Endogenous Time Preference, and Sustainable Growth and Sustainable Development: An Outlook[*][**]

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Abstract

This paper makes a distinction between the analytical viewpoints of sustainable growth and sustainable development, which are derived from the concept of sustainability, and conducts a detailed review of the two major research papers that represent the milestones of research on environmental macroeconomics in recent years. Here, the analysis focuses on the point that the research itself introduces an endogenously determined time preference function with a relatively long history, into the dynamic framework of environmental macroeconomics. Research on macroeconomic dynamics has pointed out the importance of so-called "deep parameters." The rate of time preference is one of such parameters, and we can say that it has also continued to influence macro-theoretical research in environmental economics. The first model examines the impact of environmental taxes under endogenous time preference, in the context of sustainable growth being achieved. Next, the second model derives the dynamic features of the model when complex endogenous time preferences are considered, based on definitions that are rooted in the concept of sustainable development as the constancy of utility over time.

Keywords: endogenous time preference, sustainable growth, environmental tax (Pigouvian tax), sustainable development, Hartwick Rule, Hotelling Rule
JEL Classification: H23, I31, O44, Q32, Q56

I. Introduction

This paper provides a theoretical review of the "sustainable economy" based on the framework of macroeconomic dynamics (i.e., growth theory), mainly from environmental perspectives. We focus on the concept of endogenous time preference, which has been studied intensively in this field in recent years, as a warp thread running through our paper. The author has been conducting theoretical studies on macroeconomic dynamics and related empirical studies, and has been particularly interested in the analysis that bridges environmen-

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tal factors (e.g., environmental quality, exhaustible resources, among others) and the issue of the rate of time preference. Because environmental problems encompass long-term and very long-term issues, the author has been keenly aware of the importance of research that focuses on the rate of time preference, which influences people's present and future decision-making.

Professor Hirofumi Uzawa, well known worldwide for his "Uzawa's two-sector" and "Uzawa–Lucas model," has made truly outstanding contributions to the field of growth theory, and has established the two-sector growth model that can be regarded as the prototype of the endogenous growth model. Later, Professor Uzawa advocated the concept of social common capital, which has had considerable influences on environmental economics, health economics, education economics, and so on. For this paper, which deals with themes related to the environment and growth, Professor Uzawa's contributions are naturally positioned as the starting point. Incidentally, as is also well known among experts, the modern prototype of the idea of endogenous time preference has been developed by Professor Uzawa (Uzawa 1968, 2003). As already mentioned, "growth" and "endogenous time preference" are the technical keywords of the present study. Recognizing once again that the origin of both keywords is Professor Uzawa's contributions, one can only marvel at his foresight. It goes without saying that the two papers that form the core of the present review, as well as our related description itself, are themselves standing on the shoulders of Professor Uzawa.

Our task of this paper is to introduce in depth and organize recent characteristic studies that address sustainable growth and sustainable development, both concepts somewhat concretizing the notion of sustainability. It is necessary to start by clarifying these conceptual definitions as much as possible, but it is not so easy to describe sustainable development in a concise manner. Asako et al. (2002) indicated that these two are strictly different concepts and stated that "in general, sustainable growth concerns only an increase in income levels, while sustainable development is a broad concept involving not only income but also environmental quality and cultural levels" and refuse to go deeper into these differences than necessary in their analysis. In other words, sustainable development is a broader concept that includes not only economic status but also the actual state to conserve environmental resources and the way people and nations approach environmental protection.

Although it is relatively well known that the conceptual definition of sustainable development in the modern sense is based on the Brundtland Commission of the United Nations, according to Kato and Agatsuma (2012), it can be seen as the product of a compromise agreement between developed and developing countries, given the historical background. Namely, it calls for ensuring intergenerational and intragenerational equity and appropriate resource allocation. This is somewhat abstract, but it certainly provides a basic viewpoint for the theoretical investigation developed in this paper[1]. This is further broken down and organized in Table 1.

It should be noted that, as a premise for looking at Table 1, "diachronic nondecreasing nature" is basically assumed for each of the requirements. The substitutability of production inputs also indicates that whether substitutability between natural (resource) capital and physical/human capital is allowed or not[2]. Accepting the importance of all of these concepts, it is requirements 1 through 3 that have particular significance for an economic analysis such as the present paper. Perhaps the most useful requirements to consider when using economic models are 1 and 2. Requirement 1 roughly corresponds to a positive consumption growth over time, so it seems reasonable to make it a requirement for sustainable growth. On the other hand, although some difficulties can be anticipated, one promising idea is to consider requirement 2 as a prerequisite for sustainable development, and in fact, its origin can be traced back to Solow (1974)[3].

Table 1. Classification of sustainability concepts

	Requirements	Substitutability of production inputs
1：	Consumption is nondecreasing	○
2：	Utility is nondecreasing	○
3：	Resources are managed to maintain production potential	○
4：	Natural resource capital is nondecreasing	△
5：	The use of various resource flows must be maintained	×
6：	The stability and restoration of ecosystems must be maintained	×

Note: Based on Kato and Agatsuma (2012).

With this classification in mind, this paper examines two types of model analyses which can be considered as recent achievements in the field of environmental macroeconomics, both of which are distinct from the usual growth-oriented macroeconomic models in that environmental resources are included in both instantaneous utility function and endogenous time preference function of individual agents.

[1] Kato and Agatsuma (2012) provide a detailed and critical review of basic ideas about economic development/economic growth and the environment as they relate to the concept of sustainable development and are very informative. Fleurbaey (2015) classifies definitions of sustainability and gives a comprehensive literature survey in correspondence with them. A general and comprehensive discussion of the concept of sustainability is Arrow et al. (2004), which is the result of discussions among prominent researchers with different areas of expertise. Also useful is the *Handbook of Environmental Economics* (Volume 3) as an essential basic reference on the economic analysis of sustainability, especially Chapter 23 (Xepapadeas, 2005), which discusses the environment and growth.

[2] The view that basically allows for substitutability is generally referred to as *weak sustainability*. In contrast, *strong sustainability* describes a situation in which it is necessary to maintain a certain level of natural capital in order to ensure sustainability in the future. Therefore, in this case, a constraint is imposed on substitutability with natural capital. In this connection, Dasgupta (2019) points out that a bias toward weak sustainability has led to the depletion of natural capital and argues that strong rather than weak sustainability should be sought in the analysis of economic modeling involving the environment. Dasgupta also states that what is important to society is multidimensional and that sustainability should be enhanced by minimizing the tradeoffs among them. While this requires efforts to sustain the positive aspects of growth and prevent the negative aspects, Dasgupta emphasizes the need for collective action at all levels and the institutions making such action possible. Incidentally, Dasgupta (2019) provides a highly thought-provoking and comprehensive discussion on the key issues behind economic growth and sustainability.

[3] Some of the difficulties are discussed in detail in Asako et al. (2002).

Although the first model presented in Section III has advantages in terms of tractability, it is constructed as an *AK* model, and in that respect, the persistence of growth phenomenon (i.e., endogenous growth) can basically be viewed as being assumed in advance. However, the analysis implements an endogenous time preference function and focuses primarily on the effectiveness of environmental taxation, and it is therefore an important contribution to a better understanding of the growth process. In the classification of Table 1, this analysis is related to requirement 1, which clarifies the nature of sustainable growth including the environment.

The second model presented in Section IV is a study that attempts to derive macroeconomic conditions, including the characteristics of the time preference function, that would satisfy the condition of a constant utility level over time and across generations. In other words, it is a study directly related to requirement 2 in Table 1, and in this respect, the second model can be positioned as one that considers the issue of sustainable development in the framework of environmental-macro analysis.

The remainder of this paper is organized as follows. Section II provides a brief review of the literature drawing on the content of this paper and restricting it to the most recent contributions. In Sections III and IV, we examine in detail key studies related to sustainable growth and sustainable development, respectively. In Section V, instead of a summary, some future challenges to be imposed on the types of research presented in this paper will be mentioned.

II. Literature review

This section provides a brief review of previous studies as a prelude to the analysis after Section III. Because of the limited space, the review will be fragmentary. For more comprehensive reviews on macroeconomic analysis of the environment, including economic growth, see for instance, Xepapadeas (2005), mentioned earlier. Here, we would like to focus on relatively recent contributions.

Calling the implementation of the relationship in which the rate of time preference depends on the endogenous variables of the model the *endogenous time preference model*, there is a series of theoretical studies such as Strotz (1955), Uzawa (1968), Epstein (1987), Obstfeld (1990), and Becker and Mulligan (1997). Applications associated with specific topics have also been attempted, with Agénor (2010) introducing a health factor (health service consumption) and Dioikitopoulos and Kalyvitis (2015) considering the impact of human capital stocks on the rate of time preference. In addition, in recent years, various empirical studies based on questionnaire surveys and economic experiments have also been conducted to investigate the magnitude and determinants of the rate of time preference. The general understanding that this parameter fundamentally determines people's behavior is now growing, and the depth of the overall research field is expected to continue to gain in the future[4].

[4] For more details, including empirical studies, see for instance, Das (2003) and Hosoya (2023).

Macroeconomic analysis incorporating environmental factors has been conducted extensively, especially since the mid-1990s, and is expected to become increasingly active in the future, reflecting growing social concerns about the SDGs in recent years. In relation to growth, a type of analysis taking environmental factors into account in the production activities of firms (i.e., production function) can be considered. For example, specifications have been made in which renewable and nonrenewable resources (exhaustible resources) are input factors in the production function, or the quality of the environment affects productivity indicators and is consequently related to the production of goods (Groth and Ricci, 2011; Suphaphiphat et al., 2015). On the other hand, it is needless to say that the environment has an extremely large impact on human welfare (well-being). Some studies pay attention to this point and consider the environment as one of the components of the agent's utility function. For example, Klarl (2016) is a relatively recent contribution: in this model, the production process of human health is taken into account, and the instantaneous utility function of a representative household is formulated in a non-separable form with respect to consumption and health. The mechanism by which environmental pollution affects health is then incorporated. The studies focused on time preference examined in this paper can be regarded as a different extension of this type of research[5].

In this context, Weitzman (1994) highlighted the importance of time preference in environmental issues. Weitzman pointed out that it is not appropriate to assume a constant rate of time preference in the face of growing concern and anxiety about global environmental problems. In addition to this background specific to environmental economics, the importance of deep parameters that determine individuals' preferences in the analysis of macroeconomic dynamics has been recognized more strongly than before, and environmental macro analysis under endogenous time preference is considered to have come to be performed.

Nevertheless, there are not so many model analyses including endogenous time preference functions at this time. Although not a complete, a chronological list of prominent works includes Pittel (2002), Ayong Le Kama and Schubert (2007), Yanase (2011), Vella et al. (2015), Chu et al. (2016), and Hartwick and Long (2018)[6]. In this paper, we focus on more recent ones and examine them in detail: in correspondence with the previous section, one addresses economic performance in the context of sustained growth, and the other, in the tradition of Solow (1974), Dasgupta (1974), and Cairns and Long (2006), clarifies the basic requirements for sustainable development.

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[5] Due to the nature of the environment, it is natural to assume that it affects both production and utility, and in fact there are studies of this type (e.g., Fullerton and Kim, 2008; Chu and Lai, 2014). The models presented by Sections III and IV of this paper are also of this type.

[6] Very recently published Dioikitopoulos et al. (2020) use an endogenous time preference framework to examine the issue of *poverty traps* with respect to the environment and the economy in developing countries under certain circumstances. We also recently confirmed that Hartwick and Long (2020) has been published. This is a companion paper to Hartwick and Long (2018), which will be discussed in Section IV of this paper.

