Lack of debt restructuring and lenders' credibility: A theory of nonperforming loans

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Question addressed in this paper

- consider a long-term relationship between a borrower and a lender.
    - The borrower may be a firm or a sovereign.
- What would happen if a sequence of bad shocks to the borrower raises his debt to an unrepayable level?
- Debt restructuring (reduction) involves a time-consuming procedure.
    - creates nonperforming loans.
- show that lack of delay of debt restructuring tends to depress the economic activity of the borrower.
- helpful to understand why a financial crisis is followed by a persistent recession.
    - The amount (fraction) of nonperforming loans is an important statistics to watch.

Example

- $r = 0$ and $t = 0, 1, 2, \ldots$.
- The borrower earns \$ 1 million in each period.
  - He chooses to walk away if the PDV of repayments exceeds \$ 1 million.
- D = (contractual) amount of debt in period 0.
- For $D \leq 1$ million, there is no problem with repayments.
  - e.g., the lender can offer a repayment plan: $b_0 = \$D$ and $b_t = 0,\ t \geq 1$.
  - This is a credible repayment plan.
- Suppose that $D = 2$ million, and D cannot be reduced.
  - The lender could offer a repayment plan: $b_0 = 1$ million and $b_t = 0,\ t \geq 1$.
  - But it is not credible, because, in period 1, $D_1 = 1$, and the lender can demand the borrower to repay another 1 million.
  - Expecting it, the borrower chooses to walk away in period 0.

- When the (contractual) amount of debt exceeds a certain threshold, any (dynamic) repayment plans offered by the lender becomes non-credible.
- Thus, dynamic provision of incentives becomes infeasible.
  - The equilibrium relationship between the lender and the borrower becomes static.
  - Technically, this is due to the fact that the contractual amount of debt becomes not payoff relevant when it becomes "too large."
- Inefficiency would last until debt restructuring is conducted.
- difference from the theory of debt overhang:
  - debt overhang: the supply of funds to a heavily indebted borrower is depressed.
  - our theory: the demand for funds by the borrower is depressed due to the loss of the lender's credibility.

Outline

- long-term debt contract between a borrower (firm/sovereign) and a lender (bank).
  - Albuquerque and Hopenhayn (2004).
- Borrowing constraint arises because the borrower may default at any time.
- Unlike AH, we do not allow the value of the borrower, V_t, to be used as a state variable.
  - The contractual amount of debt, D_t, is the only variable that can be used as a state variable.
- The contractual amount of debt, D_t, may deviate from the value of the lender, d_t (PDV of repayments).
- debt restructuring:
  - debt forgiveness, i.e., a reduction in the contractual amount of debt, D_t.
- Debt restructuring is not feasible in the benchmark model.
  - Later, we extend the model with stochastic debt restructuring.

Setting

- time periods: $t = 0, 1, \ldots$.
- productivity: $s \in \{s_L, \ s_H\}$, where $0 \leq s_L < s_H$.
- The lender and the borrower have a common discount factor β.
- two types of funds provided by the lender: long-term and short-term (working capital).
- D_0 = initial amount of long-term loan.
  - $r \geq \beta^{-1} - 1$ = (fixed) interest rate on the long-term loans.
- k_t = working capital (intra-period loans) in each period $t \geq 0$.
  - R = rate on short-term loans k_t.
- $F(s_t, k_t)$ = production (revenue) function of the firm.
- b_t = repayments on the long-term loans D_t in periods $t \geq 0$.
- Both the borrower and lender take β, r, and R as given.

Value of the borrower

- borrower's net income (dividends):

$$F(s_t, k_t) - Rk_t - b_t.$$

- Limited liability:

$$F(s_t, k_t) - Rk_t - b_t \geq 0, \qquad \forall t \geq 0.$$

- V_t = PDV of dividends (value of the borrower) in period t:

$$V_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} [F(s_i, k_i) - Rk_i - b_i] = F(s_t, k_t) - Rk_t - b_t + \beta \mathbb{E}_t V_{t+1}.$$

- The firm can choose to default in any period t, after receiving working capital k_t.
- The bank would receive none when the firm defaults.
- $G(s_t, k_t)$ = the value of the outside opportunity of the firm.
- Enforcement constraint:

$$V_t \geq G(s_t, k_t), \qquad \forall t \geq 0.$$

- In each period t, given an offer from the lender $\{k_i, b_i, V_i\}_{i=t}^{\infty}$, the borrower decides whether or not to accept it.

