Policy Simulation of Government Expenditure and Taxation Based on the DSGE Model *

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Abstract

This study constructs a dynamic stochastic general equilibrium (DSGE) model including four types of government expenditure (merit goods, public goods, government investment, and lump-sum income transfers) and three types of tax (consumption tax, labor income tax, and capital income tax), and estimate the model parameters. We then perform a simulation analysis based on the estimation results. The estimates, using Japanese data from the first quarter of 1981 to the fourth quarter of 2012, suggest that Japanese government expenditure and effective tax rates do not significantly respond to changes in the output gap and cumulative debt. Furthermore, from a simulation analysis based on the estimation results, we draw two main findings. First, as to the differences in taxes used for financing government expenditure, while consumption tax and labor income tax are almost indifferent, capital income tax aggravates the economy in the long term. Second, when using the increased tax revenue derived from raising the consumption tax rate for additional government expenditure, expenditure on merit goods and government investment have positive effects on the economy in the short and long term, respectively.

Keywords: DSGE model, Bayesian estimation, fiscal policy, simulation analysis
JEL Classification: C11, D58, E32, E62

I. Introduction

In recent years, Japan has accumulated a vast amount of public debt due to long-term economic instability and an increase in social security expenditures accompanying the decline in birthrate and ageing society. Hence, the revision of policies and institutions concerning both revenue and expenditure is a significant challenge. When considering policy and

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institutional change, it is important to quantitatively examine the effects of government expenditure and tax rate changes. Therefore, this study constructs a dynamic stochastic general equilibrium (DSGE) model incorporating four types of government expenditures (merit goods, public goods, government investment, lump-sum income transfers) and three types of taxes (consumption tax, labor income tax, capital income tax). We estimate the model’s structural parameters and the coefficient parameters of policy rules by using Bayesian techniques, and quantify the effects of Japan’s fiscal policy. Moreover, based on the estimation results, we conduct a simulation analysis to reveal how the differences in taxes used for financing influence the effects of merit goods expenditure and how policy effects vary with expenditure components when raising the consumption tax rate for additional government expenditure.

Following the 2008 global financial crisis, while the limitations of monetary policy using traditional interest rate manipulation were being debated, the effectiveness of fiscal policy as a stimulus package was also reconsidered. Therefore, many prior studies have analyzed fiscal policy by using DSGE models. In this context, there is a well-known “puzzle” of the relationship between government spending and private consumption. While standard dynamic general equilibrium models predict the negative effect of government spending on private consumption, previous empirical studies, such as Blanchard and Perotti (2002), indicate positive effects. Hence, how this puzzle can be overcome is one of the key points in model construction. The puzzle can be solved by incorporating non-Ricardian household finances facing liquidity constraints (Galí, López-Salido, and Vallés, 2007), government expenditure rules with a debt stabilization function (Corsetti, Meier, and Müller, 2012), and the Edgeworth complementarity between private consumption and government expenditure (Bouakez and Rebei, 2007; Ganelli and Tervala, 2009; Fève, Matheron, and Sahuc, 2013). While these studies only focus on government expenditure among fiscal policy, Forni, Monteforte, and Sessa (2009) and Leeper, Plante, and Traum (2010) focus on multiple distorted tax systems, covering the Eurozone and United States respectively, and show that the effects of government expenditure can change significantly when financed through distortionary taxes.

Addressing Japan, the studies by Iwata (2011), Hasumi (2014), and Kotera and Sakai (2017) are closely related to this study. Iwata (2011), considering consumption tax, labor income tax, and capital income tax, shows that in Japan in the 1980s and 1990s, debt-stabilizing fiscal policy rules contributed to a short-term expansion of the government expenditure multiplier. The present study reveals the characteristics of Japanese tax rules, considering a larger number of government expenditures than Iwata (2011), and examines the effects of fiscal policy. Hasumi (2014) conducts a simulation analysis based on a small open economy DSGE model including multiple government expenditures and tax systems, and quantitatively show that corporate tax cuts and the equivalent consumption tax increases leads to the short-term growth and raises inflation. Although our model is closed economy, we further separate government consumption into merit goods (individual consumption such as healthcare, long-term care, and education) and public goods (collective consumption such as de-
fense). We then simulate the effects of differences in tax finances on the effects of merit goods expenditure and the effects of differences in items of expenditure on the economy when using an increase in tax revenue from raising the consumption tax rate for fiscal expenditure.