III. Endogenous time preference and sustainable growth

III-1. Basic setting

In this section, we take a typical model of achieving sustainable growth (endogenous growth) as an approach to sustainability and consider environmental policy under endogenous time preference setting. The resulting achievement of long-term growth while taking environmental factors into account illustrates one aspect of the sustainability of an economy and society. Specifically, we examine the model of Chu et al. (2016, *Macroeconomic Dynamics*).

Suppose that the firm's production activity inevitably emits pollutants, and that the polluting input, z, is indispensable for production. Now, z is called *dirty* input. Let the output be y and the production function of individual firms be $y = \Lambda k^{\alpha} z^{1-\alpha}$. Note that k is the physical capital held by the individual firms and $0 < \alpha < 1$. In this model, the sustainability of growth can be considered to derive from this specification. That is, $\Lambda \equiv AK^{1-\alpha}$ in which K is defined as the total capital stock in the overall macroeconomy and $A > 0$ is a constant parameter representing the level of technology. Although dirty input is taken into account, this can be regarded as a Sheshinski–Romer type production function[7]. Thus, the road to sustainable growth can be explained as follows: by introducing Λ in this way, one would expect the production function to be virtually the same as in the so-called AK model. Accordingly, there is a lower bound on the marginal productivity of capital, and this leads to endogenous growth. Therefore, as will be mentioned again later, this model does not exhibit transition dynamics as in the Solow model, and the economy grows along a balanced growth path from the beginning.

Let us now describe the firm's profit maximizing behavior. The profit function, π, is

$$\pi = y - (1 + \tau_k) rk - T_p z = \Lambda k^{\alpha} z^{1-\alpha} - (1 + \tau_k) rk - T_p z, \tag{1}$$

where r, τ_k, and T_p denote the rental cost of capital (the real interest rate), the tax rate imposed on capital, and the tax rate imposed on pollution (the environmental tax rate), respectively. To prevent pollutants from continuing to increase over time and eventually diverge, the environmental tax rate is assumed to vary with the total capital. We therefore specify $T_p = \tau_p K$ and τ_p is the environmental policy variable (parameter). In terms of the government, it can influence firm behavior by varying τ_p. The first-order conditions for profit maximization lead to

$$\frac{\partial \pi}{\partial k} = 0: \quad \alpha \Lambda k^{\alpha-1} z^{1-\alpha} = (1 + \tau_k) r, \tag{2}$$

[7] See Romer (1986). What z would become in the steady-state equilibrium (on the balanced growth path) will be mentioned later.

$$\frac{\partial \pi}{\partial z} = 0: \quad (1-\alpha)\Lambda k^{\alpha} z^{-\alpha} = T_p. \tag{3}$$

Next, we explain the utility function of a representative household. When considering intertemporal utility, the endogenous time preference function, which is the key factor of this paper, plays an important role. The use of dirty input in production is assumed to affect household utility in two ways: one is a direct negative impact on instantaneous utility itself, and the other is through the rate of time preference. The outstanding contribution of Chu et al. (2016) is to attempt a diverse and comprehensive investigation of the impact of the latter. The household's intertemporal (lifetime) utility function can be specified as

$$U = \int_0^{\infty} \frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma} \exp[-\Theta]\,\mathrm{d}t, \quad \eta > 0, \quad \sigma > 1, \tag{4}$$

where

$$\Theta \equiv \int_0^t \theta(z_s)\,\mathrm{d}s, \quad \dot{\Theta} = \theta(z), \quad \theta'(z) \gtreqless 0.$$

In equation (4), σ is the inverse of the intertemporal elasticity of substitution, and assuming its value to be greater than 1 is standard in light of previous studies, both theoretical and empirical[8]. The point of interest is, of course, the time preference function part. The basic literature on such endogenous specifications in the modern sense includes Uzawa (1968), Obstfeld (1990), and Becker and Mulligan (1997), and in particular, in Chu et al. (2016), the amount of dirty input used (i.e., environmental quality) determines the agent's present preference, "how much weight does the agent place on living in the present compared to the future[9]."

To be more specific, three cases are considered. The case, $\theta'(z) = 0$, implies an exogenously constant rate of time preference, and therefore is attributed to the standard neoclassical growth case (i.e., Ramsey–Cass–Koopmans model). Of interest are the remaining two cases. The first possible situation for subjective evaluation of environmental deterioration due to the use of dirty input is when the rate of time preference increases ($\theta'(z) > 0$). This corresponds to the environmental version of increasing marginal impatience (IMI). The assumption is that the resulting environmental deterioration makes people impatient[10]. Yanase (2011) and Vella et al. (2015) have similar specifications. In contrast, we can also assume a

8 Ayong Le Kama and Schubert (2007), a seminal study that also considers the quality of the environment and employs an endogenous time preference formulation, make the same assumption. In general, deep parameters such as the rate of time preference and the intertemporal elasticity of substitution play an extremely important role in dynamic general equilibrium models. For example, depending on the magnitude of σ, dynamically interesting phenomena such as multiple equilibria and indeterminacy may arise.

9 Hosoya (2023) provides a review including related classical studies.

10 Chu et al. (2016) noted that the prediction that global warming problems will have extremely severe impacts in the near future may increase the uncertainty of future consumption. People may then act to increase their current consumption and reduce their savings. This corresponds to a case in which people's present preference is strengthened by a change in their time preference.

situation where the rate of time preference decreases ($\theta'(z) < 0$), which can be interpreted as the environmental version of decreasing marginal impatience (DMI), describes behavior in which people worried about environmental worsening become concerned about, for instance, global environmental and economic sustainability and then turn to more patient preferences (i.e., weakening present preferences). Ayong Le Kama and Schubert (2007) take a similar view.

III-2. The decentralized economy

We now consider the problem of a representative household maximizes their lifetime utility under the endogenous time preference in equation (4) while subject to the budget constraint. The budget constraint can be expressed as $c + \dot{k} = rk + R$, where R is the lump-sum transfer from the government. The key to solving this optimization problem is that the household cannot be directly involved in dirty input or pollution emission level, and acts given z and thus given the rate of time preference Θ, which is affected by z. As a preliminary note, this is a point of difference in socially planned economies, discussed later. Let $\hat{\varphi}$ be the co-state variable for capital, the corresponding present-value Hamiltonian, H^d, is as follows:

$$H^d = \frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma}\exp[-\Theta] + \hat{\varphi}(rk + R - c). \tag{5}$$

The first-order conditions for optimum are listed below, but notice that $\varphi \equiv \hat{\varphi}\exp[\Theta]$, so $\varphi\exp[-\Theta] = \hat{\varphi}$:

$$\frac{\partial H^d}{\partial c} = 0: \ (cz^{-\eta})^{-\sigma} z^{-\eta}\exp[-\Theta] = \hat{\varphi} \rightarrow c^{-\sigma} z^{-\eta(1-\sigma)} = \varphi, \tag{6}$$

$$\dot{\hat{\varphi}} = -\frac{\partial H^d}{\partial k}: \ \dot{\hat{\varphi}} = -\hat{\varphi}r \rightarrow \dot{\varphi} = \theta(z)\varphi - r\varphi, \tag{7}$$

$$\hat{\varphi}: \dot{k} = rk + R - c, \tag{8}$$

$$\text{The transversality condition: } \lim_{t\to\infty} \hat{\varphi}k = 0. \tag{9}$$

As noted above, the household cannot be involved in the social determinants of the time preference rate Θ (i.e., the rate of time preference is endogenously determined from a social context). The government transfers the revenues, R, collected through the tax on capital imposed on firms and the environmental tax to the household in a lump-sum fashion[11]. Accordingly, the flow budget constraint for the government is as follows:

$$R = \tau_k rk + \tau_p kz. \tag{10}$$

We begin with the process of specifying the equilibrium of a decentralized economy

[11] The environmental tax rate (T_p) faced by individual firms is $\tau_p k$ proportional to the amount of capital (k) for each firm.

characterized by six endogenous variables $\{c, k, r, z, \varphi, R\}$. Applying $\Lambda = AK^{1-\alpha}$, $T_p = \tau_p K$, and the equilibrium condition $K = k$ (all firms are homogeneous and the number of firms is normalized to 1) to equation (3), we can find the dirty input level at the steady-state equilibrium, $\tilde{z} = [(1-\alpha)A/\tau_p]^{1/\alpha}$ (constant). Substituting this into the similarly transformed equation (2), the real interest rate at the steady state can be obtained as $\tilde{r} = [\alpha A/(1+\tau_k)][(1-\alpha)A/\tau_p]^{(1-\alpha)/\alpha}$ (constant). The remaining endogenous variables are four. Let us define three new variables: $x \equiv c/k$, $f \equiv \varphi k^{\sigma}$, and $q \equiv R/k$. Due to $\dot{c}/c = 1/\sigma[-(\dot{\varphi}/\varphi) - \eta(1-\sigma)(\dot{z}/z)]$ from equation (6) and $\dot{k}/k = \tilde{r} + q - x$ from equation (8), the following holds:

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{1}{\sigma}\left(-\frac{\dot{\varphi}}{\varphi} - \eta(1-\sigma)\frac{\dot{z}}{z}\right) - \tilde{r} - q + x.$$

Now, applying the condition for the steady-state equilibrium and taking into account some relations, we obtain $\tilde{x}$. A simple transformation also yields $\tilde{f}$ and $\tilde{q}$. All of these are summarized below:

$$\tilde{x} = A\tilde{z}^{1-\alpha} - \frac{1}{\sigma}\left(\tilde{r} - \theta(\tilde{z})\right),$$

$$\tilde{f} = \left(\frac{k}{c}\right)^{\sigma} \tilde{z}^{-\eta(1-\sigma)} = \left(\frac{1}{\tilde{x}}\right)^{\sigma} \tilde{z}^{-\eta(1-\sigma)} = \tilde{x}^{-\sigma}\tilde{z}^{-\eta(1-\sigma)},$$

$$\tilde{q} = \tau_k \tilde{r} + \tau_p \tilde{z}.$$

All the steady-state solutions are derived above. We then investigate the properties of equilibrium dynamics in a decentralized economy. Using the expressions $\dot{c}/c = [r - \theta(z)]/\sigma$ (the Euler equation) and $\dot{k}/k = r + \tau_k r + \tau_p z - x$ (the capital accumulation equation), the dynamics of consumption−capital ratio can be expressed as follows:

$$\frac{\dot{x}}{x} = \frac{(1-\sigma)r - \theta(z)}{\sigma} - \tau_k r - \tau_p z + x. \tag{11}$$

As confirmed above, r and z are determined by constant parameters only. Therefore, from equation (11), the following holds:

$$\frac{\partial(\dot{x}/x)}{\partial x} = 1 > 0.$$

This result shows that the right-hand side of equation (11) is an increasing function of x. That is, the decentralized equilibrium is a deterministic and hence locally unstable equilibrium. Therefore, there are no transition dynamics in this economy, and it grows along the balanced growth path from the beginning. From this, the following holds:

Proposition III. 1: The decentralized equilibrium is uniquely established and locally determined. There are no transition dynamics and the economy is always along the balanced growth path.