Value of the lender

- d_t = PDV of repayments (value of the lender) in period t:

$$d_t = \mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} b_i \geq 0.$$

- D_t = contractual value (book value) of debt in period t:

$$D_t = (1+r)(D_{t-1} - b_{t-1}). \qquad (*)$$

- The lender cannot require repayment more than D_t:

$$b_t \leq D_t$$

- Debt restructuring is infeasible, i.e., D_t cannot deviate from the value determined by $(*)$.
  - This is so even when D_t exceeds the amount that the borrower can repay.

Properties of the constrained efficient contract

- AH (2004): constrained efficient contract using V as a state variable.
- $k^*(s)$ = first-best allocation of working capital: $k^*(s) = \arg\max_k F(s,k) - Rk$.
- When V is small,
  - enforcement constraint binds: $G(s, k(s, V)) = V$;
  - inefficient allocation of funds: $k(s, V) < k^*(s)$;
  - back-loading of payoffs: $F(s,k) - Rk - b = 0$.
- The first-best allocation $k^*(s)$ is achieved in finite time with probability one.
  - The inefficiency lasts only temporarily.
- This paper: Introducing (some) inflexibility to the adjustment of D can change the equilibrium drastically:
  - it may no longer be possible to provide incentives dynamically;
  - inefficiency may last forever.

Markov equilibrium

- Consider a Markov equilibrium with the state variables (s_t, D_t): $\{k(s,D), b(s,D), d(s,D), V(s,D)\}$.
- The lender's problem is, given the borrower's value function $V(s,D)$,

$$
\begin{aligned}
d(s,D) = \max_{b,k,V,D_{+1}} \quad & b + \beta \mathbb{E} d(s_{+1}, D_{+1}) \\
\text{s.t.} \quad & F(s,k) - Rk - b + \beta \mathbb{E} V(s_{+1}, D_{+1}) \geq G(s,k), \\
& F(s,k) - Rk - b \geq 0, \\
& D_{+1} = (1+r)(D-b), \\
& b \leq D.
\end{aligned}
$$

- The borrower's value $V(s,D)$ solves

$$
V(s,D) = F(s, k(s,D)) - Rk(s,D) - b(s,D) + \beta \mathbb{E} V(s_{+1}, D_{+1}).
$$

where $k(s,D)$ and $b(s,D)$ are the policy functions of the lender's problem.

NPL equilibrium

- The contractual amount of debt, D_t, evolves as $D_t = (1+r)(D_{t-1} - b_{t-1})$.
  - With a sequence of bad shocks $s_t = s_L$, D_t will exceed what the borrower can repay.
- Benchmark assumption: D_t cannot be reduced from the value given by $(1+r)(D_{t-1} - b_{t-1})$.

Proposition 2

There exists a threshold value $\bar{D}$ such that for all $D > \bar{D}$,

1. $D_{+1}(s, D) \geq D$; and
2. $\{k(s,D), b(s,D), d(s,D), V(s,D)\} = \{k^{npl}(s), b^{npl}(s), d^{npl}(s), V^{npl}(s)\}$.

- The tuple $\{k^{npl}(s), b^{npl}(s), d^{npl}(s), V^{npl}(s)\}_{s=s_H,s_L}$ is referred to as the NPL equilibrium.

- Suppose that D_t becomes too large to be repaid.
  - D_t becomes nonperforming loans.
- Then, it no longer matters how much the contractual amount of debt is.
  - D_t becomes not payoff relevant.
- There is no way to provide incentives dynamically.
  - Without reducing D_t, any dynamic offer made by the lender is not credible.
- What the lender can do is only to offer a static contract.