Kotera and Sakai (2017) not only categorize government investment and government consumption as government expenditure, but also divide government consumption into merit goods and public goods. Then, they empirically show that while the former is complementary to private consumption, the latter is a substitute and that the effects of these expenditures vary greatly. This study extends Kotera and Sakai’s (2017) model by introducing multiple tax systems focusing not only on expenditure but also on the revenue side.

The estimation results of this study show that Japanese government expenditure and tax systems are not particularly sensitive to economic fluctuations and accumulated debt. Moreover, the simulation analysis demonstrates that financing through capital income tax, compared with other taxes, significantly reduces the effects of government expenditure in the long term. In addition, the expenditures on merit goods and government investment financed by raising consumption tax have positive effects in the short and long term, respectively.

The remainder of this paper is organized as follows. In Section II, we construct a model that incorporates multiple government expenditures and taxation systems. In Section III, we estimate the parameters of the model by using Bayesian methods and clarify the characteristics of the model. In Section IV, based on the estimation results, we perform two simulation analyses. Section V concludes.

**II. Model**

Our model expands on Kotera and Sakai’s (2017) model with two types of government consumption (merit and public goods) and government investment, based on the models of Smets and Wouters (2007) and Hirose (2012), by incorporating consumption tax, labor income tax, and capital income tax as the three tax systems.

**II-1. Households**

In the economy, there is a continuum of infinitely lived households whose sum is unity. Households are divided into Ricardian households that own assets and optimize consumption across time periods and non-Ricardian households that face liquidity constraints and consume all their income in the current term, the proportion of the latter being $\omega \in [0,1)$.

The utility function of Ricardian households $h \in (\omega, 1]$ is expressed as follows:

$$
E_0 \sum_{t=0}^{\infty} \beta^t e^\gamma \left\{ \frac{(C_t^e(h) - \theta C_{t-1}^e(h))^{1-\sigma}}{1-\sigma} - \frac{Z_t^{1-\sigma} e^{\gamma} I_t^e(h) \gamma^{1+\gamma}}{1+\chi} + V_{gm}(G_{gm}) + V_{gp}(G_{gp}) \right\},
$$

(1)

where $\beta \in (0,1)$, $\theta \in (0,1)$, $\sigma > 0$, and $\chi > 0$ are the discount factor, extent of habit formation
in consumption, inverse of the elasticity of intertemporal substitution, and inverse of the elasticity of labor supply, respectively. \( l, z^b, \) and \( z^i \) represent labor supply and preference shocks to the discount factor and labor supply. \( Z_t \) is the technology level following the non-stationary stochastic processes \( \log Z_t = \log Z_{t-1} + \log z + z^i_t \), where \( z \) is the gross growth rate on a balanced growth path and \( z^i_t \) is technical shocks. Moreover, \( C^e_t \) is the effective consumption of Ricardian households and is defined as

\[
C^e_t(h) = C^R_t(h) + \nu^m G^m_t + \nu^p G^p_t .
\]

(2)

Following Iwata (2013) and Kotera and Sakai (2017), in our model, Edgeworth complementarity (substitution) exists between government consumption (merit goods \( G_t \)) and private consumption. If \( \nu^m \) is negative (positive), this implies that merit goods and Ricardian households’ consumption \( C^R_t \) have a complementary (substitutional) relationship.\(^1\) Functions \( V_{gm}(\cdot) \) and \( V_{gp}(\cdot) \) assume \( V'_{gm}>0, \ V'_{gp}>0 \), respectively, and this ensures that the marginal utility of government consumption is positive.\(^2\)

The budget constraint of Ricardian households is expressed as

\[
(1 + \tau^c_t) C^R_t(h) + I^R_t(h) + B^R_t(h) = (1 - \tau^w_t) W_t(h) l_t(h) + \frac{R^r_{t-1}}{\pi_t} B^R_{t-1}(h) + (1 - \tau^e_t) (R^e_t u_t(h) K^R_{t-1}(h) + D^K_{t}(h)) + T^R_t ,
\]

