An Effect of Population Aging on the Effectiveness of Fiscal Policy: Analysis using a panel VAR model*

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Abstract
We examine how population aging changes the macroeconomic effects of fiscal policy using prefectural-level panel data in Japan. We select two groups from prefectures that are ranked in terms of the rates of population aging, and then estimate a panel VAR (vector autoregression) model for each group. In order to identify a structural shock to government spending and analyze its effects on the macroeconomy, we use sign restrictions, which are set consistently with computed impulse responses in a dynamic stochastic general equilibrium model. A key feature of this theoretical model is that it takes into consideration the presence of retirees in order to capture changes in demographics due to population aging. The estimation results indicate that aging leads to a decline in the effectiveness of spending. Moreover, we find that a way employee compensation responses also varies with aging. This implies that some effects of aging on labor markets may have contributed to the decline in the effectiveness of spending.

Key words: population aging, fiscal multiplier, dynamic general equilibrium model, panel VAR model

JEL Classification: C32, C33, E62

I. Introduction

In this study we examine how in Japan population aging changes the effect of government spending on economic activity. In Japan implications of aging population for the macroeconomy and policy have attracted a lot of attention. In terms of fiscal policy, in particular, much of this interest has been focused on long-run issues such as fiscal sustainability.¹

* This article is based on a study first published in the Financial Review No. 145, pp. 32-48, MORITA Hiroshi and NIWA Hidekazu, 2021, “An Effect of Population Aging on the Effectiveness of Fiscal Policy: Analysis using a panel VAR model” written in Japanese. We are deeply grateful to Naoyuki Yoshino, Professor Emeritus of Keio University, and other seminar participants at the Policy Research Institute, Ministry of Finance for their valuable comments. Any remaining errors are the sole responsibility of the authors.
Indeed, an increase in expenditures for social security due to population aging is expected to continue imposing a huge fiscal burden. Nonetheless, even in such a fiscal condition, we would not still be able to disregard the role of fiscal policy expected to play in stabilizing the macroeconomy, at least to some extent. In this sense, population aging enhances the importance of our examination in this study.

To accomplish our objective we conduct a panel vector autoregression (VAR) analysis using Japanese prefectural-level panel data. Specifically, we take two groups from prefectures that are raked in terms of the rate of population aging; a group of ones with higher rates of population aging, and a group of ones with lower rates population aging. We then estimate the fiscal multipliers for each group. Due to data limitation, we use annual data for our estimates and thus are unable to identify structural shocks using the strategy developed by Blanchard and Perotti (2002). It is difficult to assume that the government does not change its spending in response to an innovation of output within the same period. To resolve this problem, we use sign restrictions to identify a structural shock to government spending. In order to decide how to set these restrictions, we construct a New Keynesian dynamic stochastic general equilibrium (DSGE) model and compute impulse responses to exogenous shocks.

The DSGE model used to identify structural shocks builds on the model of Yoshino and Miyamoto (2017), combined with that of Galí et al. (2007). A key feature of Yoshino and Miyamoto’s (2017) model is that it takes into account the presence of retirees explicitly. They investigate the effects that demographic changes due to population aging have on the effectiveness of fiscal and monetary policy. Galí et al. (2007) extend the standard New Keynesian model to incorporate the presence of households with different marginal propensities of consume to analyze the effects of government spending. Specifically, in their model households are classified into the two types: Ricardian households, which have access to asset markets to smooth their consumption over time, and non-Ricardian households, which cannot access to asset markets. In our DSGE model households are classified into three categories. First of all, as in Yoshino and Miyamoto (2017), we classify them into workers and retirees. Then, following Galí et al. (2007), workers are classified into Ricardian and non-Ricardian. Retirees are assumed to have the same lack of access to asset markets as non-Ricardian workers.

Our estimation results indicate that population aging leads to a decline in the effectiveness of government spending. More precisely, the fiscal multiplier of output for the group of prefectures with lower rates of population aging exceeded that for the group with higher ones at the median. Moreover, when focusing on the multiplier of employee compensation for the two groups, we can observe a relationship similar to that of output. This implies that some effects of population aging on labor markets would have contributed to the decline in the effectiveness of spending.

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1 See e.g., Braun and Joins (2015) and Hansen and Imrohorogiu (2016) for studies on fiscal adjustments that will be required to stabilize government debt in Japan.

2 The solution to this identification problem proposed by them is to use quarterly data, as explained in detail later.
Our study is related to recent literature that explores how economic situations change the effectiveness of fiscal policy. Auerbach and Gorodnichenko (2012) present evidence that indicates the effects of government spending are different in expansions and recessions; since then, several studies have addressed this issue (e.g., Fazzari et al., 2015; Ramey and Zubairy, 2018). More recently, Ghassibe and Zanetti (2020) demonstrate that the efficacy of fiscal policy differs depending on whether the sources of economic fluctuations are demand- or supply-side.

The previous studies most closely related to ours are Yoshino and Miyamoto (2017) and Basso and Rachedi (forthcoming). Yoshino and Miyamoto (2017) develop the New Keynesian model with the aforementioned feature and demonstrate that aging leads to a decline in the effectiveness of fiscal and monetary policy. Basso and Rachedi (forthcoming) analyze the effects of demographic changes on the effectiveness of government spending by using data from the US. They show results like those in our study. Our empirical results would be helpful to gain a further insight about this topic.