III-3. The effect of environmental tax in a decentralized economy

The model of Chu et al. (2016) implements an environmental tax (the Pigouvian tax), and how a change in the tax rate affects the economy is a typical topic in the analysis of the environment and growth. As a standard assumption, we now consider a situation in which the economy grows at a constant rate along a balanced growth path, but the use of dirty input, in other words, pollutant emissions, remains at stationary levels. Given the disutility caused by z, this is a natural assumption and a possible situation, for instance, in the context of a developed country. By definition, in the steady-state equilibrium with a balanced growth, both $\dot{k}/k = \dot{c}/c = \dot{y}/y = \tilde{g}$ (constant) and $\dot{z} = 0$ are satisfied.

Having already obtained the Euler equations, which can be obtained using equations (6) and (7) and $\dot{z}/z = 0$ (the condition for the steady-state equilibrium), and applying the expressions $\tilde{r}$ and $\tilde{z}$, the growth rate at the decentralized equilibrium, $\tilde{g}^d$ can be obtained as follows:

$$\tilde{g}^d = \frac{1}{\sigma}\left[\frac{1}{1+\tau_k} A\alpha \tilde{z}^{1-\alpha} - \theta(\tilde{z})\right]. \tag{12}$$

For this equation (12), using some relations, $\partial\tilde{g}^d/\partial\tau_p$ can be calculated[12]. As a result, the impact of changing the environmental tax rate on the growth rate is calculated as in equation (13).

$$\begin{aligned}\frac{\partial\tilde{g}^d}{\partial\tau_p} &= \frac{(1-\alpha)A}{\sigma\alpha\tau_p^2}\tilde{z}^{1-\alpha}\left(-\frac{\alpha\tau_p}{1+\tau_k}\right) + \theta'(\tilde{z})\frac{1}{\sigma\alpha\tau_p}\tilde{z} \\ &= \frac{(1-\alpha)A}{\sigma\alpha\tau_p^2}\tilde{z}^{1-\alpha}\left[-\frac{\alpha\tau_p}{1+\tau_k} + \theta'(\tilde{z})\right]. \end{aligned} \tag{13}$$

From equation (13), the following holds.

Proposition III. 2: When the properties of the time preference function are $\theta'(\cdot) < 0$ or $\theta'(\cdot) = 0$, an increase in the environmental tax rate reduces the growth rate. On the other hand, when $\theta'(\cdot) > 0$, the effect of the tax on the growth rate cannot be determined, and in some cases a growth-stimulating effect may occur.

The latter argument in Proposition III. 2 is particularly important for the result obtained under the increasing marginal impatience. It suggests that environmental protection and economic development may be compatible when agents have relatively impatient preferences (an increase in τ_p, $\tilde{z}$ decreases). Although the use of dirty input and environmental deterioration are parallel in this model, for agents whose present preferences are enhanced by deteriorating environmental quality, strengthening environmental tax to protect and improve the environment may increase the growth rate. The positive effect of environmental tax when

[12] For details, see Appendix A.

focusing on long-run growth is quite possible given the process of economic development and is a popular result in the literature on environment and growth. Nevertheless, it is the contribution of Chu et al. (2016) to show the multifaceted impacts of environmental tax under an endogenous time preference setting including environmental factors. This result also suggests that different specifications of the time preference function may lead to different consequences, this topic should continue to be explored in the future. For policy makers, the need to take into account this new factor, which should be called the *time preference effect*, emerges in response to the result that it governs the long-term outcome of tax policy.

The significance of time preference effect in environmental policy may be greater than our understanding. For example, the process of “dance” in the *hammer & dance* often discussed in the recent COVID-19 infection dynamics can be seen as a projection of the diversity of people’s time preferences. Although global environmental problems are probably a longer-term societal challenge than infectious disease epidemics, it is important to understand how to assume the collective attributions regarding people’s time horizons when making policy decisions.

III-4. The centralized (socially planned) economy

With a view to optimal environmental taxation, we now solve the optimization problem assuming the existence of an omniscient social planner. The focus of attention is on how it differs from the previous decentralized case. The objective function (utility function) is the same as before. The corresponding present-value Hamiltonian, H^{sp}, can be formulated as follows:

$$H^{sp} = \frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma}\exp[-\Theta] + \hat{\lambda}(y-c) - \hat{\mu}\theta(z), \tag{14}$$

where $\hat{\lambda} \equiv \lambda\exp[-\Theta]$ and $\hat{\mu} \equiv \mu\exp[-\Theta]$. Unlike a decentralized economy, the effect of dirty inputs is internalized. This appears in the last term on the right-hand side of equation (14). Let us derive the first-order conditions for optimum. Specifically, we require conditions on c (the flow variable), k (the stock ...), z (the flow ...), and Θ (the stock ...), and impose the transversality condition:

$$\frac{\partial H^{sp}}{\partial c} = 0:\ (cz^{-\eta})^{-\sigma}z^{-\eta}\exp[-\Theta] = \hat{\lambda} \to c^{-\sigma}z^{-\eta(1-\sigma)} = \lambda, \tag{15}$$

$$\dot{\hat{\lambda}} = -\frac{\partial H^{sp}}{\partial k}:\ \dot{\hat{\lambda}} = -Az^{1-\alpha}\hat{\lambda} \to \dot{\lambda} = \theta(z)\lambda - Az^{1-\alpha}\lambda, \tag{16}$$

$$\frac{\partial H^{sp}}{\partial z} = 0:\ (cz^{-\eta})^{-\sigma}(-\eta)cz^{-\eta-1}\exp[-\Theta] + \hat{\lambda}(1-\alpha)Akz^{-\alpha} = \hat{\mu}\theta'(z)$$

$$\to \quad -\eta c^{1-\sigma}z^{-\eta(1-\sigma)-1} + \lambda(1-\alpha)Akz^{-\alpha} - \mu\theta'(z) = 0, \tag{17}$$

$$\dot{\mu} = -\frac{\partial H^{sp}}{\partial \Theta}: \ \dot{\mu} = -(-1)\frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma}\exp[-\Theta]$$

$$\rightarrow \quad -\frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma} = -\dot{\mu} + \mu\theta(z), \tag{18}$$

$$\text{The transversality condition: } \lim_{t\to\infty} H^{sp} = 0. \tag{19}$$

The equilibrium in the centralized economy is characterized by equations (15)−(19). This form of the transversality condition is more general and is due to Michel (1982), and we use it directly later in the present study[13].

Let us examine the conditions, paying attention to the differences from the decentralized case. First, the social planner takes into account the externality of capital at the macro level included in Λ, as well as the social impact of dirty input (i.e., the social cost of pollution), to make the choice of k and z. These are shown in equations (16) and (17). The social planner then also takes into account the cumulative impact of using dirty input on the rate of time preference and treats it as an endogenous variable for decision making. The relevant condition for this is equation (18). To verify these two effects specifically, let us consider the standard neoclassical situation and set $\theta'(z) = 0$ (i.e., this implies an exogenously constant rate of time preference). Regarding equation (17), the third term on the left-hand side can be ignored, and the first term implies the negative impact of dirty input on the agent's welfare. The second term on the same side represents the positive effect of increased z on output. These are positive and negative effects for the entire economy, and can therefore be interpreted as the social planner faced with the difficult tradeoff of achieving sustainable development while embracing the concern for environmental preservation. Although this composition itself is the same in the case of exogenous time preference, in Chu et al. (2016), the level of z is an endogenous determinant of the rate of time preference, so the decision must be made under more complicated circumstances taking such a factor into account. Naturally, this has a significant impact on the optimal environmental taxation investigated below.

III-4-1. Existence and stability of socially optimal growth path

We now proceed to analyze the characteristics of the optimal growth path[14]. In preparation, we define the elasticity parameter as $\epsilon(z) \equiv z\theta'(z)/\theta(z)$, because the rate of time preference is a function of z. Note that there are now two state variables and the situation is more complex than in the decentralized economy. Following the original paper, we proceed to solve the problem using the Michel-style transversality condition in equation (19) as a breakthrough. From the condition that the present−value Hamiltonian becomes zero in the limit ($t\rightarrow\infty$), we obtain the following equation[15].

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[13] For details, see Acemoglu (2009), Barro and Sala-i-Martin (2003), and others.

[14] The original paper explains this subject in a little more than one page, including the main text and the Appendix, but one must be prepared for a bit of voluminous calculations. In this paper, we use Appendix A and attempt to explain them while clarifying the key points.

$$H^{sp} = \frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma} + \lambda(Akz^{1-\alpha} - c) - \mu\theta(z) = 0. \tag{20}$$

Here, λ and μ can be expressed as

$$\lambda = c^{-\sigma} z^{-\eta(1-\sigma)}, \tag{21}$$

$$\mu = \frac{-\eta c^{1-\sigma} z^{-\eta(1-\sigma)-1} + c^{-\sigma} z^{-\eta(1-\sigma)}(1-\alpha)Akz^{-\alpha}}{\theta'(z)}. \tag{22}$$

Substituting equations (21) and (22) into equation (20) and rearranging through a rather lengthy calculation yields equations (23)–(25)[16].

$$\frac{\dot{z}}{z} = \frac{x}{\Delta}\left[\left\{\eta + \frac{\sigma}{1-\sigma}\epsilon(z)\right\}\theta(z) - \{\sigma(1-\alpha) + \eta(1-\sigma)\}Az^{1-\alpha}\right], \tag{23}$$

where x and Δ are expressed as

$$x = \frac{[1-\alpha-\epsilon(z)]Az^{1-\alpha}}{\eta + \left(\frac{\sigma}{1-\sigma}\right)\epsilon(z)}, \tag{24}$$

$$\Delta \equiv \frac{\epsilon'(z)}{\epsilon(z)} z\sigma[(1-\alpha)Az^{1-\alpha} - \eta x] + \{\epsilon(z) - 1 + \alpha\}[\sigma(1-\alpha) + \eta(1-\sigma)]Az^{1-\alpha}. \tag{25}$$

By equations (23)–(25), the dynamic system for the socially planned economy is characterized. As previously mentioned, Chu et al. (2016) is nothing but an *AK* model in its model structure, so the properties of its equilibrium dynamics and balanced growth equilibrium are predictable, and thus we would like to confirm them explicitly. First, from equation (24) we obtain the condition which guarantees the positivity of x, that is $\epsilon(z) < \min[\eta(\sigma - 1)/\sigma, 1 - \alpha]$. In the model of Chu et al. (2016), this condition is assumed to be satisfied. Then, linearizing equation (23) around the steady-state equilibrium, the following expression is obtained:

$$\dot{z} = \tilde{x} \cdot (z - \tilde{z}), \tag{26}$$

where the tilde represents the steady-state value. Now $\tilde{x}$ is determined by equation (24), which is positive according to the previous discussion. Let ξ be an eigenvalue of the current dynamic system, then $\xi = \tilde{x} > 0$ from equation (26). Since the eigenvalue is positive and the dirty input, z, becomes a control variable due to it is a flow variable, the balanced growth equilibrium of the present socially planned economy is locally determinate (i.e., dynamically unstable). In other words, this economy is always on a uniquely determined balanced growth path. The result is summarized as follows:

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15 Using the expressions $\hat{\lambda}$ and $\hat{\mu}$, we obtain $H^{sp} = [\cdot]\exp[-\Theta]$. Based on the fact that $\exp[-\Theta] = 1$ in the limit, H^{sp} is attributed to equation (20).

16 See Appendix A for the derivations of equations (23)–(25).

Proposition III. 3: An equilibrium in a socially planned economy is uniquely determined and therefore exhibits local determinacy. There are no transition dynamics and the economy is always along the balanced growth path.

