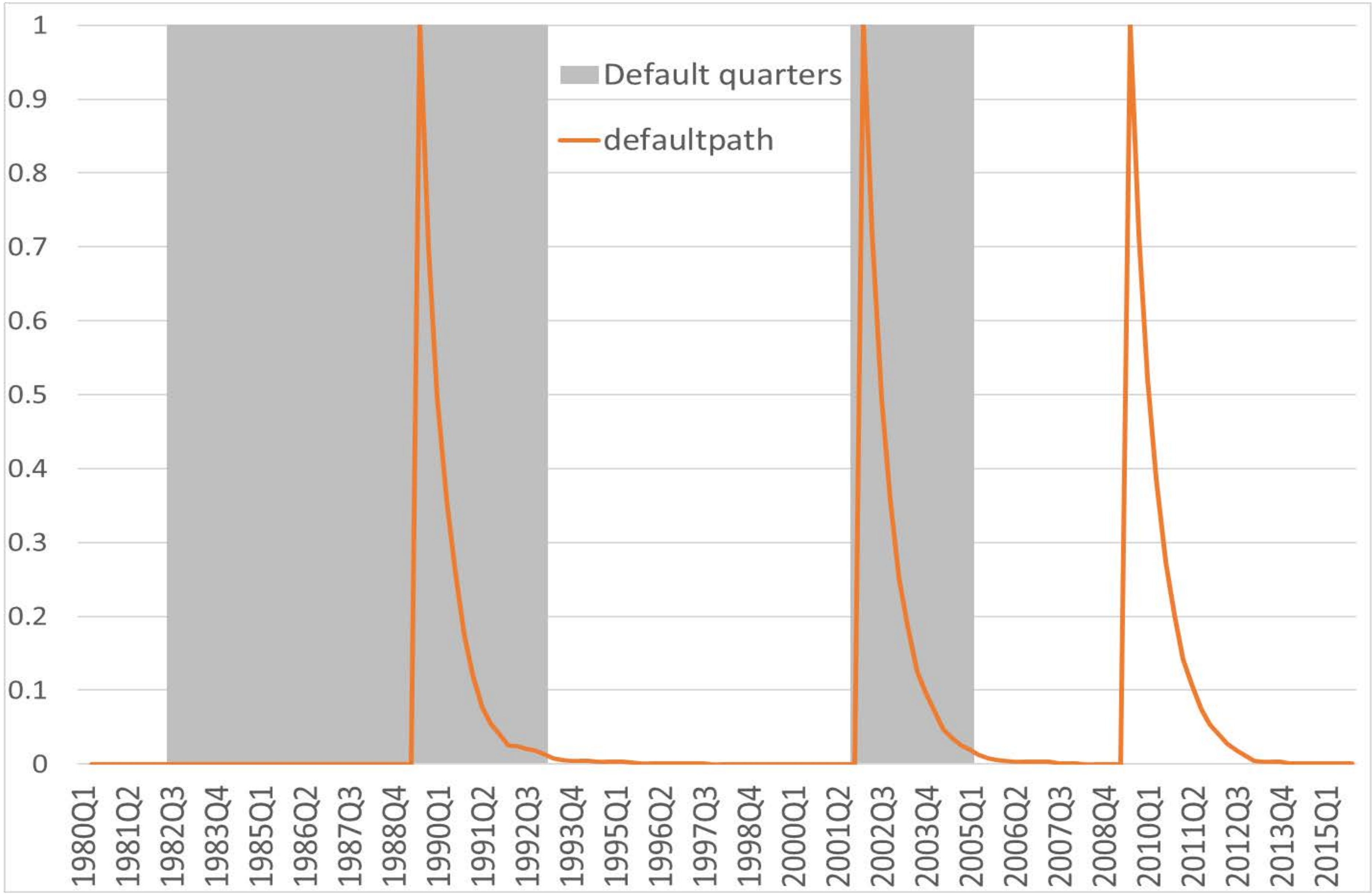
$$\approx \left[\sum_i P(Z^o | D^o, Z_i) P(D^o | D_i) P(Z_i, D_i | Y) \right] P(Y)$$