The rest of the paper is organized as follows. In Section 2, we describe a DSGE model and then compute impulse responses to shocks in order to decide how to choose sign restrictions. In Chapter 3, after explaining the data, we construct a panel VAR model and give estimation results. In Section 4, we present our conclusions.

II. A New Keynesian DSGE Model

In this section we construct a New Keynesian DSGE model. The economy consists of households, firms, the fiscal authority, and the central bank. The main difference from a standard New Keynesian model is the presence of three types of households; Ricardian workers, non-Ricardian workers, and retirees, as explained above. The model economy is perturbed by four types of exogenous shock; a productivity shock, a discount factor shock, a government spending shock, and a monetary policy shock. We compute impulse responses to these shocks, which provide guidance on how to impose sign restrictions on responses of endogenous variables in the VAR model that we will estimate in the next section.

II-1. Households

II-1-1 Ricardian workers

Ricardian workers derive utility from their consumption $c_{w,t}^R$ and disutility from their labor supply $h_{w,t}^R$. Their expected lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \nu_t^\beta \{ \ln c_{w,t}^R + \chi \ln (1 - h_{w,t}^R) \},$$

where $E_t$ is the expectation operator conditional upon the information set available at time $t$, $\beta \in (0, 1)$ is the subjective discount factor, and $\nu_t^\beta$ is an exogenous shock to the subjective discount factor.

In line with Galí et al., (2007) we assume that the real wages are set by labor unions in a
centralized manner, and workers supply enough labor to meet the demand given these real wages. Ricardian workers are subject to a sequence of flow budget constraints:

\[ P_t c_{w,t}^R + P_t r_{w,t}^R + \frac{B_{w,t}}{R_t} = P_t w_t h_{w,t} + P_t r_t^R k_{w,t-1}^R + B_{w,t-1}^R + P_t D_t - P_t \tau_{w,t}^R , \]  

(2)

where \( P_t \) is the price level, \( w_t \) is the real wage, \( B_{w,t}^R \) is a risk-free one-period nominal government bond, \( R_t \) is the risk-free gross nominal interest rate, \( D_t^R \) is the dividends from firms, and \( \tau_{w,t}^R \) is lump-sum taxes imposed by the fiscal authority, \( k_{w,t-1}^R \) is the stock of private capital held by Ricardian workers, and \( r_t^R \) is the rental rate. Private capital accumulates according to:

\[ k_{w,t}^R = (1 - \delta) k_{w,t-1}^R + i_{w,t}^R , \]  

(3)

where \( \delta \) is the depreciation rate and \( i_{w,t}^R \) is investment in private capital.

Ricardian workers maximize their expected utility (1) subject to (2) and (3), which yields the following first-order conditions:

\[ \frac{V_t^\beta}{C_{w,t}^R} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} \frac{V_{t+1}^\beta}{C_{w,t+1}^R} \right] , \]  

(4)

\[ E_t [1 + r_{t+1}^R - \delta] = E_t \left[ \frac{R_t}{\Pi_{t+1}} \right] , \]  

(5)

where \( \Pi_t = P_t / P_{t-1} \) denotes gross inflation. Equation (4) is the consumption Euler equation and equation (5) is implied by absence of arbitrage opportunities.

### II-1-2 Non-Ricardian workers

Consumption and labor supply of non-Ricardian workers are represented by \( c_{w,t}^N \) and \( h_{w,t}^N \) respectively. Their period utility is given by

\[ \ln c_{w,t}^N + \chi \ln (1 - h_{w,t}^N) . \]  

(6)

They are subject to a sequence of flow budget constraints:

\[ P_t c_{w,t}^N = P_t w_t h_{w,t}^N - P_t \tau_{t}^N , \]  

(7)

where \( \tau_{t}^N \) is lump-sum taxes that non-Ricardian workers pay. Since they cannot access to assets market, they consume all of their labor income after taxes; \( c_{w,t}^N = w_t h_{w,t}^N - \tau_{t}^N \). We set a steady-state value of lump-sum taxes \( \tau_{t}^N \) so that the levels of consumption of all households are equalized in a steady state.

### II-1-3 Retirees

Each period, retirees receive social security benefits \( s_t \) from the fiscal authority and con-

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3 The structure of the labor market and how labor unions behave are explained in Section 2.3. In our model there is continuum of unions in the unit interval, and each of them set wages. We focus attention to a symmetric equilibrium in which they choose a same real wage, so that an index of unions to which workers belong is omitted.

4 Private and public capitals are assumed to depreciate with a same rate.
sume all of them:
\[ c_{t} = s_{r}. \]  
(8)

Social security benefits are assumed to remain constant over time. We choose its value to equalize levels of consumption between three types of households.