III-5. Optimal environmental tax structure

Using the equilibrium conditions of the socially planned economy, we can consider an optimal environmental tax structure. Comparing equation (15) with equation (6) for the decentralized economy case, we find that the necessary condition for obtaining the optimal growth path is $\lambda = \varphi$. The first step of the procedure is to obtain the steady-state growth rate (balanced growth rate) in the planning economy and compare it with the one in the decentralized economy. If a difference occurs, the tax structure as a means of adjustment is clarified and implications regarding the optimal taxation are derived. The growth rate in the present economy can be obtained as follows[17].

$$\tilde{g}^{sp} = \frac{1}{\sigma}[A\tilde{z}^{1-\alpha} - \theta(\tilde{z})] \tag{27}$$

Let us compare equation (27) with the corresponding result of equation (12) in the decentralized economy. That is, the condition for $\tilde{g}^d = \tilde{g}^{sp}$ can be expressed as

$$\tau_k^* = \alpha - 1 \tag{28}$$

The implication of (28) will be comprehensively examined later, and since $0 < \alpha < 1$, it follows that $\tau_k^* < 0$, which implies that a subsidy rather than a tax on capital is required to reach the social optimum. Namely, this result is related to the fact that capital is assumed to be a *clean* input as opposed to a dirty input.

We then consider specifically the optimal environmental tax structure. From the Euler equation for consumption and the budget constraint ($y = c + \dot{k}$), the following holds in the socially planned economy[18].

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{r - \theta(z)}{\sigma} - Az^{1-\alpha} + x \tag{29}$$

Applying the various conditions to equation (29) and rearranging them appropriately, the optimal environmental tax rate can be derived as follows[19].

$$\tau_p^* = \eta\frac{\tilde{x}}{\tilde{z}} + \frac{\theta'(\tilde{z})}{1-\sigma} \tag{30}$$

[17] For the derivation of equation (27), see Appendix A.
[18] Equation (29) corresponds to equation (D.5) in APPENDIX D of Chu et al. (2016).
[19] See Appendix A for the derivation of equation (30). In models with endogenous time preference, it is frequently the case that the main results, including the stability of the long-term equilibrium, depend on the intertemporal elasticity of substitution as well as on the rate of time preference. See, for example, Hosoya (2023).

Based on equation (30), let us examine the effectiveness of the Pigouvian taxation in light of its important goal of internalizing negative environmental externalities. To do so, we now employ the concept of marginal environmental damage (MED) as developed by Bovenberg and Goulder (1996)[20]. By contrasting declining utility due to the deterioration of environmental quality caused by pollution with increasing utility due to consumption activities, Chu et al. (2016) derive an optimal environmental tax rate to compensate (correct) for the worsening of utility level. The MED of pollution due to dirty input use in the current setting is defined as

$$D \equiv -\frac{\partial u / \partial z}{\lambda}. \tag{31}$$

Based on the definition of (31), we set $\widetilde{D} \equiv D/k$ to evaluate the MED at the steady-state equilibrium (along the balanced growth path). Using the relation of $D = \eta(c/z)$ from equation (15), the resulting MED is derived as

$$\widetilde{D} = \eta \frac{\tilde{x}}{\tilde{z}}. \tag{32}$$

From equations (30) and (32), we obtain $\tau_p^* = \widetilde{D} + \theta'(\tilde{z})/(1-\sigma)$. Accordingly, for the relationship between the optimal environmental tax rate, τ_p^*, and the MED, each case is determined according to the property of the time preference function (where $\sigma > 1$):

$$\begin{cases} \tau_p^* > \text{MED} & \text{if } \theta'(\tilde{z}) < 0, \\ \tau_p^* = \text{MED} & \text{if } \theta'(\tilde{z}) = 0, \\ \tau_p^* < \text{MED} & \text{if } \theta'(\tilde{z}) > 0. \end{cases}$$

These results are summarized as follows:

Proposition III. 4: In the case where the rate of time preference is exogenously given, the optimal environmental tax rate coincides with the Pigouvian tax rate (i.e., MED). In contrast, when the case is $\theta'(\tilde{z}) < 0$, the optimal tax rate is above the MED, whereas when the case $\theta'(\tilde{z}) > 0$, the tax rate is below the MED.

Comparing the two economic systems (decentralized and socially planned economies) so far, we can see that *distortions* arise from three types of externalities, which generate divergence between the two economic systems. The first is an externality caused by capital accumulation, which can be interpreted as Marshallian externality in the broad sense[21]. The

[20] In Bovenberg and Goulder (1996), the tax rate, τ_D^x, imposed on dirty inputs (intermediate inputs) is expressed as follows:

$$\tau_D^x = \left[\frac{\partial U/\partial Q(-\partial q/\partial x_D)}{\partial U/\partial C_C}\right]\frac{1}{\eta},$$

where Q, x_D, and C_C denote an environmental quality, dirty inputs, and clean consumption goods, respectively. Within this result, the square brackets part on the right-hand side represents the MED resulting from the use of dirty inputs.

second is an externality of pollution, which is derived from the use of dirty input, on an agent's instantaneous utility. Lastly, the third is an externality originated from pollution as well as the second, but which affects the rate of time preference[22]. As obtained as equation (28), we can conclude that the government should subsidize individual firms to correct for the first externality because the optimal tax rate on capital is negative. In other words, capital accumulation has a positive externality at a social level, but individual firms fail to recognize this benefit, resulting in insufficient capital accumulation under the decentralized economy.

Of particular interests in Chu et al. (2016) are the second and third externalities related to equation (30). For the moment, the second externality can be corrected if the policy maker sets the Pigouvian tax rate to compensate for the MED. This is a well-known general result. On the other hand, however, this has the following implication: even if the Pigouvian tax rate could be set based on the MED, it would be impossible to properly correct for the third distortion derived from the time preference. Thus, taking into account the situation where the rate of time preference is endogenously determined ($\theta'(z)\neq 0$), the Pigouvian tax cannot completely eliminate inefficiencies occurring in the current environment of the economy. This is a novel result from the introduction of endogenous time preference to the model and has important implications for the subsequent discussion of environmental policy.

However, the following view on dealing with externalities can be taken. Since Chu et al. (2016) depict a particular situation where only two policy instruments, subsidies for capital accumulation and the normal Pigouvian taxation, are used to address the three policy issues that require internalization, we can see that we are missing one instrument, as indicated by the "Tinbergen theorem." If additional policy measures could be implemented that could control for pollution externality stemming from time preference function, it should theoretically be possible to bring the performance of the decentralized economy to an optimal level. However, as one might expect from the previous discussion, it is quite difficult to take such an appropriate measure in practice. In any case, one must be willing to observe and evaluate the aggregate attribution concerning people's time preference and to think about how to deal with the facing issue based on the accumulated knowledge. In the case of using taxation as a measure of internalization, for example, it may require a Baumol−Oates style of idea, responding flexibly to environmental goals (Baumol and Oates, 1971).

In concluding this section, we should further elaborate on Proposition III. 4. The optimal determination of z by an omniscient social planner is straightforwardly expressed in equation (17). To begin, we consider the case of increasing marginal impatience ($\theta'(z) > 0$). In $-\eta c^{1-\sigma} z^{-\eta(1-\sigma)-1} + \lambda(1-\alpha)Akz^{-\alpha} - \mu\theta'(z) = 0$, the third term on the left-hand side is positive, since $\mu < 0$, as shown in Appendix A. This implies a benefit from an increase in z, and hence the social planner chooses a high level of z. If the optimal tax rate on capital $\tau_k^* = \alpha - 1$ is now implemented, the planner can make the growth rate in the decentralized economy

[21] In modern context, it is also called MAR (Marshall−Arrow−Romer) externality.

[22] As is clear from the previous explanations, agents in the decentralized economy take their optimal behavior given these externalities.

match the optimal growth rate. Given that the time path of consumption is identical, a higher rate of time preference should result in a higher welfare level for households. To account for this effect, the planner chooses a higher z, and thus a higher $\theta(z)$. As a result, in this case the optimal environmental tax rate is at a relatively low level relative to the Pigouvian tax level. In contrast, the case of decreasing marginal impatience, $\theta'(z) < 0$, can be explained as follows. Due to the fact that the third term on the left-hand side of equation (17) is negative, there will be a disadvantage caused by an increase z. The social planner takes this into account and chooses a low level of z. This suppresses the negative impact of an increase in dirty input on utility and allows households to reach a relatively high level of economic welfare. Consequently, the optimal environmental tax rate is at a relatively higher level relative to the Pigouvian level (i.e., the higher tax rate reduces the use of z)[23].

The most important point that emerges from the above analysis is that the effectiveness of the Pigouvian tax as an optimal tax scheme may be questionable under the endogenous time preference, which seems to be a more realistic way to describe the agent's economic behavior. This should be a new consideration for government to keep in mind when formulating their environmental policies. The model of Chu et al. (2016) is based on the *AK* -type production technology and is therefore classified as a sustainable growth (endogenous growth) model without transition dynamics. Within the context of the interests of this paper, this model makes an important contribution to understand one aspect of sustainability in that it depicts sustainable growth while taking into account the damages caused by environmental pollution[24]. In the research expected in this extension, analysis under different production technologies is naturally an interesting challenge, and at the same time, it is necessary to devise a more extended treatment of the situation in which the rate of time preference is influenced only by dirty input flow.

IV. Endogenous time preference and sustainable development

IV-1. Basic setting

In the model examined in Section III, there was only one type of capital, physical capital. In Hartwick and Long (2018, *Mathematical Social Sciences*), presented in this section, in contrast to conventional physical capital, which is positioned as man-made capital, natural capital is considered as the second type of capital (Krautkraemer, 1985; d'Autume and Schubert, 2008)[25]. According to Hartwick and Long (2018), two roles are assigned to natural

[23] As a matter of course, the point is that the functional form of the time preference still remains the same upon the choice of the social planner. Thus, in an economy composed of impatient people, the planner can enhance their welfare by choosing a higher level of z for those who have *strong* present preferences. In contrast, in an economy composed of patient and future-oriented people, the planner can enhance their welfare by choosing a lower level of z for those who have *weak* present preferences.

[24] The next section presents another type of study included in the macroeconomic analysis of sustainability with environmental considerations that focuses directly on the welfare level expressed by the utility function with natural resource.

[25] Hartwick and Long (2018) share much in common with the model structure of Chu et al. (2016) in the previous section, but for the convenience of readers in actually addressing the original papers, the notations follow those of the respective papers.

capital: one is an input factor to production, the natural resource input (flow variable) extracted from that stock and the other is the effects generated by that stock in aggregate, which provide amenity services to humankind in the sense of "comfort" and "gift of nature." We can enjoy these services by conserving the natural environment. Following the literature, natural capital is treated as a non-renewable and exhaustible resource in Hartwick and Long (2018)[26].

A continuum of infinitely lived individual agents, indexed by θ, are densely populated on $\theta \in [0, 1]$. Each agent is assumed to be endowed with a capital stock, $k(0, \theta)$, and an exhaustible natural resource stock, $x(0, \theta)$, at $t = 0$[27]. Although an exhaustible resource is inherently public good in nature, for simplicity, we assume that it is owned separately by each individual. Therefore, the sum of individual ownership is equal to the stocks in the economy as a whole. Since the agent obtains utility from final good consumption and amenity services provided by the owned resource stocks, we specify the utility function as $u[c(t, \theta), x(t, \theta)]$. This function is strictly increasing and strictly concave, satisfying $u_{cx} > 0$, $u_c[0, x] = \infty$, and $u_x[c, 0] = \infty$[28].