(3)

where, \( I^R_t, B^R_t, u_t, K^R_{t-1}, D^K_t, \) and \( T^R_t \) are private investment, government bonds, the capital utilization rate, the capital stock held at the beginning of period \( t \), dividends, and the net income transfer to Ricardian households, respectively. In addition, \( \pi_t, W_t, R^e_t, R^r_{t-1}, \tau^w_t, \tau^c_t, \) and \( \tau^e_t \) are the gross inflation rate of final goods price \( \pi_t \), real wage, gross rental rate of capital, nominal gross interest rate of government bonds, consumption tax rate, labor income tax rate, and capital income tax rate, respectively. The first-order conditions for \( C^q_t \) and \( B^R_t \) are given by

\[
(1 + \tau^c_t) \Lambda_t = e^{\nu^b_t} (C^e_t - \theta C^c_t)^{\nu^g} - \beta \theta \mathbb{E}_t e^{\nu^b_t+1} (C^e_{t+1} - \theta C^c_{t+1})^{\nu^g},
\]

(4)

\[
\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} \frac{R^r_t}{\pi_t}. \]

(5)

where \( \Lambda_t \) is the Lagrange multiplier associated with the budget constraint in period \( t \).

Under monopolistic conditions, households will provide differentiated labor given the labor demands of intermediate goods producers. In such cases, as in Galí, López-Salido, and Vallés (2007), intermediate goods firms make uniform labor demands from the two types of households.

Demand for labor services \( i \in [0,1] \) is expressed as

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\(^1\) The Edgeworth complementarity (substitutability) between government consumption and private consumption means that the marginal utility of private consumption increases as government consumption increases and that if the utility functions are differentiable, it can be expressed by the fact that thecross-differential is positive (negative).

\(^2\) Strictly speaking, \( V'_{gm}>0, V'_{gp}>0 \) is a sufficient (necessary) condition for positive marginal utility if government consumption is a substitute (complementary) for private consumption.
Here, $l_i$ is total labor demand defined by the aggregate technology $l_i = \int_0^1 l_i(i)^{\theta_i-1} d\theta_i$ and $\theta_i > 1$ is the elasticity of substitution between differentiated labor services. $W_i$ represents the aggregate wage satisfying

$$W_i = \left( \int_0^1 W_i(i)^{1-\theta_i} di \right)^{1/(1-\theta_i)}$$  \hspace{1cm} (7)$$

Ricardian households make the optimal wage-setting decisions following Calvo (1983). Thus, they can optimally determine wages with probability $1 - \zeta^w$ in each period, in which

$$E \sum_{j=0}^{\infty} (q^w)^j \left[ \Lambda_{r;j}(1 - \tau^w_i) l_{r;j}(i) \right] W_i(i) \left\{ \frac{\Lambda_{r+k-1}}{\Lambda_{r+k}} \right\} \frac{\pi}{\pi+1} - e^{\zeta^w} e^{\gamma w} Z_{r+1}^{1-\gamma} \Lambda_{r+1}$$

is maximized subject to Equation (6). The optimal wage is shown by $W_i^*$ and the first-order condition for $W_i(i)$ is

$$E \sum_{j=0}^{\infty} (q^w)^j \left[ \frac{z^j W_i^*}{W_{r+j}} \prod_{k=1}^{j} \left\{ \frac{\Lambda_{r+k-1}}{\Lambda_{r+k}} \right\} \frac{\pi}{\pi+1} \right] \frac{1 + \zeta^w_{r+j}}{\zeta^w_{r+j}} \Lambda_{r+j} \right\} \prod_{k=1}^{j} \left\{ \frac{\Lambda_{r+k-1}}{\Lambda_{r+k}} \right\} \frac{\pi}{\pi+1} = 0 \right\} \frac{1 + \zeta^w_{r+j}}{\zeta^w_{r+j}} \Lambda_{r+j} \right\} \prod_{k=1}^{j} \left\{ \frac{\Lambda_{r+k-1}}{\Lambda_{r+k}} \right\} \frac{\pi}{\pi+1} = 0 \right\}$$

where $\lambda^w_i \equiv 1/(\theta_i^w - 1)$ represents the wage markup. In addition, assuming that non-Ricardian households earn aggregate wages in each period, Equation (7) is shown by

$$W_i = (1 - \zeta^w) \left( W_i^* \right) \sum_{j=1}^{\infty} \zeta^w(j) \left\{ z^j W_i^* \prod_{k=1}^{j} \left\{ \frac{\Lambda_{r+k-1}}{\Lambda_{r+k}} \right\} \frac{\pi}{\pi+1} \right\} \frac{1}{\zeta^w}.$$  \hspace{1cm} (10)$$