II-1-4 Economy-wide aggregation

Aggregate consumption is defined as the weighted sum of consumption of the three types of household:
\[ c_{t} \equiv (1 - \zeta) (1 - \zeta) c_{w,t}^{R} + \zeta (1 - \zeta) c_{w,t}^{N} + \zeta c_{r,t}. \]  
(9)

Aggregate hours worked is given by
\[ h_{t} \equiv (1 - \zeta) h_{w,t}, \]  
(10)

where \( h_{w,t} \equiv (1 - \zeta) h_{w,t}^{R} + \zeta h_{w,t}^{N} \). Since assets are held by Ricardian workers only, investment, private capital, outstanding government bonds, and dividends can be expressed as follows:
\[ i_{t} = (1 - \zeta) (1 - \zeta) i_{w,t}^{R}, \]  
(11)

\[ k_{t} = (1 - \zeta) (1 - \zeta) k_{w,t}^{R}, \]  
(12)

\[ B_{t} = (1 - \zeta) (1 - \zeta) B_{w,t}^{R}, \]  
(13)

\[ d_{t} = (1 - \zeta) (1 - \zeta) d_{w,t}^{R}, \]  
(14)

Finally, lump-sum taxes and social security benefits are given by:
\[ \tau_{t} = (1 - \zeta) \left[ (1 - \zeta) \tau_{w,t}^{R} + \zeta \tau_{w,t}^{N} \right], \]  
(15)

\[ s = \zeta s_{r}. \]  
(16)

II-2. Firms

II-2-1 Final goods producers

The final goods market operates under perfect competition. Final goods producers combine intermediate goods \( y_{j,t} \) into a homogeneous goods \( y_{t} \) according to a CES technology:
\[ y_{t} = \left( \int_{0}^{1} y_{j, t}^{\varepsilon_{p}} \right)^{\frac{1}{\varepsilon_{p}}}, \]  
(17)

where \( \varepsilon_{p} > 1 \) is the elasticity of substitution. Profit maximization yields the demand for the \( j \) th intermediate goods:
\[ y_{j,t} = \left( \frac{P_{j,t}}{P_{t}} \right)^{-\varepsilon_{p}} y_{t}, \]  
(18)

where \( P_{j,t} \) is the price of intermediate goods. We can also obtain the zero-profit condition:
\[ P_{t} = \left( \int_{0}^{1} P_{j,t}^{1 - \varepsilon_{p}} \right)^{1 - \varepsilon_{p}}. \]  
(19)

II-2-2 Intermediate goods producers

There is a continuum of intermediate goods producers in the unit interval. A variety \( j \) is
produced according to the production function:

$$y_{jt} = \exp\{z_t\} h_{jt}^a k_{jt}^{a_z} k_{gjt}^{a_k},$$

(20)

where $h_{jt}$ is labor input, $z_t$ is economy-wide productivity, and $k_{gt}$ is public capital. The cost minimization problem by intermediate goods producers implies the following condition:

$$\frac{k_{jt}}{h_{jt}} = \frac{aw_t}{(1-\alpha)r_t^k}.$$  

(21)

In addition, the marginal cost $mc_t$ which is common to all intermediate goods producers, is given by

$$mc_t = \frac{w_t}{(1-\alpha)\exp\{z_t\}} \frac{(1-\alpha)r_t^k}{aw_t}.$$  

(22)

We incorporate nominal price rigidities into the model following Calvo (1983). In each period, a fraction $1-\theta$ of firms are allowed to re-optimize their prices, while the remaining fraction $\theta$ of firms leave their prices unchanged. All firms revising their prices in period $t$ set the same price, denoted by $P^*_t$. They choose $P^*_t$ to maximize the present discounted value of profits:

$$E_t \sum_{t=0}^{\infty} \theta^t \left[ \Lambda_t \theta L_{t+1} y_{jt+1} \left( \frac{P^*_{jt}}{P_{t+1}} \right) - mc_{t+1} \right],$$

subject to the downward-sloping demand function (18), where $\Lambda_t \equiv \beta^l (c_{t+1}^R/c_t^R)^{1-\epsilon} \equiv \lambda_t$ is the stochastic discount factor. The first-order condition for this problem is:

$$P^*_{jt} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{E_t \sum_{t=0}^{\infty} \theta^t \Lambda_{t+1} \theta L_{t+1} y_{jt+1} mc_{t+1}}{E_t \sum_{t=0}^{\infty} \theta^t \Lambda_{t+1} \theta L_{t+1} y_{jt+1}}.$$  

(23)

Finally, it can be seen from equation (19) that the price level $P_t$ evolves according to:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{1-\epsilon}.$$  

(24)

II-3. Wage-setting by labor unions

In line with Galí et al. (2007), we assume that the labor market operates under imperfect competition. There is a continuum of labor unions in the unit interval. Each of them represents the interests of type $l$ worker and chooses the real wage they face, where $l \in [0,1]$. It is assumed that Ricardian and non-Ricardian workers are uniformly distributed across all unions. We define efficient labor employed by an intermediate goods producer $j$, $h_{jt}$, by using a CES function:

$$h_{jt} = \left( \int_0^1 h_{jt}(l) \frac{\epsilon_l}{\epsilon_r} dl \right)^{\epsilon_l^{-1}/\epsilon_r^{-1}},$$