Let $q(t, \theta)$ now be defined as the flow of extraction from the individually held natural resource stock $x(t, \theta)$. Hence, the dynamic equation for the resource stock can be expressed as $\dot{x}(t, \theta) = -q(t, \theta)$[29]. In other words, because exhaustible resource is non-renewable, the extraction flow is subtracted from the stock. The individual agent sells the extracted resource at the market price $p(t)$.The firm purchases the resource and uses it as an input to final goods production $Y(t)$. Physical capital is also acquired by the firm at the rental price $r(t)$ and used in production activity. From the above, the total amount of extracted exhaustible resource $Q(t)$ and the total man-made capital stock $K(t)$ in the macroeconomy are represented as $Q(t) \equiv \int_0^1 q(t, \theta)\,\mathrm{d}\theta$ and $K(t) \equiv \int_0^1 k(t, \theta)\,\mathrm{d}\theta$ (i.e., the holdings of individuals are aggregated). Accordingly, the aggregate production function is $Y(t) = F[K(t), Q(t)]$. This is standard Cobb−Douglas form satisfying homogeneous of degree 1. In a perfect competitive economy, it is zero profit and the product exhaustion theorem holds. Each factor price is thus determined as $r(t) = F_K[K(t), Q(t)]$ and $p(t) = F_Q[K(t), Q(t)]$.

The income of each individual is obtained by the supply of capital and the sale of extractive resource to the firm, and thus $y(t, \theta) = r(t)k(t, \theta) + p(t)q(t, \theta)$ holds. Consequently, $\dot{k}(t, \theta) = y(t, \theta) - c(t, \theta)$ is the capital accumulation equation at the individual level. Incidentally, it is clear from the previous description that the total stock of natural resource in this economy can be expressed by definition as $X(t) \equiv \int_0^1 x(t, \theta)\,\mathrm{d}\theta$. In addition, given the individual capital accumulation equation above, the total consumption is $C(t) = F[K(t), Q(t)] - \dot{K}(t)$, and its growth rate is defined as $g(t) \equiv \dot{C}(t)/C(t)$.

[26] Old-growth forests and sand-stone cliffs are mentioned as examples in their paper.
[27] In the following, exhaustible natural resource stocks are sometimes referred to as "exhaustible resource," "natural resource stocks," "resource stocks," and so on, all of which are used with the same meaning.
[28] From $u_c[\cdot] = \partial u[c, x]/\partial c$ and $u_x[\cdot] = \partial u[c, x]/\partial x$, $\lim_{c\to 0} u_c[c, x] = \infty$ and $\lim_{x\to 0} u_x[c, x] = \infty$ are obtained.
[29] Note that the negative sign on the right-hand side of this equation.

IV-2. The decentralized economy

As in Section III, let us first focus on the case of the decentralized economy. Specifically, the individual agent chooses the time paths of consumption, $c(t, \theta)$, and extracted resource flow input, $q(t, \theta)$. Based on the above, the intertemporal utility function is formulated as follows:

$$U = \int_0^\infty u[c(t, \theta), x(t, \theta)]\beta(t)\,\mathrm{d}t,$$

where

$$\beta(t) \equiv \exp\left(-\int_0^t \phi[K(\tau), Q(\tau), X(\tau), C(\tau), g(\tau)]\,\mathrm{d}\tau\right).$$

Here, $\beta(t)$ is a utility discount factor, of which an instantaneous time preference function, $\phi(\cdot)$, is a function of the five macroeconomic variables $\{K, Q, X, C, g\}$[30]. As noted before, the definition of sustainability in Hartwick and Long (2018) is that a certain utility level is maintained across generations, which is one standard view in the relevant field. Accordingly, the main interest is to clarify the nature of the time preference such that this is satisfied.

To solve the corresponding decentralized optimization problem, we set up the following present-value Hamiltonian[31]. Note that the co-state variables associated with the state variables k and x are denoted by π and ψ, respectively.

$$J^d = u[c, x]\beta + \pi(rk + pq - c) - \psi q$$

The first-order conditions for optimization can be represented as follows. Note that equations (37) are the transversality conditions for the two state variables.

$$\frac{\partial J^d}{\partial c} = 0:\ u_c[c, x]\beta - \pi = 0, \tag{33}$$

$$\frac{\partial J^d}{\partial q} = 0:\ \pi p - \psi = 0, \tag{34}$$

$$\dot{\pi} = -\frac{\partial J^d}{\partial k}:\ \dot{\pi} = -\pi r, \tag{35}$$

$$\dot{\psi} = -\frac{\partial J^d}{\partial x}:\ \dot{\psi} = -u_x[c, x]\beta, \tag{36}$$

30 In the original paper, the dynasty model is mentioned here, and the utility discount rate is discussed in some detail, but this is omitted in the present paper for reasons of space limitation. Note that the original paper includes R as one of the five variables, but it is believed to be Q.

31 Hereafter, we simplify the notation and omit t and θ unless necessary.

$$\text{Transversality conditions: } \lim_{t\to\infty} \pi(t)k(t) = 0,\ \lim_{t\to\infty} \psi(t)x(t) = 0. \tag{37}$$

IV-2-1. The competitive equilibrium

Using the conditions obtained above, we clarify the properties established in the competitive equilibrium. First, for equation (33), taking a logarithm of both sides, we obtain (using the definition of β)

$$\ln u_c - \int_0^t \phi[K(\tau), Q(\tau), X(\tau), C(\tau), g(\tau)]\,\mathrm{d}\tau = \ln \pi. \tag{38}$$

Differentiating equation (38) with respect to time, the following characteristic relation can be derived[32].

$$\frac{u_{cc}\dot{c}}{u_c} + \frac{u_{cx}\dot{x}}{u_c} - \phi[K(t), Q(t), X(t), C(t), g(t)] = -r = -F_K. \tag{39}$$

By analogy with the Ramsey model, equation (39) is just the Euler equation for consumption.

Then, from equation (34), $\ln \pi + \ln p = \ln \psi$ holds. We can therefore derive the following relation[33].

$$\frac{\dot{\pi}}{\pi} + \frac{\dot{p}}{p} = \frac{\dot{\psi}}{\psi} \Leftrightarrow -r + \frac{1}{F_Q}\frac{\mathrm{d}}{\mathrm{d}t}F_Q = -\frac{1}{F_Q}\left(\frac{u_x}{u_c}\right) \tag{40}$$

Equation (40) can be regarded as a modified Hotelling Rule. We find that the rate of increase in the price of the extracted resource $(1/F_Q)(\mathrm{d}F_Q/\mathrm{d}t)$ is equal to the interest rate minus the adjusted marginal rate of substitution between consumption and resource amenity services. Based on Dasgupta and Heal (1974), for example, the basic literature in the field, the Hotelling Rule for exhaustible resource use asserts that the rate of increase in resource price equals to the marginal productivity of man-made capital (i.e., interest rate). Therefore, the new adjustment term added in the Hartwick and Long (2018) is $-(1/F_Q)(u_x/u_c)$. Compared to the standard model, this can be considered an adjustment due to the addition of resource amenity services to the utility function.

IV-2-2. Requirement for utility level to be constant across time

The concept of sustainability in Hartwick and Long (2018) is that the utility level is maintained constant over time, as described in Section I. That is, utility constancy generally requires a variable rate of time preference, and their work is positioned as an investigation to derive some requirements in this case. Denoting the constant utility level by $\overline{u}$, it is necessary to establish $u[c(t, \theta), x(t, \theta)] = \overline{u}$ at each point in time. Differentiating this with respect

[32] Equation (35) and $r = F_K$ were applied.
[33] For the derivation of equation (40), see Appendix B.

to time yields $u_c\dot{c} + u_x\dot{x} = 0$ with the right-hand side being zero, and the following equation holds:

$$\dot{x} = -\frac{u_c}{u_x}\dot{c}. \tag{41}$$

Substituting equation (41) into (39) and rearranging it using the notations of the elasticity of marginal utility (i.e., $\epsilon_{cc} \equiv cu_{cc}/u_c$ and $\epsilon_{xc} \equiv cu_{xc}/u_x$), we obtain $(\epsilon_{cc} - \epsilon_{xc})(\dot{c}/c) + r = \phi[\cdot]$. Letting $G(x, c) \equiv \epsilon_{xc} - \epsilon_{cc}$, the following is consequently derived:

$$-G(x, c)\frac{\dot{c}}{c} + r = \phi[K, Q, X, C, g]. \tag{42}$$

Since ϵ_{cc} is the elasticity of marginal utility of consumption with respect to individual consumption and ϵ_{xc} is the elasticity of marginal utility of amenity services with respect to individual consumption, taking into account $u_c > 0$, $u_{cc} < 0$, $u_x > 0$, and $u_{xc} > 0$, we find that $\epsilon_{cc} = cu_{cc}/u_c < 0$ and $\epsilon_{xc} = cu_{xc}/u_x > 0$. From the above properties, $G(x, c) > 0$ is obtained. These expressions of the elasticity can be rewritten as $cu_{cc}/u_c = \partial\ln u_c/\partial\ln c = (u_{cc}/u_c)\{1/(1/c)\}$ and $cu_{xc}/u_x = \partial\ln u_x/\partial\ln c = (u_{xc}/u_x)\{1/(1/c)\}$. Thus, with respect to the function $G(\cdot)$, the following is obtained:

$$G(x, c) = \frac{\partial(\ln u_x - \ln u_c)}{\partial\ln c} = \frac{\partial\ln\left(\frac{u_x}{u_c}\right)}{\partial\ln c}. \tag{43}$$

From equation (43), $G(\cdot)$ represents the elasticity of the marginal rate of substitution between the final consumption good and the amenity services with respect to individual consumption.

Now assume that the preferences of individuals are homogeneous and symmetric. Then, $G(x, c) = G(X, C)$ is established. This clarifies the property that the utility discount function (endogenous time preference function) needs to satisfy. Specifically, from equation (42), the utility level is constant over time if and only if the instantaneous time preference function, $\phi[K, Q, X, C, g]$, is equal to $F_K - G(X, C)g$, which corresponds to the marginal productivity of physical capital (man-made capital) at the macro level minus the product of the elasticity of the marginal rate of substitution and the consumption growth rate at the macro level. This result is summarized in the following proposition.

Proposition IV. 1: In a competitive equilibrium where the utility of all individuals is constant over time, the instantaneous time preference function, $\phi[\cdot]$, should be equal to $F_K - G(X, C)g$. Note that $G(\cdot)$ is the elasticity of the marginal rate of substitution between final consumption good and amenity services with respect to consumption, and g is the growth rate of consumption at the macro level.

In other words, utility is constant at $g = 0$ if and only if $\phi[\cdot]$ equals to F_K.