Intuition

- The NPL equilibrium is given by the best static contract for the lender: For each $s = s_H, s_L$,

$$\begin{aligned} d^{npl}(s) = \max_{\{k(s), b(s), V(s)\}} \quad & b(s) + \beta \mathbb{E}[d^{npl}(s_{+1})] \\ \text{s.t.} \quad & V(s) = F(s, k(s)) - Rk(s) - b(s) + \beta \mathbb{E}[V(s_{+1})], \\ & F(s, k(s)) - Rk(s) - b(s) \geq 0, \\ & V(s) \geq G(s, k(s)). \end{aligned}$$

  The policy functions for this problem are $\{k^{npl}(s), b^{npl}(s), V^{npl}(s)\}$.

- As long as there is a benefit of dynamic contracts, the NPL equilibrium is a bad outcome.

Lemma 1

$k^{npl}(s)$ is the minimum level in equilibrium:

$$k(s, D) \geq k^{npl}(s), \quad \text{for all } s \in \{s_L, s_H\} \text{ and } D \in \mathbb{R}_+.$$

- Nonperforming loans lead to an inefficiently low level of output.
- Without the possibility of debt restructuring, the level of output remains there forever.

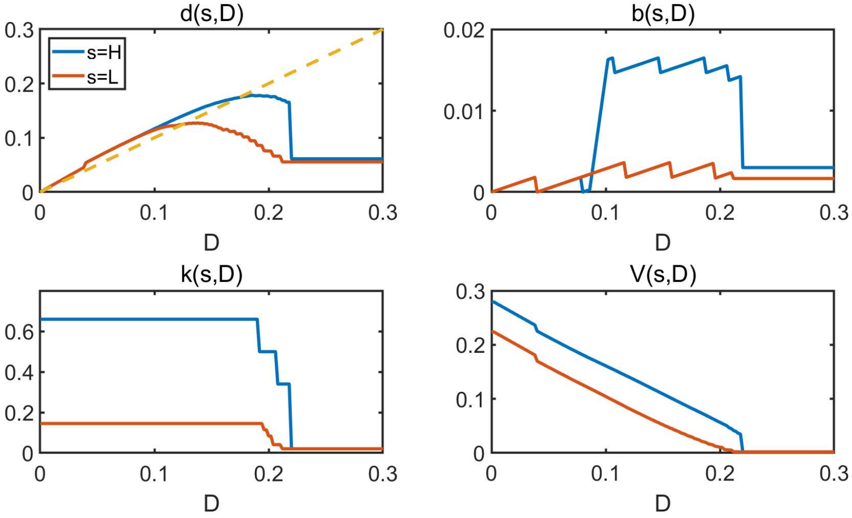
- For further analysis (both theoretically and numerically), we consider a discretized version of the model.
    - Specifically, we assume k, b, and D can take only a finitely many values.
- Without discretization, the equilibrium functions $\{k(s, D), b(s, D), d(s, D), V(s, D)\}$ would exhibit discontinuous jumps at many (possibly infinitely many) points.
- The existence of an equilibrium is proved in the discretized model (Theorem 11).

Parameter values

- Functional forms: $F(s,k) = sAk^{\alpha}$ and $G(s,k) = Bk$.

	Description	Value
A	Normalization	0.1
B	Outside option	0.1
α	Production function	0.8
R	Rental rate of capital	0.1
β	Discount factor	0.96
r	interest rate	0.05
s_H, s_L	Productivity	1.15, 0.85
π_{HH}, π_{LL}	Transition probability	0.9, 0.9

Figure 3


d(s,D)
s=H
s=L
0.3
0.2
0.1
0
0
0.1
0.2
0.3
D
b(s,D)
0.02
0.01
0
0
0.1
0.2
0.3
D
k(s,D)
0.6
0.4
0.2
0
0
0.1
0.2
0.3
D
V(s,D)
0.3
0.2
0.1
0
0
0.1
0.2
0.3
D

Properties of the equilibrium

- NPL equilibrium occurs when $D > 0.218$ for $s = s_H$ and when $D > 0.210$ for $s = s_L$.
    - In this region, $d(s, D)$, $V(s, D)$, $b(s, D)$, and $k(s, D)$ are all trapped in very low levels.
- Debt Laffer curve:
    - The value of the lender is hump-shaped.
- In the NPL equilibrium, it is beneficial for both the borrower and the lender to reduce the contractual amount of debt.