(26)

where $\epsilon_e$ is the elasticity of substitution between two types of labor. All firms $j \in [0,1]$ demand equal labor for type $l$ workers. We thus can express the demand function for type $l$ labor, $h_l(l)$, as follows:
In addition, firms are assumed to uniformly allocate their labor demand for workers in a same union, regardless of whether they are Ricardian or non-Ricardian. Given the structure of the labor market explained above, a labor union \( l \) chooses the real wage \( w(l) \) to maximize the weighted average of the period utility of type \( l \) workers:

\[
(1 - \xi) \left[ \frac{w_l(l) h_l(l)}{c^R_{w,l}(l)} + \ln(1 - h_l(l)) \right] + \frac{\xi}{\chi} \ln\left( \frac{w_l(l) h_l(l)}{c^N_{w,l}(l)} + \chi \ln(1 - h_l(l)) \right),
\]

subject to (27). Deriving the first-order condition and imposing the symmetricity conditions \( w_l(l) = w_i \) and \( h^R_l(l) = h^R_i(l) = h_i \), we obtain the following optimal wage schedule:

\[
\frac{1 - h_{w,t}}{\chi} \left[ \frac{1 - \xi}{c^R_{w,t}} + \frac{\xi}{c^N_{w,t}} \right] w_t = \frac{\varepsilon_w}{\varepsilon_w - 1}.
\]

### II-4. Fiscal and monetary authorities

The fiscal authority raises lump-sum taxes from workers and issues bond to finance government spending \( g_t \) and the payments for social security benefits. The government budget constraint is then given by:

\[
P_t \tau_t + \frac{B_t}{R_t} = B_{t-1} + P_t g_t + P_s.
\]

(30)

Defining \( \hat{\tau}_t \) and \( \hat{b}_t \) as \( \hat{\tau}_t \equiv (\tau_t - \tau)/Y \) and \( \hat{b}_t \equiv [(B_t/P_t) - (B/P)]/Y \) respectively, we set the tax rule as follows:

\[
\hat{\tau}_t = \phi \hat{b}_{t-1}.
\]

(31)

Variables without the subscript \( t \) indicate a steady-state value of the corresponding variable.

The government is assumed to allocate all of its spending to investment in public capital, implying that public capital \( k_{g,t} \) accumulates according to:

\[
k_{g,t} = (1 - \delta) k_{g,t-1} + g_t.
\]

(32)

In addition, the government decides its spending according to the rule:

\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + \phi \hat{y}_t + e^g_t.
\]

(33)

where \( \rho_g \) satisfies \( 0 < \rho_g < 1 \). \( e^g_t \) is an exogenous shock to government spending, which follows a normal distribution with mean 0 and variance \( \sigma^2_g \). Here, spending is assumed to be countercyclical. Since we use annual data for the empirical analysis in the next section, we cannot deny the possibility that the government changes its spending in response to an innovation of output within the same period. In order to incorporate such a possibility into the DSGE model, we adopt this specification.

Finally, we assume that the central bank controls the short-term nominal interest rate ac-

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5 Except for these variables, for any generic variable \( X \), we define \( \hat{X} \) as the deviation of the logarithmic of that variable from its steady state, i.e., \( \hat{X} \equiv \log (X_t/X) \).
according to the simple rule:

\[ R_t/R = (\Pi_t/\Pi)^\phi \exp(v_t^m), \tag{34} \]

where \( \phi \) denotes the sensitivity of the nominal interest rate to inflation and \( v_t^m \) is an exogenous shock to monetary policy.

II-5. Market-clearing conditions

The final goods market clearing requires:

\[ y_t = c_t + i_t + g_t, \tag{35} \]

and the equilibrium conditions of labor and private capital markets are given by:

\[ h_t = \int_0^1 h_{jt} dj \tag{36} \]
\[ k_t = \int_0^1 k_{jt} dj \tag{37} \]

II-6. Dynamics of exogenous shocks

We assume that the exogenous shocks other than government spending shock in the DSGE model, \( z_t, v_t^m \) and \( v_t^\beta \), follow the below stochastic processes respectively:

\[ \dot{z}_t = \rho_z \dot{z}_{t-1} + \epsilon_t^z \tag{38} \]
\[ \dot{v}_t^m = \rho_v \dot{v}_{t-1}^m + \epsilon_t^m \tag{39} \]
\[ \dot{v}_t^\beta = \rho_v \dot{v}_{t}^\beta + \epsilon_t^\beta \tag{40} \]

II-7. Parameter settings

In this study, in order to identify structural shocks, we use sign restrictions. To do this we use the method called “robust sign restriction” that Pappa (2009) develops. We randomly generate the values of some parameters in the DSGE model within certain ranges and compute impulse responses to the exogenous shocks for each combination of parameter values. In order to decide how to set sign restrictions, we use robust signs in the sense that under any combination of parameter values same sign of responses are implied. As explained above, we use annual data and thus are unable to identify structural shocks by imposing a constraint that the government does not change its spending in response to an innovation in output within the same period. This prevents us from identifying structural shock to spending by using the strategy developed by Blanchard and Perotti (2002). The use of sign restrictions is an effective way to resolve this identification problem.

A further advantage of this methodology is that by changing parameter values within wide ranges, we can mitigate possible effects that misspecification of the DSGE model has on the precision of the estimate of the VAR model, at least to some extent. For example, by changing the degree of price stickiness and the proportion of non-Ricardian households within wide ranges, we can decide how to impose sign restrictions taking into account not
only the Galí et al.’s (2007) parameterization but also other various possibilities.