IV-2-3. An example

We would like to obtain more detailed results on the above general discussion by specifying the functional form. Let the utility function be a constant elasticity of substitution (CES) type. Consequently, in this case, $G(x, c)$ can be expressed as the inverse of the elasticity of substitution (σ) between amenity services and consumption. To confirm this, we set $u(c, x) = U[B(c, x)]$. Now assume $U[B(\cdot)]$ to be an increasing and concave function, and $B(c, x)$ to be a composite commodity expressed as follows:

$$B(c, x) = \left(\alpha x^{\frac{\sigma-1}{\sigma}} + \beta c^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \equiv Z^{\frac{\sigma}{\sigma-1}}, \tag{44}$$

where $\sigma > 0$ (constant). From equation (44), we can derive $u_x = U'[B]\alpha x^{-(1/\sigma)} Z^{1/(\sigma-1)}$ and $u_c = U'[B]\beta c^{-(1/\sigma)} Z^{1/(\sigma-1)}$. Taking the logarithm of both sides for these equations, we obtain

$$\ln u_x = \ln U'[B] + \ln \alpha - \frac{1}{\sigma}\ln x + \left(\frac{1}{\sigma-1}\right)\ln Z, \tag{45}$$

$$\ln u_c = \ln U'[B] + \ln \beta - \frac{1}{\sigma}\ln c + \left(\frac{1}{\sigma-1}\right)\ln Z. \tag{46}$$

Subtracting equations (45) and (46) at every side, we have

$$\ln u_x - \ln u_c = -\frac{1}{\sigma}(\ln x - \ln c) + \ln\left(\frac{\alpha}{\beta}\right). \tag{47}$$

Using equations (43) and (47), the following is derived:

$$\frac{\partial(\ln u_x - \ln u_c)}{\partial \ln c} = \frac{1}{\sigma} = G(x, c). \tag{48}$$

From equation (48), it can be seen that what was stated at the beginning of this section holds. In addition, when $\sigma = 1$, the utility function is reduced to a Cobb−Douglas form and $G(x, c) = 1$.

IV-2-4. On the obtained results

From the results of employing the CES-type utility function assumed in the above example, equation (42), and the assumption for symmetricity of agents, we obtain

$$\phi[K, Q, X, C, g] = F_K - \frac{1}{\sigma} g. \tag{49}$$

Sustainable development (i.e., the constancy of utility over time) requires that $\phi[\cdot]$ is determined so as to satisfy equation (49) at the macro level. The previous discussion suggests that, in order for society to compensate for the use of exhaustible resource at each point in time, investment in man-made capital must be undertaken to maintain productive capacity and guarantee sustainability (sustainable development). Sufficient savings to make this possible are also indicated.

Incidentally, from the perspectives of sustainability discussed in Section I, the analysis of Hartwick and Long (2018) provide one possible answer to the possibility of sustainable development. However, whether the constant utility level achieved is the highest feasible level is not known at this time. To check this, we need to solve an optimization problem assuming an omniscient social planner, as in Chu et al. (2016), discussed previously (Section III-4). The next section will do just that.

IV-3. Socially planned economy: The case of maximum constant utility level

The social planner seeks to maximize a constant stream of utility over time for the representative agent. Specifically, it is formulated as the problem of choosing the total consumption level, C, and the amount of exhaustible resource extraction, Q, to achieve a certain utility level, $\overline{u}$[34]. The corresponding optimization problem is formally presented as follows:

$$\max \overline{u},$$

subject to

$$\dot{K} = F[K, Q] - C, K(0) = K_0 > 0,$$

$$\dot{X} = -Q, X(0) = X_0 > 0,$$

$$u[C, X] - \overline{u} \geq 0.$$

Additionally, for the two stock variables, we impose $\lim_{t\to\infty} K(t) \geq 0$ and $\lim_{t\to\infty} X(t) \geq 0$.

In this optimization problem, the objective function is not in the usual integral sum form, but as shown in Cairns and Long (2006), the corresponding Hamiltonian, J^{sp}, is the sum of the time derivatives of the two state variables weighted by the related co-state variables. Let the co-state variables now be π^{sp} (for K) and ψ^{sp} (for X), we can then formulate. $J^{sp} = \pi^{sp}\dot{K} + \psi^{sp}\dot{X}$. Since $J^{sp} = 0$ for the Hamiltonian on the optimal path, it follows that

$$\pi^{sp}[F[K, Q] - C] - \psi^{sp}Q = 0. \tag{50}$$

From equation (50), the optimal investment level in man-made capital needs to satisfy the

[34] Although the term "maximin principle" never appears in Hartwick and Long (2018), readers who have studied microeconomics or game theory easily recognize that the optimization concept here is closely related to this principle. And in this background, there is a Rawlsian standard of value or fairness. In such a case, "the utility of future generations should not decrease in comparison with the present generations" is probably the most general concept of sustainability, and in response to this, it is technically reasonable to "choose the highest level from the paths where the utility level is constant across generations (i.e., maximin principle)." This paper does not deal with this issue head-on due to space limitation, but a more detailed understanding on this issue can be obtained by Cairns and Long (2006), citing directly below, as well as Asako (1980), Asako et al. (2002), Cairns and Martinet (2014), and Asako et al. (2015). In particular, Asako (1980) critically examines the theoretical issues in light of the early contributions by Solow (1974), Dasgupta (1974), and others on the application of maximin principle to environmental problems. In addition, the comprehensive discussion by Kato and Agatsuma (2012), already mentioned in Section I, is highly informative in deepening our understanding of the concept of sustainability.

following condition:

$$I \equiv \dot{K} = F[K, Q] - C = \frac{\psi^{sp}}{\pi^{sp}} Q. \tag{51}$$

Equation (51) is an important result given the economic interpretation of the co-state variable. Namely, we find that the economy-wide investment in physical capital must equal to the value of extracted exhaustible resource, requiring that any decrease in the exhaustible resource be compensated for by an investment in man-made capital[35]. This outcome is the Hartwick Rule itself, and the principle holds in the present endogenous time preference framework as well as in standard models approaching problems related to the environment and sustainable development (Hartwick, 1977; Asako et al., 2002)[36]. The point of arrival in Hartwick and Long (2018) is summarized as follows[37].

Proposition IV. 2: The highest level of utility path across time that a society can reach is theoretically possible in the decentralized economy (i.e., the equivalence of performance between decentralized and socially planned economies). Specifically, the intended objective is achieved when the rate of time preference of the representative agent is equal to the marginal product of capital ($F_K = r$) adjusted by the growth rate of aggregate consumption. At this point, the Hartwick Rule also holds on the optimal path.

IV-3-1. An example

In Hartwick and Long (2018), both the utility and production function are specified in Cobb−Douglas form, and the time paths of $\{K(t), Q(t), C(t)\}$, consumption growth rate g, and endogenous time preference rate φ are explicitly obtained that satisfy the Hartwick Rule and the modified Hotelling Rule and have the constant highest utility level over time (omitted in the present paper).

V. Concluding remarks

Based on the ideas of sustainable growth and sustainable development supporting the concept of sustainability, this paper has examined in detail two models that implement the endogenous time preference function and correspond to each of these ideas. As we have already discussed the important points everywhere, this section provides a few comments on the expected future developments based on these models.

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[35] From equation (34) in the decentralized economy, $\psi^{sp}/\pi^{sp} = \psi/\pi = p = F_Q$ holds.

[36] Within a path in which efficient resource allocation is achieved in the Pareto sense, the Hartwick Rule is a necessary condition for constant (nondecreasing over time) utility across generations. The Rule is certainly an important formal condition faithfully derived from theoretical models. However, as Asako et al. (2015) argue, it is also true that careful handling is required in the policy application of the Rule when natural capital, including exhaustible resource, is regarded as social common capital (Uzawa, 2003).

[37] In connection with Proposition IV. 2, see Appendix B for the details of the present optimization problem and its relation to the Hartwick Rule and the Hotelling Rule.

The results in Section III (Chu et al., 2016) using the *AK* model symbolizing sustainable growth, for example, would be shared in many aspects even if based on other endogenous growth models. However, other models would surely yield more interesting dynamic properties that could not be obtained with the simple *AK* structure without transition dynamics. One point where the results may differ significantly in some cases is when factors other than dirty input are also taken into account in the endogenous time preference function. In such a case, the choice of households may directly extend to the rate of time preference. As an example, there is a case in which the time preference function depends on the consumption level of individual households. In this respect, it is expected to extend the analysis[38].

The model in Section IV, presented by Hartwick and Long (2018), provides a lean and fundamental analytical framework for examining sustainable development under endogenous time preference, and is expected to become a basic reference in the relevant field. Therefore, various extensions can be expected, such as explicitly allowing for environmental externality. It would also be very fruitful to make the endogenous time preference function, which depends on five endogenous variables, more concrete and to connect it to empirical studies by utilizing numerical analysis.

What is common to the two studies examined in the present paper is how individual and collective factors interact in people's time preference, and the possible formulations that can be inferred from them are wide ranging. In the words of Dasgupta (2019) mentioned above, their preferences, which are directly related to human welfare, are multidimensional, and perhaps at their core lies the multidimensionality of time preference. The models introduced and closely discussed in this paper directly approach such a difficult problem and are of great significance. Nonetheless, we would like to point out that there are still many issues that need to be addressed from various perspectives, including the specification of the functional form of the time preference function, in order to advance more specific analysis for the purpose of deriving policy implications[39]. Exploration of such interesting topics should be an essential part of the future developments of environmental macroeconomics.

References

Acemoglu, D. (2009), *Introduction to Modern Economic Growth*, Princeton University Press

[38] A review of recent studies shows that even when introducing endogenous time preference, the rate of time preference is often assumed to be exogenously given for optimizing behavior of individual agents, perhaps from the viewpoint of simplifying the analysis. Although it depends on the situation setting, it is a somewhat strong assumption to believe that this parameter is completely exogenous, given people's actual perception and behavior. Exogeneity is certainly an appropriate assumption when only macro indicators such as environmental quality are incorporated into the functional relationship, but different specifications are possible in situations where the microfoundation of the agent's behavior should be considered. Out of the present context, but in a general sense with an empirical perspective, this is also indirectly involved in the issue of the relationship between individual time preference rate and social discount rate.

[39] As a collection of individual preferences, what kind of "functional form" the prevailing conditions of society are expressed, including politics and environmental policy, is an extremely important issue. For example, this issue has a great deal to do with the composition of the conflict between developed and developing countries. As a starting point for analyzing such an issue, one of the basic questions is how to empirically assess impatience and its extent.

Agénor, P.-R. (2010), “A Theory of Infrastructure-led Development”, *Journal of Economic Dynamics and Control*, Vol. 34 No. 5, pp. 932−950

Arrow, K., P. Dasgupta, L. Goulder, G. Daily, P. Ehrlich, G. Heal, S. Levin, K.-G. Mäler, S. Schneider, D. Starrett and B. Walker (2004), “Are We Consuming Too Much?” *Journal of Economic Perspectives*, Vol. 18 No. 3, pp. 147−172

Asako, K. (1980), “Economic Growth and Environmental Pollution under the Max-Min Principle”, *Journal of Environmental Economics and Management*, Vol. 7 No. 3, pp. 157−183

Asako, K., S. Kawanishi and T. Ono (2002), “Exhaustible Resources, the Environment, and Sustainable Growth” (in Japanese), *Economic Review*, Vol. 53 No. 3, pp. 236−246 (Institute of Economic Research, Hitotsubashi University)

Asako, K., K. Ochiai and Y. Ochiai (2015), *Graphic Environmental Economics* (in Japanese), SHINSEI-SHA Co., Ltd.