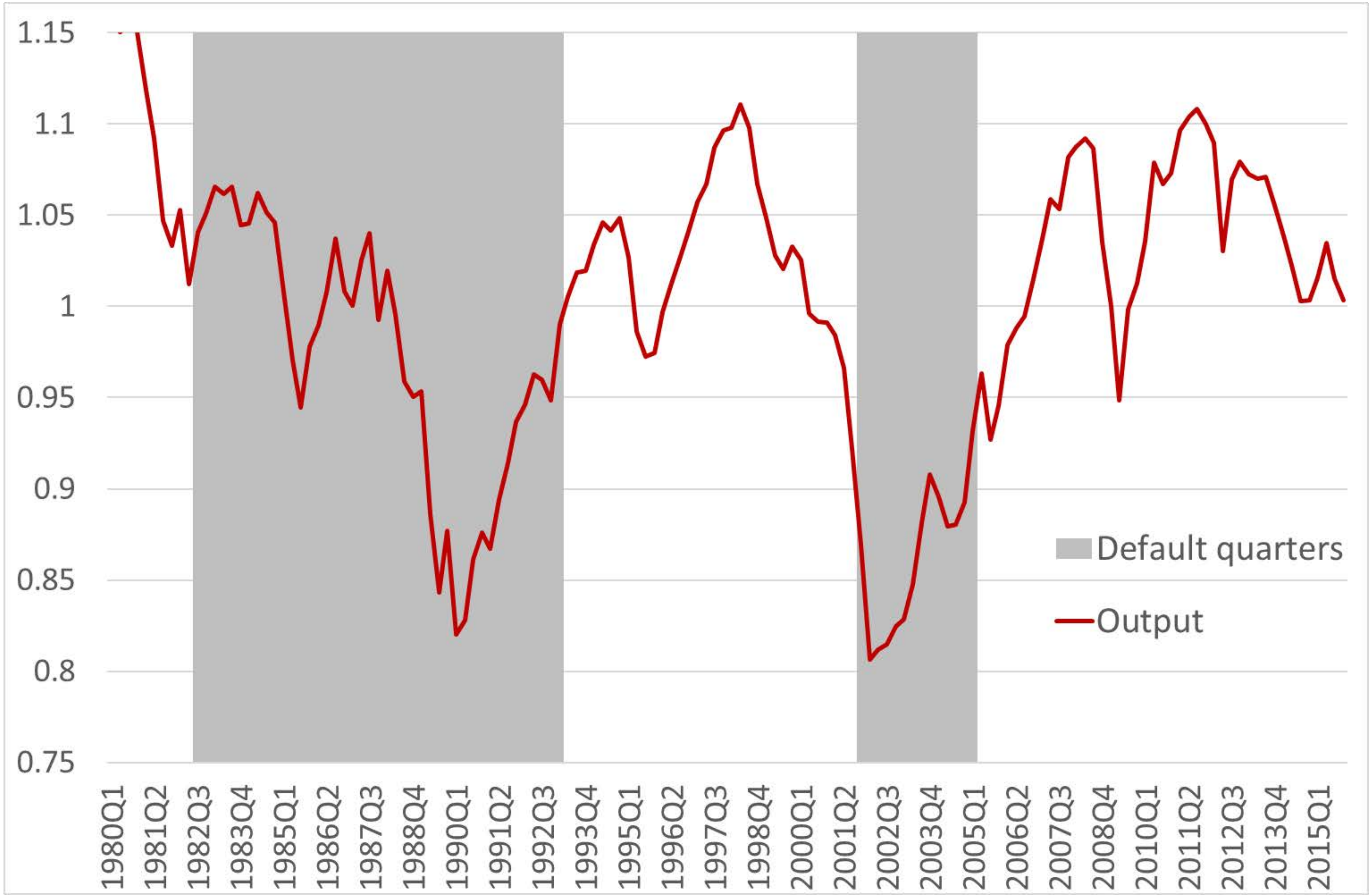
The log-likelihood function is given by

ln$P(Z^o, D^o, Y; \boldsymbol{\theta})$=ln$[\sum_i P(Z^o | D^o, S_i) P(D^o | D_i) P(S_i, D_i | Y)]$+ln$P(Y)$