Table 1 summarizes the values or ranges of the parameters used in the estimate. Within these parameters, we choose particular values for the discount rate $\beta$, the depreciation rate of capitals $\delta$, the share of private capital $\alpha$, the elasticity of taxes to outstanding debt $\phi_b$, debt-to-output ratio denoted by $\gamma_b$, government spending-to-output ratio denoted by $\gamma_g$, and the proportion of retirees $\zeta$. $\beta$ is chosen so that the discount rate is approximately 0.99 in models with quarterly data. The depreciation rate is set to $\delta = 0.1$, referring to Esteban-Pretel et al. (2011). This is in line with Yoshino and Miyamoto (2017). $\phi_b = 0.1$ is also used by them. We set the proportion of retirees to $\zeta = 0.25$, which is the average of all prefectures in the sample periods. The values of the other parameters are set to be roughly consistent with Japanese data.

On the other hand, we permit the remaining parameters to vary within plausible ranges. The value of $\alpha$, which determines the marginal productivity of public capital, ranges from 0 to 0.2. The upper bound is set referring to the value reported by Miyagawa et al. (2013), which is 0.16. The proportion of non-Ricardian households ranges from 0.1 to 0.5. Kohara and Horioka (2006) report that this value lies between 0.08 and 0.15 by using micro data, while Morita (2015) estimates this by using macro data to report a much higher value 0.47.

Table 1. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tbody>
<tr>
<td>$\beta$</td>
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<tr>
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<tr>
<td>$\alpha$</td>
<td>0.33</td>
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<tr>
<td>$\phi_b$</td>
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<td>$\gamma_b$</td>
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<tr>
<td>$\gamma_g$</td>
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</tr>
<tr>
<td>$\zeta$</td>
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</tr>
<tr>
<td>$\alpha_g$</td>
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<tr>
<td>$\xi$</td>
<td>[0.1, 0.5]</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>[6, 11]</td>
</tr>
<tr>
<td>$\theta$</td>
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<tr>
<td>$\phi_{\pi}$</td>
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<tr>
<td>$\phi_{y}$</td>
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<tr>
<td>$\rho_i, i \in (g, z, v, \beta)$</td>
<td>[0.6, 0.8]</td>
</tr>
</tbody>
</table>

Note: Parameter values and its ranges are set to be consistent with the literature.
The markup rate $\frac{\epsilon_p}{(\epsilon_p - 1)}$ ranges in values often used in the literature. The upper bound of $\theta$ is chosen so that the Calvo probability is approximately 0.875 in models with quarterly data, which is the value estimated by Sugo and Ueda (2008). Its lower bound is chosen so that the probability is approximately 0.75 in models with quarterly data, which is the value often used in the literature.\(^6\)

**II-8. Sign restrictions**

In the simulation, we randomly generate each parameter value 1,000 times from the aforementioned ranges and compute the impulse response functions for each combination of parameter values.\(^7\)

Figure 1 shows the responses of (a) government spending, (b) output, (c) the price level, and (d) employee compensation in response to the four types of exogenous shock. The structural shock in which we are interested is government spending shock only, but following Peersman (2005) we confirm the impulse responses to other types of shock in the DSGE

![Figure 1. Theoretical impulse response functions](image)

Note: The figure shows the theoretical responses of the variables to the four types of structural shock. The shaded areas are corresponding to the 90% bands for theoretical impulse response functions.

\(^6\) To be precise, \(0.875^4 \approx 0.58\) and \(0.75^4 \approx 0.31\).

\(^7\) Since we randomly generate the combinations of parameter values, under some of them unrealistic impulse responses are implied. We therefore removed the top and bottom 5% of all impulse responses.
model in order to identify structural shocks. The shaded areas show the ranges bounded by the 5% and 95% quantiles of the impulse response functions. Employee compensation is defined as the sum of the responses of the real wage and labor. Owing to date limitation only employment compensation rate is available, so that we also calculate its response to exogenous shocks.

The simulated results are summarized as follows. First, an increase in government spending leads to increases in output, the price level, and employee compensation. Second, a positive productivity shock raises output and employee compensation and lowers the price level. An increase in output due to a productivity shock leads to a decrease in government spending. Third, in response to a monetary tightening shock, we can observe an increase in spending and decreases in output, the price level, and employee compensation. Finally, in response to a shock to the discount factor output and employee compensation fall and expenditure and the price level increase.

These results provide sufficient information to identify the four types of shock. For example, the result that output responds positively to shocks to both government spending and productivity but the price level responds negatively to the former and positively to the later allows us to distinguish the two shocks. We can also identify the other types of shock in a similar way. In the next section, we estimate a VAR model by using sign restriction in order to examine how the efficacy of government spending changes with population aging. Table 2 summarizes how to set these restrictions. In addition, we impose these restrictions on the responses of the variables in a VAR model only in the first period.

Table 2. Sign restrictions

<table>
<thead>
<tr>
<th></th>
<th>gov. spending</th>
<th>output</th>
<th>prices</th>
<th>compensation</th>
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</thead>
<tbody>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
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<td>-</td>
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<td>$\varepsilon^y_t$</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon^\beta_t$</td>
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<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Sign restrictions are set based on the theoretical impulse responses. We impose them on the variables for one period (one-year) in the empirical analysis we conduct in Section 3.

### III. Empirical Analysis

#### III-1. Data

In this subsection, we explain data used to estimate the VAR model described later. The prefecture-level data used in this study contains the following elements: government spending, real gross domestic product (output), deflator (the price level), and employee compen-
sation. The source of the data is Prefectural Economic Accounts (*Kenmin Keizai Keisan*). Government spending is defined as the sum of government consumption and government investment. Deflator is defined as the ratio of prefectural nominal GDP to prefectural real GDP, which is used to express employee compensation in real terms. The data are annual from 1990 to 2014, taken from 93 SNA. During this period the benchmark year revision has been conducted in 1995, 2000, and 2005. We construct time-series data from 1990 to 2014 by using the growth rates of variables in each benchmark year.