Meanwhile, Ricardian households cannot make the optimal wage revisions with probability $\zeta^w_i$. In that event, they choose their nominal wage on the basis of both a gross steady-state growth rate $z$ and a weighted average of the past and steady-state inflation $\pi$. Specifically, an un-optimized nominal wage rule is denoted by

$$P_iW_i(h) = z^\gamma \pi^{1-\gamma} P_i W_{i-1}(h), \gamma \in [0,1].$$  \hspace{1cm} (11)$$

\footnote{Under this assumption, Ricardian households' decision making concerning wages and labor supply is the same as though non-Ricardian households were not included in the model. Similar conditions can be seen in Forni, Monteforte, and Sessa (2009).}
Ricardian households under Equation (3) as well as the law of motion of capital stock

\[ K^R_t(h) = (1 - \delta(u_t(h))) K^R_{t-1}(h) + \left( 1 - S \left( \frac{I^R_t(h)}{I^R_{t-1}(h)} \frac{e^z}{z} \right) \right) I^R_t(h) \]  

(12)

optimally choose \( u_t, I^R_t, \) and \( K^R_t \). Here, \( \delta(\cdot) \) is the capital depreciation rate satisfying \( \delta' > 0, \delta'' > 0, \delta(1) = \delta \in (0,1), \delta'(1)/\delta''(1) = \mu. \) Thus, as the capital utilization rate increases, the capital stock depreciates further. \( S(\cdot) \) is the function expressing the adjustment cost of investment, defined by \( S(x) = (x-1)^2/(2\zeta). \) Moreover, \( z_i \) is a shock to the adjustment cost of investment. The first-order conditions of \( u_t, I^R_t, \) and \( K^R_t \) are given by

\[ (1 - \tau^k_t) R^k_t = q_t \delta'(u_t), \]  

(13)

\[ 1 = q_t \left\{ 1 - S \left( \frac{I^R_t}{I^R_{t-1}} \frac{e^z}{z} \right) - S' \left( \frac{I^R_t}{I^R_{t-1}} \frac{e^z}{z} \right) \frac{I^R_t}{I^R_{t-1}} \frac{e^z}{z} \right\} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1} S' \left( \frac{I^R_{t+1}}{I^R_t} \frac{e^{z_{t+1}}}{z} \right)^2 \frac{e^{z_{t+1}}}{z}, \]  

(14)

\[ q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left| (1 - \tau^k_{t+1}) R^k_{t+1} u_{t+1} + q_{t+1} (1 - \delta(u_{t+1})) \right|, \]  

(15)

respectively. \( q_t \) is defined by \( q_t \equiv \Lambda^k_t/\Lambda_t \) expressing Tobin’s \( q \) (\( \Lambda^k_t \) is the Lagrange multiplier relating to Equation (12)).

The fraction \( \omega \) of households are non-Ricardian households who do not possess any assets because of liquidity constraints, and their budget constraint is given by

\[ (1 + \tau^c_t) C^NR_t = (1 - \tau^c_t) W_t l_t + T^NR_t, \]  

(16)

where \( C^NR_t \) and \( T^NR_t \) denote the private consumption and net income transfer of non-Ricardian households. As assumed above, because all non-Ricardian households provide labor services equal to aggregate labor and receive aggregate wages, their disposable income and consumption are equal. In other words, non-Ricardian households can be viewed as homogeneous “rule of thumb” households that do not make decisions. Since non-Ricardian households consume all the temporarily increased disposable income arising from expansionary fiscal policy, the higher the share of such households, the greater is the effect of fiscal expansion. Moreover, for simplicity, it is assumed that net income transfers between Ricardian and non-Ricardian households are equal: \( T^R_t = T^NR_t = T_t. \)