- Debt restructuring is possible, but only stochastically.
- $p =$ (exogenous) probability that the lender obtains an option to reduce D.
- When the option is exercised, the debt is reduced to $\hat{D}(s, D)$ (endogenously determined).
- bargaining models with similar features:
    - Abreu and Gul (2000); Fuchs and Skrzypacz (2010); Pitchford and Wright (2012), etc.

- Markov equilibrium: $\{d(s,D), b(s,D), k(s,D), \hat{D}(s,D), V(s,D)\}$:
  - The lender's problem:

$$
\begin{aligned}
d(s,D) = \max\ & b + \beta\mathbb{E}[(1-p)d(s_{+1}, D_{+1}) + pd(s_{+1}, \hat{D}_{+1}(s_{+1}, D_{+1}))] \\
\text{s.t.}\ & D_{+1} = (1+r)(D-b) \\
& F(s,k) - Rk + b + \beta\mathbb{E}[(1-p)V(s_{+1}, D_{+1}) + pV(s_{+1}, \hat{D}(s_{+1}, \hat{D}] \geq G(s,k), \\
& F(s,k) - Rk - b \geq 0,
\end{aligned}
$$

  - The borrower's value, $V(s,D)$, solves

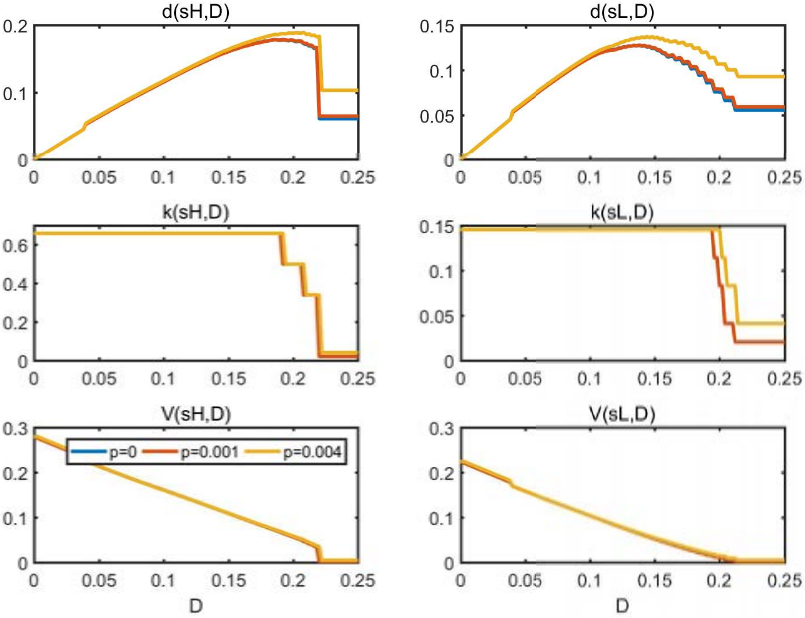
$$
V(s,D) = F(s, k(s,D)) - Rk(s,D) - b(s,D) + \beta\mathbb{E}[(1-p)V(s_{+1}, D_{+1}) + pV(s_{+1}, \hat{D}(s_{+1}, \hat{D})],
$$

    where $k(s,D)$ and $b(s,D)$ are the policy functions of the lender's problem.

  - $\hat{D}(s,D)$ is

$$
\hat{D}(s,D) = \arg\max_{D' \leq D} d(s, D')
$$

Figure 4



Concluding remarks

- illustrate how the lack/delay of debt restructuring leads to persistent inefficiency.
- due to the fact that too much debt makes dynamic repayment plans offered by the lender non-credible.
- Our theory would be helpful to understand why a financial crisis is typically followed by a prolonged recession (secular stagnation).