A notable feature of this study is that we divide the sample of prefectures in advance and then estimate a VAR model as in Ilzetzki et al., (2013) in order to examine how the macro-economic effects of government spending change with population aging. Specifically, we calculate a sample average of ratio of the of population aged 65 and older to the total population for each prefecture using the data from the Statistics Bureau of the Ministry of Internal Affairs and Communications. We then take two groups from them: the group of the 12 highest-ranked prefectures, and the group of the 12 lowest-ranked prefectures. From now on, the former is called the group of the top 12, and the latter is called the group of the bottom 12. The group of the top 12 includes Akita, Yamagata, Nagano, Wakayama, Tottori, Shimane, Yamaguchi, Tokushima, Ehime, Kochi, Oita, and Kagoshima. The group of the bottom 12 includes Miyagi, Ibaraki, Tochigi, Saitama, Chiba, Tokyo, Kanagawa, Aichi, Shiga, Osaka, Hyogo, and Okinawa.

### III-2. A panel VAR model with sign restrictions

As explained above, we estimate a panel VAR model for each group. The VAR model includes the following endogenous variables: government expenditure ($g_t$), output ($y_t$), the price level ($p_t$), and employee compensation ($w_t$). The variables other than the price level are measured in per capita. In addition, all variables are expressed in logarithm.

Denoting the vector of the endogenous variables for each prefecture $i$, $i = 1, 2, \ldots, N$ as $Y_{it} = (g_{it}, y_{it}, p_{it}, w_{it})$, we describe the panel VAR model as

$$
Y_{it} = \sum_{s=1}^{p} B_s Y_{it-s} + u_{it} (t = 1, \ldots, T), \quad u_{it} \sim N(0, \Sigma),
$$

where $B_s$ is the vector of coefficients on the lagged variables, $u_{it}$ is the vector of reduced-form residuals, and $\Sigma$ is the variance-covariance matrix of the residuals. We assume that all prefectures share the same coefficients and variance-covariance structure. In addition, we incorporate fixed effects and linear terms into the model, but to simplify the expression they are omitted. The lag order is set to $p = 2$. As in the standard VAR analysis, we assume that there is a linear relationship between the reduced-form residuals and the structural shocks:

$$
u_{it} = A \epsilon_{it},
$$

where $\epsilon_{it}$ is the vector of the structural shocks, uncorrelated with each other. The variance of each structural shock is normalized to one.

To simplify the notations, we define $\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{u}$ as follows. First of all, $\mathbf{Y} \equiv [Y'_1, \ldots, Y'_T]'$, $Y_t = [Y'_1t, \ldots, Y'_Nt]$, $\mathbf{X} \equiv [X'_1, \ldots, X'_T]'$, $X_t = [X'_1t, \ldots, X'_Nt]'$, $X_{it} = I_k \otimes (Y_{it-1}, \ldots, Y_{it-p})$, $\mathbf{u} \equiv [u'_1, \ldots, u'_T]'$, and $u_{it} \equiv [u'_1t, \ldots, u'_Nt]'$. We further define the coefficients in the VAR model as $\Theta \equiv$
Using the newly defined variables, the panel VAR model system can be rewritten as the form of the linear regression model:

\[ \Psi = X\Theta + \mu, \quad \mu \sim N(0, I_{T \times N} \otimes \Sigma). \]  

(43)

In this study, following Uhlig (2005), we use the sign restrictions to identify structural shocks to government spending. The identification process consists of the following two steps. First, we randomly generate the VAR coefficient \( \Theta \) and the variance-covariance matrix of the reduced-form residuals \( \Sigma \) to obtain candidate draws for them:

\[ \Sigma^{-1} \sim W \left( \hat{\Sigma}_{ols}^{-1}, T \times N \right), \]

(44)

\[ \Theta \sim N \left( \hat{\Theta}_{ols}, \left( X' (I_{T \times N} \otimes \Sigma) \right)^{-1} X \right), \]

(45)

where \( \hat{\Theta}_{ols} \) and \( \hat{\Sigma}_{ols} \) denote the coefficients and variance-covariance matrices obtained by estimating (43) using the ordinary least squares, and \( W(\cdot) \) denotes the Wishart distribution.

In the second step, we randomly generated a matrix \( W \) from the standard normal distribution, which is QR-decomposed to gives a matrix \( Q \) such that \( W = QR \). \( Q \) is an orthogonal matrix that satisfies \( QQ' = I \). We can then express the relationship between the endogenous variables in a period

\[ u_{it} = A_0 Q \hat{\varepsilon}_{it}, \]

(46)

where \( A_0 \) is the lower triangular matrix obtained by the Cholesky decomposition of the variance-covariance matrix \( \Sigma \), and \( \hat{\varepsilon}_{it} \) is the vector of the structural shocks obtained by the decomposition. Finally, defining \( A = A_0 Q \) and \( \varepsilon_{it} = Q' \hat{\varepsilon}_{it} \) produces a relationship between the reduced-form residuals and the structural shocks identical to equation (42). Note that it can be confirmed that under \( A \) and \( \varepsilon_{it} \) constructed here the original variance-covariance structure is maintained. Indeed, the fact that \( QQ' = I \) guarantees that:

\[ E[A\varepsilon_{it} \varepsilon_{it}' A'] = E[A_0 Q\hat{\varepsilon}_{it} \hat{\varepsilon}_{it}' Q Q'A_0'] = A_0 A_0' = \Sigma. \]

(47)

Under these generated combinations (\( \Theta \), \( \Sigma \), \( A \)), we calculate the impulse response functions and check whether or not they satisfy the sign restriction. Only when all of them are satisfied, we regard the combination as the valid sample. For every combination of \( \Theta \) and \( \Sigma \) generated in the first stage, we generate \( A \) 5,000 times in the second stage. This process is repeated until we can obtain 500 valid samples.

