Deflationary equilibrium
– an unintended consequence of expansionary policies

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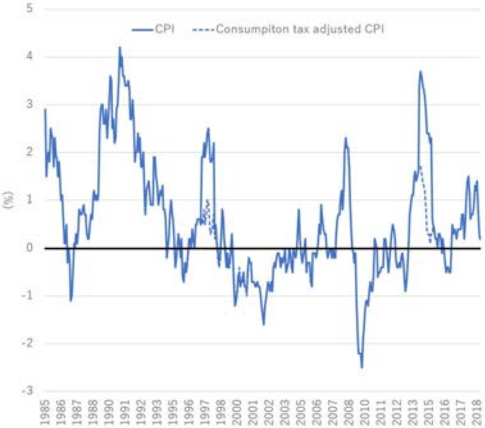
Motivation (1/2)

- We observe decade-long deflationary stagnation
  - The last three decades in Japan
- Puzzling fact (*Deflationary equilibrium*):
  Coexistence of disinflation and expansionary policy (i.e., zero nominal interest rate and zero fiscal surplus)
- Can we have the deflationary equilibrium as a stationary equilibrium outcome?
  - Why expansionary policy does not raise the inflation rate?

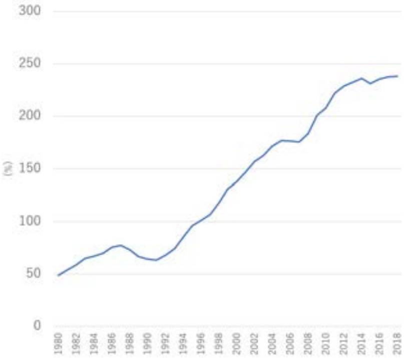
Deflation in Japan

Figure: CPI inflation rate in Japan



Debt growth in Japan

Figure: Debt to GDP ratio in Japan



Motivation (2/2)

- Our experience seems inconsistent with popular prescriptions for deflation
  - Prescription: expansionary (fiscal) policy for an extended period
    - Benhabib, et al. (2002)
  - The expansionary policy is effective, in the representative agent model.
    - In the representative agent model, $1 + r = \beta^{-1}$.
    - The government commit to continue expansionary policy, and to stop it only if inflation is observed.
    - If the expansionary policy (i.e., primary surplus = 0) continues forever, the TVC will be violated:

$$\lim_{t \to \infty} \beta^t b_t = \lim_{t \to \infty} \beta^t (1 + r)^t b_0 = b_0 \neq 0,$$

      where the TVC is $\lim_{t \to \infty} \beta^t b_t = 0$.
    - Thus, the inflation must occur to stop the expansionary policy

Strategy

- Our strategy to explain coexistence of disinflation and expansionary policy:
  - to make $1 + r < \beta^{-1}$, in a model à la Aiyagari (1994)
- Then, the expansionary fiscal policy is compatible with TVC:

$$\lim_{t \to \infty} \beta^t b_t = \lim_{t \to \infty} \beta^t (1 + r)^t b_0 = 0.$$

- We show that
  the government commitment can trap the economy in a persistent disinflation unintentionally,
  - where the commitment is
    - to set zero nominal interest rate and zero fiscal surplus, if $\pi_t \leq 0$,
    - to set a positive nominal interest rate and a positive fiscal surplus, if $\pi_t > 0$.

Literature on persistent disinflation

- Benhabib, Schmitt-Grohè, and Uribe (2002)
  - Persistent deflation with contraction of the government debt
  - Our model: persistent disinflation and expansionary policy
- Eggertsson and Woodford (2003)
  - Liquidity trap due to large shock. Forward guidance has inflationary effect.
  - Our model: forward guidance may create persistent disinflation, even without negative shocks.
- Hagedorn, Luo, Manovskii (2019)
  - Forward guidance do not have large effect on output and employment. Silent about the effect on the inflation rate
  - Our theory is complementary to theirs in that we focus on the inflation rate.