The average of simulated default paths

With Arellano’s (2008) parameter values

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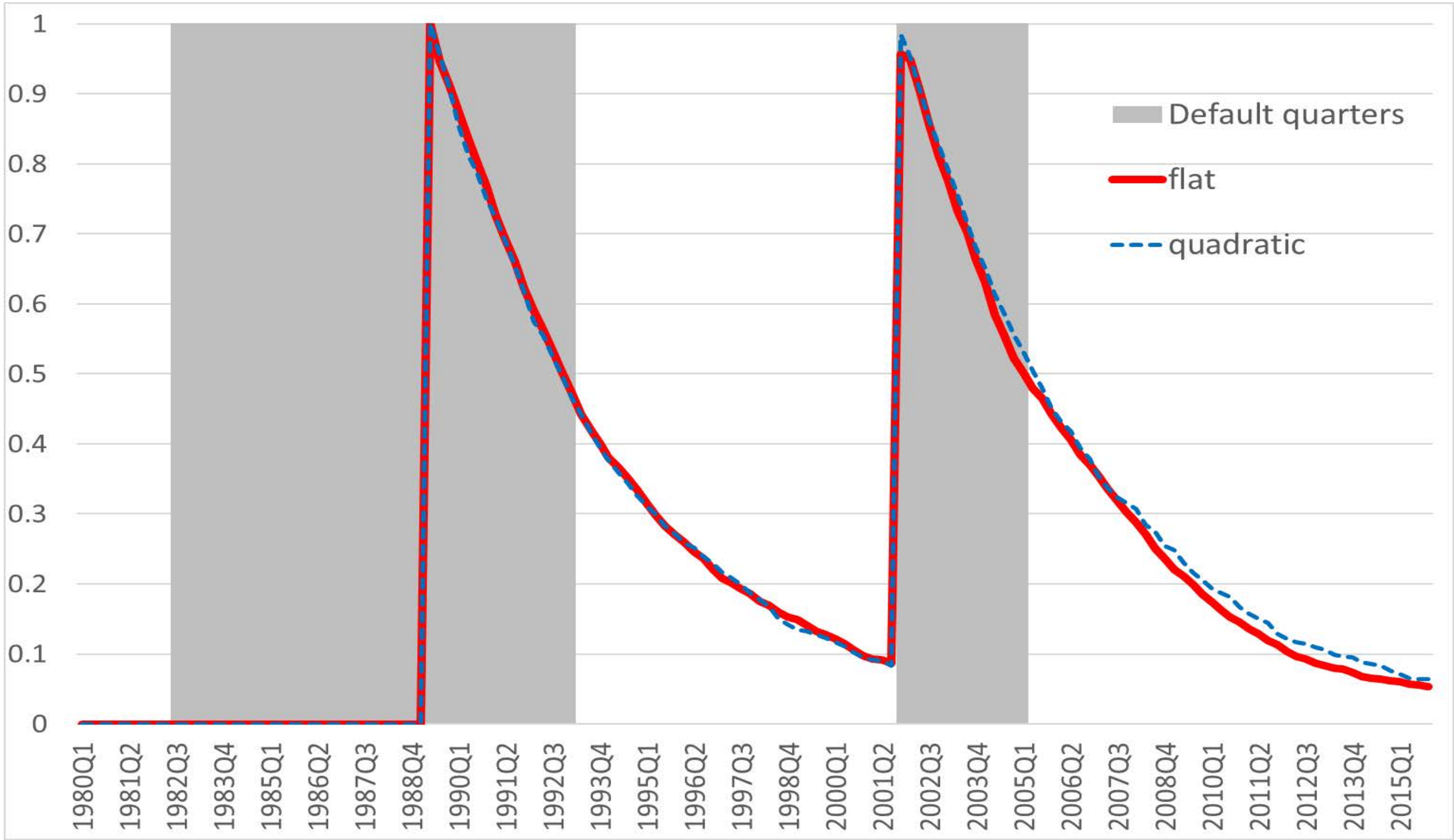
2005Q1
2006Q2
2007Q3
2008Q4
2010Q1
2011Q2
2012Q3
2013Q4
2015Q1
Default quarters
Output

The average of simulated default paths

The sample period for estimation: **1993Q2-2015Q4**



Model parameter estimates

Parameter	Definition	Flat Cost	Quadratic Cost	Arellano (2008)	Chatterjee and Eyigungor (2012) (1-period debt)
σ	Risk aversion coefficient	2	2	2	2
r	Risk-free interest rate	0.01	0.01	0.017	0.010
β	Discount factor	0.95 (0.004)	0.75 (0.005)	0.953	0.67
$\bar{y}$	Output cost	0.85 (0.001)	—	0.969	—
d_0	Quadratic output cost parameter	—	-0.50 (0.002)	—	-0.46
d_1	Quadratic output cost parameter	—	0.70 (0.002)	—	0.57
λ	Reentry probability	0.05 (0.002)	0.05 (0.002)	0.282	0.039
ρ	Output persistence	0.956 (0.001)	0.954 (0.002)	0.945	0.949
η	Std. deviation of output shock	0.025 (0.002)	0.023 (0.002)	0.025	0.027
η^s	Measurement error parameter	0.02 (0.001)	0.02 (0.001)	—	—
a_1	Measurement error parameter	0.00	0.00	—	—
a_0	Measurement error parameter	1.00	1.00	—	—

Business cycle statistics

	Data (C&E, 2012) 1993.1-2001.4	Flat Cost	Quadratic Cost	Arellano (2008)	Chatterjee and Eyigungor (2012) (1-period debt)
$\sigma(c)/\sigma(y)$	1.09	1.67	1.61	1.10	1.59
$\sigma(nx/y)/\sigma(\text{y})$	0.17	1.01	1.14	0.26	1.06
$\sigma(spread)$	4.43	1.83	2.67	6.36	4.43
$\text{corr}(c,y)$	0.98	0.80	0.73	0.97	0.73
$\text{corr}(nx/y,y)$	-0.88	-0.29	-0.12	-0.25	-0.16
$\text{corr}(spread,y)$	-0.79	-0.48	-0.69	-0.29	-0.55
Average spread	0.08	0.01	0.02	0.0358	0.0815
Default frequency	0.13	0.01	0.02	0.03	0.073

Conclusion

- We apply a structural estimation method to a sovereign default model for the first time.
- The estimated output cost is significantly higher than the calibrated values used in the literature
- The estimated model can
  - Correctly predict no default in 2009
  - Generate business cycle properties that are broadly in line with the literature