III-3. Estimate results

Figure 2 displays the impulse responses given by the estimate of the panel VAR model for each group. The solid lines plot the medians of the sampled impulse response functions, and the shaded regions indicate the 68% credibility intervals.\(^8\) The first column shows the impulse response functions in the group of the top 12 and the second column shows the impulse response functions in the group of the bottom 12.

Estimation results can be summarized as follows. First, the responses of all variables in

\(^8\) It should be noted that since we use the Bayesian techniques for the estimation, the regions are not “confidence intervals”.
period 0 to the structural shock to government spending, to which the sign restrictions are imposed, are positive. Focusing on output from period 1 onward, in the group of the bottom 12 the significantly positive response is observed until period 3. On the other hand, in the group of the top 12, this turns to be insignificant in period 3. Similar trends are also evident.

Figure 2. Estimated impulse response functions

Note: The figure shows the impulses of each variable in response to shocks to government spending (i) in the group of the top 12 and (ii) in the group of the bottom 12. The solid lines indicate the medians, and the shaded areas represent the 68% credibility intervals.
in the responses of employee compensation. In the group of the bottom 12 employee compensation shows a significant response until period 3. In the group of the top 12, though its response is significantly positive until period 2, it becomes insignificant in period 3 and 4, and significantly negative in period 5.

The analysis so far shows that the positive responses of output and employee compensation are more persistent in the group of the bottom 12 than in the group of the top 12. However, since the dynamic responses of government spending themselves differ between the two groups, a direct comparison between the impulse responses in Figure 2 is inappropriate for understanding how differently the structural shocks to spending affect the other variables. To perform this comparison in an appropriate way, we calculate the fiscal multipliers for output and employee compensation. The cumulative fiscal multiplier after $J$ period from the onset of the structural shock to spending is defined as

$$\frac{\sum_{j=0}^{J} IR_j(x)}{\sum_{j=0}^{J} IR_j(g)} \times \frac{\bar{x}}{\bar{g}},$$

(48)

where $x$ denotes output or employee compensation, $IR_j(\cdot)$ indicates the impulse response of the corresponding variable in period $j$, and $\bar{x}$ and $\bar{g}$ are the average of corresponding variables over simulated samples. The fiscal multiplier measures how many units the corresponding variable changes in response to one unit increase in government spending. This enables us to compare the effects of the structural shock to spending in the two groups even when the dynamic responses of spending differ between them.

Figure 3 illustrates the fiscal multipliers of output and employee compensation. The dotted lines with circles represent the medians of multipliers for the group of the top 12, and areas between dotted lines indicate 68% credibility intervals. Those for the group of the bottom 12 are represented by solid lines and shaded areas, respectively. Focusing on their medians, the fiscal multiplier for the group of the bottom 12 is always larger than that for the group of the top 12. More specifically, the fiscal multiplier of output for the group of the

![Figure 3. Fiscal Multipliers](image)

Note: The figure shows the fiscal multipliers for (i) output and (ii) employee compensation. The dotted lines with circles and the areas between the dotted lines represent the medians and 68% credibility intervals of the fiscal multipliers in the group of the top 12, respectively. Those in the group of bottom 12 are represented by solid lines and shaded areas, respectively.
bottom 12 is always greater than one, while those for the group of the top 12 are below one from period 3 onward. This result implies that population aging would have a negative effect on the effectiveness of government spending.

To understand the logic behind this result, it is instructive to consider how population aging affects the fiscal multiplier of employee compensation with the help of the DSGE model described in Section 2. The fiscal multipliers for employee compensation have trends qualitatively similar to those for output. This implies the possibility that some effects of population aging on labor markets would have contributed to the decline in the effectiveness of spending. Indeed, in the DSGE model described in Section 2, a rise in proportion of retirees dampens the response of total labor to changes in wages. Therefore, we can interpret that our empirical results imply that the population aging weakens the responses of consumption and labor supply to government spending, thereby leading to the decline in its macroeconomic effect. Strictly speaking, it should be noted that since employee compensation is defined as the product of hours worked times the real wage, we cannot capture effects of population aging only on hours worked.

In addition, we need reservations about statistical significance. Since the credibility intervals for the two groups overlap, we cannot find statistically significant difference between the responses in the two groups. In this study for simplicity we assume that in the VAR model all prefectures share the same coefficients on the lagged variables and variance-covariance structure. A possible way to improve statistical precision is to estimate a panel VAR model with a hierarchical structure following Pappa (2009). We leave this for future research.