Literature on $1 + r < \beta^{-1}$

- Incomplete market models
  - Huggett (1993)
    Idiosyncratic income risk without capital
  - Aiyagari (1994)
    Idiosyncratic income risk with capital
- Bond-in-the-Utility model
  - Hansen and Imrohoroglu (2016)
  - Hagedorn (2018)

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Setting (1/2)

- Aiyagari (1994) with the government bond
- A closed economy with heterogeneous households.
- Output is produced from capital and labor.
- Each household supplies one unit of labor, the productivity of which is ϵ.
- There is an idiosyncratic risk in the productivity:

$$\log(\epsilon') = \rho \log(\epsilon) + e,$$
$$e \sim N(0, \sigma^2).$$

- Households own asset, a, which consists of capital and the government bond.

Setting (2/2)

- The optimization problem of a household is given by

$$V(a,\epsilon) = \max_{a',c} \{u(c) + \beta E[V(a',\epsilon)|\epsilon]\} \tag{1}$$

$$\text{s.t.} \quad c + a' = (1+r)a + w\epsilon + \tau \tag{2}$$

$$a' \geq \underline{a}. \tag{3}$$

- Firms solve $\max_{K,L} K^{1-\theta}L^{\theta} - wL - (r+\delta)K$, which implies the following FOCs

$$r = F_K(K,L) - \delta, \tag{4}$$

$$w = F_L(K,L). \tag{5}$$

- The government budget constraint is $\frac{B'}{P} = \frac{1+i}{1+\pi}\frac{B}{P_{-1}} + \tau$.
  The arbitrage implies $1 + r = \frac{1+i}{1+\pi}$. It can be rewritten as

$$\frac{B'}{P} = (1+r)\frac{B}{P_{-1}} + \tau. \tag{6}$$

Stationary Equilibrium (SE)

Given a policy (i, τ), a stationary equilibrium is a list of functions $\{V(a,\epsilon), g_c(a,\epsilon), g_a(a,\epsilon), \mu(a,\epsilon), K(r)\}$, quantity of debt, B/P_{-1}, and a set of prices $\{r, w\}$, which satisfies

1. V solves household problem, where g_c (g_a) is the policy function for c (a').
2. $K(r)$ and L solve the firm's problem, given the prices (r, w).
3. Government budget constraint (6) is satisfied: $b' = (1+r)b + \tau$, with $b' = b$.
4. Markets clear:

$$A(r;\tau) \equiv \int a d\mu(a,\epsilon) = b + K(r,w), \tag{7}$$

$$L = \int \epsilon d\mu(a,\epsilon), \tag{8}$$

$$\int g_c(a,\epsilon)d\mu(a,\epsilon) + K' - (1-\delta)K = F(K,L), \qquad \text{where } K' = K = K(r). \tag{9}$$

5. The distribution is stationary

$$\mu(a',\epsilon') = \int P(\epsilon'|\epsilon)\mathbf{1}\{a' = g_a(a,\epsilon)\}d\mu. \tag{10}$$

Numerical experiments

- Parameters
  - The parameter values are mostly the same with the one used in Aiyagari(1994).
  - $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$ with $\alpha = 2.0$ or 3.0.
  - $\beta = 0.96$.
  - For the AR(1): $\rho = 0.9$ and $\sigma = 0.275$.
  - No Borrowing: $\underline{a} = 0.0$.
  - The production function: $Y = K^{1-\theta}L^{\theta}$ with $\theta = 2/3$
  - $L = \int e^x d\phi(x) = 1.5242$.
- Denote $b \equiv \frac{B}{P_{-1}}$.
- We will show that there exists the SE with $r = \tau = 0$,
  - for $b < 0$, in the case where α is small,
  - for $b > 0$, in the case where α is large.
  - $r = 0$ implies that $i = 0$ induces $\pi = 0$.

Numerical result

If $\alpha = 3.0$, the SE with $r = \tau = 0$ exists, for $b > 0$.