Ayong Le Kama, A. and K. Schubert (2007), “A Note on the Consequence of an Endogenous Discounting Depending on the Environmental Quality”, *Macroeconomic Dynamics*, Vol. 11 No. 2, pp. 272−289

Barro, R.J. and X. Sala-i-Martin (2003), *Economic Growth*, 2nd ed., MIT Press

Baumol, W.J. and W.E. Oates (1971), “The Use of Standards and Prices for Protection of the Environment”, *Swedish Journal of Economics*, Vol. 73 No. 1, pp. 42−54

Becker, G. and C. Mulligan (1997), “The Endogenous Determination of Time Preference”, *Quarterly Journal of Economics*, Vol. 112 No. 3, pp. 729−758

Bovenberg, L. and L.H. Goulder (1996), “Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses”, *American Economic Review*, Vol. 86 No. 4, pp. 985−1000

Cairns, R.D. and N.V. Long (2006), “Maximin: A Direct Approach to Sustainability”, *Environment and Development Economics*, Vol. 11 No. 3, pp. 275−300

Cairns, R.D. and V. Martinet (2014), “An Environmental-Economic Measure of Sustainable Development”, *European Economic Review*, Vol. 69, pp. 4−17

Chu, H. and C.-C. Lai (2014), “Abatement R&D, Market Imperfections, and Environmental Policy in an Endogenous Growth Model”, *Journal of Economic Dynamics and Control*, Vol. 41, pp. 20−37

Chu, H., C.-C. Lai and C.-H. Liao (2016), “A Note on Environment-dependent Time Preferences”, *Macroeconomic Dynamics*, Vol. 20 No. 6, pp. 1652−1667

Das, M. (2003), “Optimal Growth with Decreasing Marginal Impatience”, *Journal of Economic Dynamics and Control*, Vol. 27 No. 10, pp. 1881−1898

Dasgupta, P. (1974), “On Some Alternative Criteria for Justice between Generations”, *Journal of Public Economics*, Vol. 3 No. 4, pp. 405−423

Dasgupta, P. and G. Heal (1974), “The Optimal Depletion of Exhaustible Resources”, *Review of Economic Studies*, Vol. 41 No. 5, pp. 3−28

Dasgupta, P. (2019), “Economic Growth and Sustainability: The Multidimensionality of Social Progress” (in Japanese), *Economic Review*, Vol. 70 No. 3, pp. 253−270 (Institute of

Economic Research, Hitotsubashi University)

Dioikitopoulos, E.V. and S. Kalyvitis (2015), “Optimal Fiscal Policy with Endogenous Time Preference”, *Journal of Public Economic Theory*, Vol. 17 No. 6, pp. 848−873

Dioikitopoulos, E.V., S. Ghosh, C. Karydas and E. Vella (2020), “Roads to Prosperity without Environmental Poverty: The Role of Impatience”, *Economics Letters*, Vol. 186, 108870

d’Autume, A. and K. Schubert (2008), “Hartwick’s Rule and Maximin Paths When the Exhaustible Resource Has an Amenity Value”, *Journal of Environmental Economics and Management*, Vol. 56 No. 3, pp. 260−274

Epstein, L.G. (1987), “A Simple Dynamic General Equilibrium Model”, *Journal of Economic Theory*, Vol. 41 No. 1, pp. 68−95

Fleurbaey, M. (2015), “On Sustainability and Social Welfare”, *Journal of Environmental Economics and Management*, Vol. 71, pp. 34−53

Fullerton, D. and S.-R. Kim (2008), “Environmental Investment and Policy with Distortionary Taxes, and Endogenous Growth”, *Journal of Environmental Economics and Management*, Vol. 56 No. 2, pp. 141−154

Groth, C. and F. Ricci (2011), “Optimal Growth When Environmental Quality is a Research Asset”, *Research in Economics*, Vol. 65 No. 4, pp. 340−352

Hartwick, J.M. (1977), “Intergenerational Equity and the Investing of Rents from Exhaustible Resources”, *American Economic Review*, Vol. 67 No. 5, pp. 972−974

Hartwick, J.M. and N.V. Long (2018), “Sustainability with Endogenous Discounting When Utility Depends on Consumption and Amenities”, *Mathematical Social Sciences,* Vol. 95, pp. 31−36

Hartwick, J.M. and N.V. Long (2020), “Sustainability with Endogenous Discounting”, *International Journal of Economic Theory*, Vol. 16 No. 2, pp. 216−221

Hosoya, K. (2022), “Endogenous Time Preference, and Sustainable Growth and Sustainable Development: An Outlook” (in Japanese), *Financial Review*, No. 150, pp. 93−116

Hosoya, K. (2023), “The Effects of Patience in a Growth Model with Infrastructure and a Related Externality”, in S. Takahashi, M. Yanagihara, K. Hosoya and T. Shinozaki (eds), *Modern Macroeconomics with Historical Perspectives,* Springer Nature (forthcoming)

Kato, H. and N. Agatsuma (2012), “Economic Development, Growth and the Environment: Theory and Practice in Japanese Economic Plans and OECD’s Analyses” (in Japanese), *Journal of Tokyo Keizai University: Economics*, No. 275, pp. 19−57

Klarl, T. (2016), “Pollution Externalities, Endogenous Health and the Speed of Convergence in an Endogenous Growth Model”, *Journal of Macroeconomics*, Vol. 50, pp. 98−113

Krautkraemer, J.A. (1985), “Optimal Growth, Resource Amenities and the Preservation of Natural Environments”, *Review of Economic Studies*, Vol. 52 No. 1, pp. 153−169

Michel, P. (1982), “On the Transversality Condition in Infinite-Horizon Optimal Problems”, *Econometrica*, Vol. 50 No. 4, pp. 975−985

Obstfeld, M. (1990), “Intertemporal Dependence, Impatience, and Dynamics”, *Journal of*

Monetary Economics, Vol. 26 No. 1, pp. 45−75
Pittel, K. (2002), *Sustainability and Endogenous Growth,* Edward Elgar
Romer, P. (1986), "Increasing Returns and Long-Run Growth", *Journal of Political Economy*, Vol. 94 No. 5, pp. 1002−1037
Solow, R.M. (1974), "Intergenerational Equity and Exhaustible Resources", *Review of Economic Studies*, Vol. 41 No. 5, pp. 29−45
Strotz, R.H. (1955), "Myopia and Inconsistency in Dynamic Utility Maximization", *Review of Economic Studies*, Vol. 23 No. 3, pp. 165−180
Suphaphiphat, N., P. Peretto and S. Valente (2015), "Endogenous Growth and Property Rights over Renewable Resources", *European Economic Review*, Vol. 76, pp. 125−151
Uzawa, H. (1968), "Time Preference, the Consumption Function, and Optimum Asset Holdings", in J.N. Wolfe (ed), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh University Press
Uzawa, H. (2003), *Economic Analysis: Development Part* (in Japanese), Iwanami Shoten, Publishers
Vella, E., E.V. Dioikitopoulos and S. Kalyvitis (2015), "Green Spending Reforms, Growth, and Welfare with Endogenous Subjective Discounting", *Macroeconomic Dynamics,* Vol. 19 No. 6, pp. 1240−1260
Weitzman, M.L. (1994), "On the Environmental Discount Rate", *Journal of Environmental Economics and Management*, Vol. 26 No. 2, pp. 200−209
Xepapadeas, A. (2005), "Economic Growth and the Environment", in K.-G. Mäler and J.R. Vincent (eds), *Handbook of Environmental Economics*, Vol. 3, North−Holland
Yanase, A. (2011), "Impatience, Pollution, and Indeterminacy", *Journal of Economic Dynamics and Control*, Vol. 35 No. 10, pp. 1789−1799

Appendix

Appendix A. Details of Chu−Lai−Liao model

Derivation of equation (13)

Notice the expression, $\tilde{z} = [(1-\alpha)A/\tau_p]^{1/\alpha}$, we obtain

$$\frac{\partial \tilde{g}^d}{\partial \tau_p} = -\frac{\tau_p}{1+\tau_k}\frac{1}{\sigma}(1-\alpha)A\tilde{z}^{1-\alpha}\frac{1}{(\tau_p)^2} + \frac{1}{\sigma}\theta'(\tilde{z})\frac{1}{\alpha}\tilde{z}\left(\frac{(1-\alpha)A}{\tau_p}\right)^{-1}\frac{(1-\alpha)A}{(\tau_p)^2}$$

$$= \frac{(1-\alpha)A}{\sigma\alpha(\tau_p)^2}\tilde{z}^{1-\alpha}\left(-\frac{\alpha\tau_p}{1+\tau_k}\right) + \theta'(\tilde{z})\frac{1}{\sigma\alpha\tau_p}\tilde{z}.$$

Herein, using $[(1-\alpha)\mathrm{A}/\sigma\alpha(\tau_p)^2]\tilde{z}^{1-\alpha} = (1/\sigma\alpha\tau_p)\tilde{z}$, we arrive at equation (13) in the main text.

Derivation of equations (23)−(25)

Substituting the two co-state variables into equation (20) of the transversality condition and rearranging, we have

$$\frac{c^{-\sigma}z^{-\eta(1-\sigma)}}{1-\sigma}+c^{-\sigma}z^{-\eta(1-\sigma)}A\frac{1}{x}z^{1-\alpha}-c^{-\sigma}z^{-\eta(1-\sigma)}$$

$$+\frac{\theta(z)\eta c^{-\sigma}z^{-\eta(1-\sigma)}}{z\theta'(z)}-\frac{\theta(z)c^{-\sigma}z^{-\eta(1-\sigma)}(1-\alpha)Az^{-\alpha}}{\theta'(z)x}=0.$$

Expanding as, $B\equiv c^{-\sigma}z^{-\eta(1-\sigma)}$, we obtain

$$\frac{B}{\epsilon(z)}\left[\frac{\eta x-A(1-\alpha)z^{1-\alpha}}{xz}\right]=\frac{B[-\sigma x-A(1-\sigma)z^{1-\alpha}]}{(1-\sigma)xz}$$

$$\rightarrow\ -\eta x+\beta Az^{1-\alpha}=\epsilon(z)\left(\frac{\sigma}{1-\sigma}x+Az^{1-\alpha}\right), \tag{A1}$$

where $\beta\equiv 1-\alpha$. Moving to the next step. For this equation (A1), taking a logarithm of both sides and differentiating with respect to time, the following holds:

$$\frac{\epsilon'(z)}{\epsilon(z)}\frac{\dot{z}}{z}z+\frac{A(1-\alpha)z^{1-\alpha}}{\frac{\sigma}{1-\sigma}x+Az^{1-\alpha}}\frac{\dot{z}}{z}-\frac{\beta A(1-\alpha)z^{1-\alpha}}{\beta Az^{1-\alpha}-\eta x}\frac{\dot{z}}{z}=-\frac{\eta\dot{x}}{\beta Az^{1-\alpha}-\eta x}-\frac{\frac{\sigma}{1-\sigma}\dot{x}}{\frac{\sigma}{1-\sigma}x+Az^{1-\alpha}}.$$

Now from equation (A1) it follows that $\sigma x/(1-\sigma)+Az^{1-\alpha}=(\beta Az^{1-\alpha}-\eta x)/\epsilon(z)$, and by grouping them together in terms of $\dot{z}/z$ and $\dot{x}/x$, we obtain (using $\beta=1-\alpha$)

$$\left[\frac{\epsilon'(z)}{\epsilon(z)}z+\frac{\{\epsilon(z)-1+\alpha\}(1-\alpha)Az^{1-\alpha}}{(1-\alpha)Az^{1-\alpha}-\eta x}\right]\frac{\dot{z}}{z}=-\frac{x}{(1-\alpha)Az^{1-\alpha}-\eta x}\left[\eta+\frac{\sigma}{1-\sigma}\epsilon(z)\right]\frac{\dot{x}}{x}. \tag{A2}$$

Next, multiplying both sides of equation (15) by k^{σ} and using x, we have $-\sigma(\dot{x}/x)=(\dot{\lambda}/\lambda)+\sigma(\dot{k}/k)+\eta(1-\sigma)(\dot{z}/z)$. Substituting equation (16) and the budget constraint into this equation and rearranging, we obtain the following equation[40]. That is,

$$\frac{\dot{x}}{x}=-\frac{1}{\sigma}\left[\theta(z)-(1-\sigma)Az^{1-\alpha}-\sigma x+\eta(1-\sigma)\frac{\dot{z}}{z}\right]. \tag{A3}$$

In brief, using equations (A1)−(A3) and rearranging them, equation (23) is derived. To do so, equation (A2) is slightly rewritten[41]. This is denoted as equation (A2)′ below:

$$\left[\frac{\epsilon'(z)}{\epsilon(z)}z+\frac{\{\epsilon(z)-1+\alpha\}(1-\alpha)Az^{1-\alpha}}{(1-\alpha)Az^{1-\alpha}-\eta x}\right]\frac{\dot{z}}{z}$$

[40] Equations (A1)−(A3) correspond to equations (C.1)−(C.3) in APPENDIX C of Chu et al. (2016).