### IV. Conclusion

By conducting a sign-restriction VAR analysis using Japanese prefectural-level panel data we examine how population aging changes the effectiveness of government spending. In order to decide how to set these restrictions we construct a DSGE model that takes into account the presence of retirees explicitly and compute impulse responses to exogenous shocks. The estimation results imply that the population aging leads to a decline in the effectiveness of spending. In addition, we find that the population aging also weakens the response of employee compensation to spending. This implies that some effects of population aging on labor markets would have contributed the decline in the macroeconomic effects of spending.

Future research can address how the population aging affects labor markets in more detail in order to clarify a mechanism behind the decline in fiscal multipliers. It is also an important task to develop a theoretical model that embeds this mechanism. Such a model would enable us to investigate implications for fiscal policy in an aging society.

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9 This can be seen by substituting the definition of total hours worked (10) into the optimal wage schedule (29).

10 Nonetheless, some notable difference can be found. One example is that, in the group of the top 12 the response of employee compensation turns to insignificant in period 5, while this is not the case in the group of the bottom 12.
References


Appendix I. Steady state

We log-linearize the equilibrium conditions of the DSGE model around the steady state with zero inflation. In the steady state the following relationships hold.

Gross nominal interest rate
\[ R = 1/\beta \]

Real rental rate
\[ r^k = R - 1 + \delta \]

Aggregate output
\[ y = \left( \left( \frac{\varepsilon_p - 1}{\varepsilon_p} \right)^{\alpha} \frac{\gamma_g}{\delta} \beta \right)^{1/\alpha} \]

Public capital
\[ k_g = \left( \frac{\gamma_g}{\delta} \right) y \]

Real wage
\[ w = \left( \left( \frac{\varepsilon_p - 1}{\varepsilon_p} \right) (1 - \alpha)^{1 - \alpha} \frac{\gamma_g}{\delta} (r^k)^{1 - \alpha} k_g^\beta \right)^{1/1 - \alpha} \]

Private capital
\[ k = \left( \frac{\alpha w}{1 - \alpha} r^k \right) h \]

Private investment
\[ i = \delta k \]

Government spending
\[ g = \gamma_g y \]

Government bond
\[ b = \gamma_h y \]

Private consumption
\[ c = y - i - g \]
Appendix II. Log-linearized equilibrium conditions

Log-linearized equilibrium conditions are summarized as follows. Variables with hats denote log-deviation from its steady-state value except the fiscal variables, i.e., for a generic variable $X_t, \hat{X}_t \equiv \log(X_t/X)$. The fiscal variables with hats denote the ratio of deviation from its steady-state value to output. For example, $\hat{g}_t = (g_t - g)/y$.

Euler equation
\[
\hat{c}^R_{w,t} = E_t \hat{c}^R_{w,t+1} - \hat{r}_t + E_t \hat{\pi}_{t+1} - (E_t \hat{v}^\beta_t - \hat{v}^\beta_t)
\]

No-arbitrage condition
\[
E_t \hat{r}^k_t = \left( \frac{1}{1 - \beta + \beta \delta} \right) \hat{r}_t - \left( \frac{1}{1 - \beta + \beta \delta} \right) E_t \hat{\pi}_{t+1}
\]

Consumption of non-Ricardian
\[
\hat{c}^N_{w,t} = \left( \frac{wh}{c} \right) \hat{w}_t + \left( \frac{wh}{c} \right) \hat{h}_t - \left( \frac{v}{c} \right) \hat{\tau}_t
\]

Aggregate consumption
\[
\hat{c}_t = (1 - \xi) (1 - \zeta) \hat{c}^R_{w,t} + \zeta (1 - \zeta) \hat{c}^N_{w,t}
\]

Optimal wage schedule
\[
\hat{w}_t = \frac{1}{(1 - \zeta)} \hat{c}_t + \frac{h}{1 - h} \hat{h}_t
\]

Cost minimization condition
\[
\hat{r}^k_t = \hat{w}_t + \hat{h}_t - \hat{k}_{r-1}
\]

New Keynesian Phillips Curve
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \hat{m}_t
\]

Marginal cost
\[
\hat{m}_t = \alpha \hat{r}^k_t + (1 - \alpha) \hat{w}_t - \beta \hat{k}_{g,t} - \hat{\zeta}_t
\]

Evolution of private capital
\[
\hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \delta \hat{h}_t
\]

Evolution of public capital
\[
\hat{k}_{g,t} = (1 - \delta) \hat{k}_{g,t-1} + \delta \hat{g}_t
\]

Government budget constraint
\[
\hat{b}_t = R (\hat{g}_t - \hat{\tau}_t + \hat{\beta}_{t-1}) + \gamma (\hat{r}_t - R \hat{\pi}_t)
\]

Tax rule
\[
\hat{\pi}_t = \phi \hat{\pi}_{t-1} + \phi \hat{v}_t
\]

Government spending rule
\[
\hat{g}_t = \rho \hat{g}_{t-1} + \phi \hat{y}_t + \hat{e}_t
\]

Taylor rule
\[
\hat{r}_t = \phi \hat{r}_{t-1} + \phi \hat{v}_t
\]

Goods market clearing
\[
\hat{y}_t = \gamma_1 \hat{c}_t + \gamma_2 \hat{h}_t + \hat{g}_t
\]

Aggregate production function
\[
\hat{y}_t = \hat{z}_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{h}_t + \beta \hat{k}_{g,t}
\]