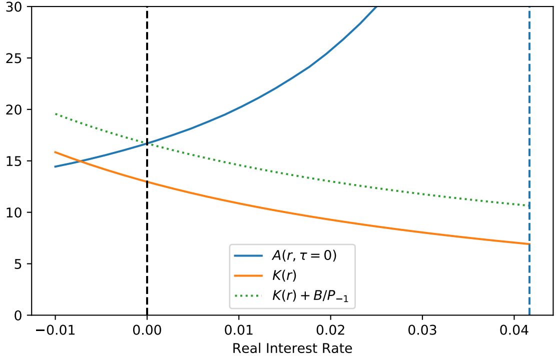

Figure: The supply and demand for assets under $\tau = 0$ and $\alpha = 3.0$.

Features of SE

- Fiscal policy τ decides r and b.
- Monetary policy i decides π by the Fisher equation ($i = r + \pi$).

Monetary policy and steady-state inflation

- In the steady state, r does not depend on i: $r = r(\tau)$ and $r'(\tau) < 0$.
- The steady-state inflation is given by

$$1 + \pi = \frac{1 + i}{1 + r(\tau)}.$$

Suppose that the fiscal policy (τ) is fixed,

- monetary easing (a decrease in i) is disinflationary, or decreases π.
- In particular, when $\tau = 0$,

$$r = 0 \text{ and } \pi = i.$$

In this case the zero nominal interest rate policy ($i = 0$) induces zero inflation:

$$\pi = 0.$$

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Shock in period 0

- The economy was initially in the zero inflation steady state for $t < 0$.
  - $\alpha = 2.0$,
  - $\pi = 0$,
  - $r = i > 0$,
  - b is given by $b = -\frac{\tau}{r}$,
  - primary surplus ($\tau < 0$).

- Two unexpected shocks in period 0
  1. Parameter α changes from $\alpha = 2.0$ to 3.0, so that demand for savings increased.
  2. In response to the change in α, the government makes a new policy commitment (next page)

- Price P_0 jumps in response to the unexpected shocks in period 0.

Policy to be assessed

- **Forward Guidance:** Gov't makes the following commitment in period 0
  - as long as observed $\pi_t \leq 0$, the government sets $i_{t+1} = \tau_t = 0$,
  - if $\pi_t > 0$, the government sets

$$i_{t+1+j} = (1 + \pi^*)(1 + r^*) - 1,$$

$$\tau_{t+j} = \tau^* = -r^* b^*,$$

  for all $j \geq 0$, where
    - $\pi^* > 0$ is the inflation target,
    - $r^* > 0$ is also a given target.

Complete market model: immediate inflation

- In this case, the inflation target is immediately attained.
  - The following logic is the same as Bernanke (2000), Benhabib, et al. (2002)

- Given the government commitment, P_0 jumps up such that $\pi_0 = \frac{P_0}{P_{-1}} - 1 > 1$.
  - Note that $1 + r = \beta^{-1}$ in the complete market model.
  - Suppose $\pi_0 \leq 0$.
  - Then, the government chooses $i_1 = 0$.
  - Then, $\pi_1 = \beta(1 + i_1) - 1 = \beta - 1 < 0$. Thus, the government chooses $i_2 = 0$.
  - By induction, $i_{t+1} = 0$ and $\pi_{t+1} = \beta - 1$ for all $t \geq 0$.
  - Then, the TVC is violated: $\lim_{t\to\infty} \beta^t b_t = \lim_{t\to\infty} \beta^t \frac{B_0}{(1+\pi)^t P_0} = \frac{B_0}{P_0} > 0$.
  - As TVC should be satisfied in equilibrium, π_0 should be larger than 0.
- The only possible equilibrium is $\frac{P_0}{P_{-1}} > 1$ and $\pi_t = \pi^*$ for all $t \geq 0$.

Incomplete market model: stationary equilibrium with $r = \tau = 0$

- In the incomplete market model, the SE with zero inflation can exist.
  - $i = \tau = r = 0 \quad \Rightarrow \quad \pi = 0$
  - $b = b_\infty$ is the value in SE with $\alpha = 3.0$ and $\tau = r = 0$.
  - The initial b_0 is the value in SE with $\alpha = 2.0$ and $r = 0.01$, where $b_0 < b_\infty$.