[41] For equation (A3) as well, the term $\dot{z}/z$ should be outside the square brackets.

$$= -\left[\frac{x}{(1-\alpha)Az^{1-\alpha}-\eta x}\cdot\frac{\eta(1-\sigma)+\sigma\epsilon(z)}{1-\sigma}\right]\frac{\dot{x}}{x}. \tag{A2$'$}$$

Substitute equation (A3) into the right-hand side of equation (A2)′ and proceed with the policy of putting terms that can be bracketed by $\dot{z}/z$ on the left-hand side:

$$\frac{\dot{z}}{z}\left[\frac{\epsilon'(z)}{\epsilon(z)}z+\frac{\{\epsilon(z)-1+\alpha\}(1-\alpha)Az^{1-\alpha}}{(1-\alpha)Az^{1-\alpha}-\eta x}-\frac{\eta\{x\eta(1-\sigma)+x\sigma\epsilon(z)\}}{\sigma\{(1-\alpha)Az^{1-\alpha}-\eta x\}}\right]$$

$$=\frac{\{x\eta(1-\sigma)+x\sigma\epsilon(z)\}\{\theta(z)-(1-\sigma)Az^{1-\alpha}-\sigma x\}}{\sigma(1-\sigma)\{(1-\alpha)Az^{1-\alpha}-\eta x\}}.$$

From equation (A1), we have $x\eta(1-\sigma)+x\sigma\epsilon(z)=(1-\sigma)Az^{1-\alpha}\{1-\alpha-\epsilon(z)\}$, and applying this to the above equation and solving for $\dot{z}/z$ yield

$$\frac{\dot{z}}{z}=\frac{\{x\eta(1-\sigma)+x\sigma\epsilon(z)\}\{\theta(z)-(1-\sigma)Az^{1-\alpha}-\sigma x\}}{1-\sigma}$$

$$\times\frac{1}{\frac{\epsilon'(z)}{\epsilon(z)}z\sigma[(1-\alpha)Az^{1-\alpha}-\eta x]+\{\epsilon(z)-1+\alpha\}[\sigma(1-\alpha)+\eta(1-\sigma)]Az^{1-\alpha}}.$$

The denominator of the fractional part of the latter half of the right-hand side is identical with the expression in equation (25) in the main text. From this, we obtain

$$\frac{\dot{z}}{z}=\frac{x}{\Delta}\left[\left\{\eta+\frac{\sigma}{1-\sigma}\epsilon(z)\right\}\theta(z)-\left\{\eta+\frac{\sigma}{1-\sigma}\epsilon(z)\right\}[(1-\sigma)Az^{1-\alpha}+\sigma x]\right].$$

Since $(1-\sigma)Az^{1-\alpha}+\sigma x=[(1-\sigma)/\{\eta(1-\sigma)+\sigma\epsilon(z)\}]\cdot[\sigma(1-\alpha)+\eta(1-\sigma)]Az^{1-\alpha}$ in this expression, equation (23) in the main text can be derived. Also, x is represented by equation (24).

Derivation of equation (27)

Find μ from the transversality condition of equation (19) for the socially planned economy:

$$\mu=\frac{1}{\theta(z)}\left[\frac{(cz^{-\eta})^{1-\sigma}}{1-\sigma}+\lambda(Akz^{1-\alpha}-c)\right]. \tag{A4}$$

Chu et al. (2016), as well as Ayong Le Kama and Schubert (2007), assumed $\sigma>1$ for the inverse of the intertemporal elasticity of substitution and stated $\mu<0$ holds. This is correct but strictly restated: for $\mu<0$, $\sigma>Akz^{1-\alpha}/(Akz^{1-\alpha}-c)>1$ is required. We will use later the negativity condition for this co-state variable. Substituting equations (15) and (A4) into equation (17), we obtain

$$-\eta\frac{\frac{c}{k}}{z}+(1-\alpha)Az^{-\alpha}=\frac{\theta'(z)}{\theta(z)}\left[\frac{\sigma}{1-\sigma}\frac{c}{k}+Az^{1-\alpha}\right],$$

where $c/k = x$. Evaluating this along the balanced growth path leads to equation (A5)[42].

$$-\eta\frac{\tilde{x}}{\tilde{z}}+(1-\alpha)A\tilde{z}^{-\alpha}=\frac{\theta'(\tilde{z})}{\theta(\tilde{z})}\left[\frac{\sigma}{1-\sigma}\tilde{x}+A\tilde{z}^{1-\alpha}\right] \tag{A5}$$

From equation (16), it follows $Az^{1-\alpha} = -(\dot{\lambda}/\lambda) + \theta(z)$, and considering $\dot{\lambda}/\lambda = -\sigma(\dot{c}/c) - \eta(1-\sigma)(\dot{z}/z)$ in equation (15), we apply them to equation (A5). Since $\dot{z}/z = 0$ on the balanced growth path, we use the notation for that case, resulting in equation (27) in the main text.

Derivation of equation (30)

Since consumption and capital grow at the same rate and therefore $\dot{x} = 0$ along the balanced growth path, we apply this to equation (29) and solve for $\tilde{x}$. Here, considering $\Lambda = AK^{1-\alpha}$ and the equilibrium condition $K = k$ to equation (2) and in light of the condition in equation (28), we obtain $r = A\tilde{z}^{1-\alpha}$. Substituting this into the formula for determining $\tilde{x}$, the following holds:

$$\tilde{x}=-\frac{A\tilde{z}^{1-\alpha}-\theta(\tilde{z})}{\sigma}+A\tilde{z}^{1-\alpha}. \tag{A6}$$

On the other hand, as we are analyzing here the relationship with the environmental tax rate, we can rewrite equation (3) using $T_p = \tau_p K$, the expression of Λ used above, and the equilibrium condition for capital as follows:

$$(1-\alpha)A\tilde{z}^{-\alpha}=\tau_p. \tag{A7}$$

Substituting equation (A7) for the left-hand side of equation (A5) and equation (A6) for $\tilde{x}$ on the right-hand side to rearrange, we finally obtain equation (30) in the main text, because the numerator in the square brackets of equation (A5) is $\theta(\tilde{z})$.

Appendix B. Details of Hartwick−Long model

Derivation of equation (40)

As $(\dot{\pi}/\pi) + (\dot{p}/p) = \dot{\psi}/\psi$, we apply equation (35) to the first term of the left-hand side. The second term follows from $p = F_Q$, so that $\dot{p}/p = (1/F_Q)(\mathrm{d}F_Q/\mathrm{d}t)$. On the right-hand side, $\dot{\psi}/\psi = -u_x\beta/\psi$ from equation (36) and $\beta = \pi/u_c$ from equation (33) lead to $\dot{\psi}/\psi = \{-u_x(\pi/u_c)\}/\psi$. In addition, since $\pi = \psi/p$ from equation (34), by using this, the following can be obtained:

[42] Equations (A4) and (A5) correspond to equations (D.1) and (D.3) in APPENDIX D of Chu et al. (2016). Equation (D.2) is omitted here. In addition, equation (D.4) in Chu et al. (2016) is the same as equation (27) in this paper.

$$\frac{\dot{\psi}}{\psi} = \frac{-\dfrac{u_x}{u_c}\dfrac{\psi}{p}}{\psi} = -\frac{u_x}{u_c}\frac{1}{p} = -\frac{1}{F_Q}\left(\frac{u_x}{u_c}\right).$$

From the above, we can confirm that equation (40) in the main text holds.

Details of the optimization problem associated with Proposition IV. 2

This is based on Cairns and Long (2006). The Hamiltonian, J^{sp}, and its associated Lagrangian, L, are formulated as follows:

$$J^{sp} = \pi^{sp}[F[K, Q] - C] - \psi^{sp}Q,$$

$$L = J^{sp} + \lambda[u[C, X] - \overline{u}],$$

where λ is the Lagrange multiplier and $\overline{u}$ is treated as a selectable control variable. From these, the necessary conditions for optimum can be shown as equations (B1)−(B7) below[43].

$$\frac{\partial L}{\partial C} = 0:\ -\pi^{sp} + \lambda u_C = 0 \tag{B1}$$

$$\frac{\partial L}{\partial Q} = 0:\ \pi^{sp}F_Q - \psi^{sp} = 0 \tag{B2}$$

$$\dot{\pi}^{sp} = -\frac{\partial L}{\partial K}:\ \dot{\pi}^{sp} = -\pi^{sp}F_K \tag{B3}$$

$$\dot{\psi}^{sp} = -\frac{\partial L}{\partial X}:\ \dot{\psi}^{sp} = -\lambda u_X \tag{B4}$$

$$1 - \int_0^\infty \frac{\partial L}{\partial \bar{u}}\,\mathrm{d}t = 0:\ 1 - \int_0^\infty \lambda(t)\,\mathrm{d}t = 0 \Leftrightarrow \int_0^\infty \lambda(t)\,\mathrm{d}t = 1 \tag{B5}$$

$$\lambda \geq 0,\ u[C, X] - \overline{u} \geq 0,\ \lambda[u[C, X] - \overline{u}] = 0 \tag{B6}$$

$$J^{sp}(t) = \pi^{sp}(t)[F[K(t), Q(t)] - C(t)] - \psi^{sp}(t)Q(t) = 0 \quad \forall t \tag{B7}$$

We obtain $F[\cdot] - C = (\psi^{sp}/\pi^{sp})Q$ from equation (B7) and $\psi^{sp}/\pi^{sp} = F_Q$ from equation (B2). Based on these, as equation (B8) below, one can derive the Hartwick Rule, which claims that investment in man-made capital is equal to the value of extracted exhaustible resource:

$$F[K, Q] - C = Q \cdot \left(\frac{\psi^{sp}}{\pi^{sp}}\right) = Q \cdot F_Q. \tag{B8}$$

Subsequently, we obtain $(\dot{\pi}^{sp}/\pi^{sp}) + (\dot{F}_Q/F_Q) = (\dot{\psi}^{sp}/\psi^{sp})$ from equation (B2). For the first

[43] Equations (B1)−(B7) correspond to equations (A.1)−(A.7) in the Appendix of Hartwick and Long (2018).

term on the left-hand side, we plug in equation (B3). For the right-hand side, we first derive $\dot{\psi}^{sp}/\psi^{sp} = -\lambda u_X/\psi^{sp}$ from equation (B4), and then substitute equation (B2) to this to obtain $\dot{\psi}^{sp}/\psi^{sp} = -\lambda u_X/\pi^{sp}F_Q$. Applying λ in equation (B1) to this, we have $\dot{\psi}^{sp}/\psi^{sp} = -(u_X/u_C)\cdot(1/F_Q)$. From the above, a modified Hotelling Rule is derived, which coincides with equation (40) in the main text.

$$-F_K + \frac{\dot{F}_Q}{F_Q} = -\frac{1}{F_Q}\left(\frac{u_X}{u_C}\right)$$