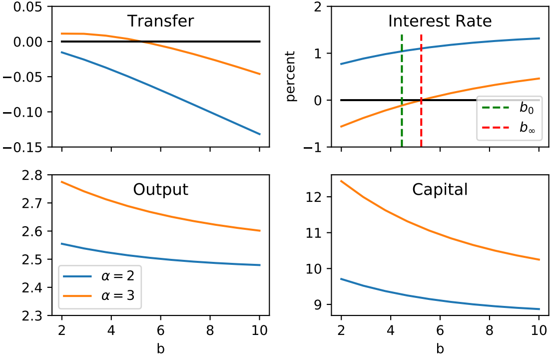

Figure: Stationary Equilibrium under different levels of real debt, for $\alpha = 2.0$ and 3.0

Incomplete market model: an unintended deflation (1/2)

- We can show numerically that there exists the following equilibrium path:
  - The economy was initially in Ω_{-1},
    where $b = b_0$, $\tau < 0$ (fiscal surplus > 0), $r > 0$, and $i = r$ and $\pi = 0$.
  - In response to the unexpected shocks (α changes from 2.0 to 3.0. Government makes the commtiment of Forward Guidance),
    the price P_0 jumps down
  - The economy converges to Ω_∞,
    where $b = b_\infty$, $\tau = 0$ (fiscal surplus $= 0$), $r = 0$, and $i = 0$ and $\pi = 0$.

Incomplete market model: an unintended deflation (2/2)

- In the incomplete market model, the following path can be the equilibrium:
  - $\frac{P_0}{P_{-1}} = 1 + \pi_0 \leq 1$,
  - $\pi_t < 0$ for all $t \geq 0$,
  - The government chooses $i_{t+1} = \tau_t = 0$ for all $t \geq 0$.

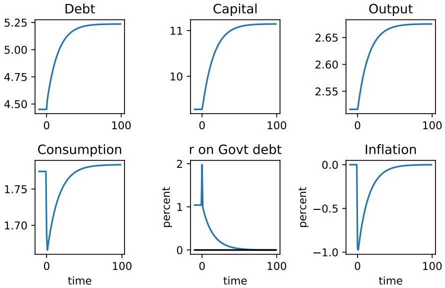

Figure: Transition dynamics to zero real interest rate equilibrium

- The inflation target π^* can never be attained.
  Moderate deflation or zero inflation continues.

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Conclusion

- Extreme monetary easing ($i = 0$) may induce persistent disinflation (i.e., zero inflation)
- Government deficit ($\tau > 0$) can continue indefinitely in the stationary equilibrium,
    - as long as τ is not too large.
- Low inflation (i.e., low π) and expansionary policy (i.e., low i and a nonnegative fiscal deficit) can coexist in the stationary equilibrium

Appendix

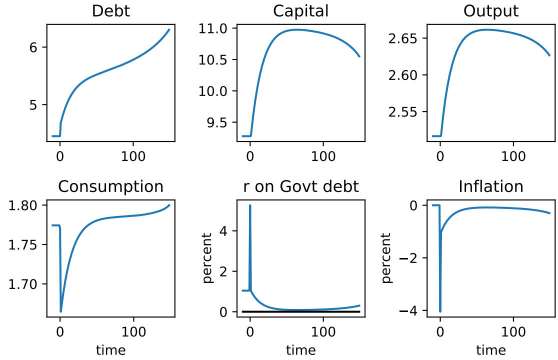

Figure: The case when P_0 is too low.

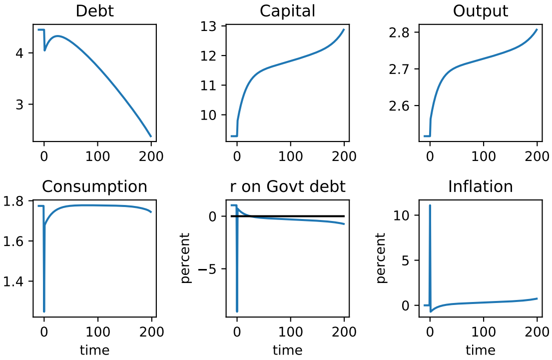

Figure: The case when P_0 is too high.