II-2. Firms

The final goods market is perfectly competitive and the final goods producer produces under the following constant returns technology:
where \( Y_t \) is a final good that can be used for both consumption and investment, \( Y_t(f) \) is an intermediate good produced by an intermedia goods firm \( f \), which is continuously and uniformly distributed on \([0,1]\), and \( \theta_p > 1 \) is the elasticity of the substitution across intermediate goods. As a result of final goods firms’ profit maximization given the intermediate goods of price \( P_t(f) \), demand for intermediate goods \( Y_t(f) \) is derived as

\[
Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\theta_p} Y_t, \tag{18}
\]

and the relationship between the price of final goods and intermediate goods is expressed as

\[
1 = \left( \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{1-\theta_p} df \right)^{\frac{1}{1-\theta_p}}. \tag{19}
\]

The production function for intermediate goods producers under monopolistic competition is as follows:

\[
Y_t(f) = Z_t^{1-\alpha} \left( u_t K_{t-1}(f) \right)^{\alpha} l_t(f)^{1-\alpha} (K_{t-1})^\nu - \Phi Z_t, \tag{20}
\]

where \( K_{t-1}^{\nu} \) is the public capital stock at the beginning of period \( t \) and \( \Phi > 0 \) denotes the fixed cost. This formulation is used in many previous studies including Baxter and King (1993) and Iwata (2013), and indicates that the constant returns to scale exist in privately supplied factors and public capital has a positive external effect.

The cost minimization condition of intermediate goods firms is given by

\[
mc_t = \left\{ \frac{W_t}{(1-\alpha)Z_t} \right\}^{1-\alpha} \left( \frac{R_t}{\alpha} \right)^\alpha \left( \frac{K_{t-1}^{\nu}}{Z_t} \right)^{-\nu}, \tag{21}
\]

where \( mc_t \) is the Lagrange multiplier with respect to cost minimization, and this is interpreted as the marginal cost of intermediate goods production.

Further, from Equations (18), (20), and (21), aggregate output is expressed as:

\[
Y_t \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\theta_p} df = Z_t^{1-\alpha-\nu} \left( u_t K_{t-1} \right)^{\alpha} l_t^{1-\alpha} (K_{t-1}^{\nu})^\nu - \Phi Z_t, \tag{22}
\]

where \( K_{t-1} = \int_0^1 K_{t-1}(f) df \) and \( l_t = \int_0^1 l_t(f) df \).

As in Calvo (1983), intermediate goods firms set the prices for intermediate goods. In other words, they can set optimal prices in each period with probability \( 1-\xi_p \), in the event of
which prices are set to maximize

\[
E_t \sum_{j=0}^{\infty} (\xi_p^j) \left( \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} \right) \left[ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left( \frac{\pi_{t+k-1}}{\pi_t} \right)^{\gamma} \right] Y_{t+j}(f),
\]

subject to Equation (18). When the optimal price is expressed as \( P_t^* \), the first-order condition for \( P_t(f) \) is

\[
E_t \sum_{j=0}^{\infty} (\xi_p^j)^j \left( \frac{\Lambda_{t+j}}{\Lambda_t \gamma_p^{j+1}} \right) \left[ \frac{P_t^*}{P_t} \prod_{k=1}^{j} \left( \frac{\pi_{t+k-1}}{\pi_t} \right)^{\gamma} \right] Y_{t+j} \left[ \frac{P_t^*}{P_t} \prod_{k=1}^{j} \left( \frac{\pi_{t+k-1}}{\pi_t} \right)^{\gamma} \right] - (1 + \lambda_{t+j}^p) mc_{t+j} = 0.
\]

where \( \lambda_{t+j}^p \equiv 1/(\theta_{p}^t-1) \) denotes the price markup. By using this, Equation (19) can be rewritten as

\[
1 = (1 - \xi_p^p) \left( \frac{P_t^*}{P_t} \right)^{-1} \prod_{j=1}^{\infty} (\xi_p^j)^j \left[ \frac{P_t^{*j}}{P_t} \prod_{k=1}^{j} \left( \frac{\pi_{t+k-1}}{\pi_t} \right)^{\gamma} \right] \left( \frac{1}{\xi_t^p} \right).
\]

Meanwhile, intermediate goods producers cannot set optimal prices with probability \( \xi_p^p \). In such a case, the price of intermediate goods is set according to the following rule

\[
P_t(f) = \pi_t^{\gamma_p} \pi_t^{1-\gamma_p} P_{t-1}(f), \gamma_p \in [0,1].
\]

The monopolistic profit of intermediate goods firms is distributed to Ricardian households as dividends. Therefore, aggregate dividend \( D_t \) is expressed as

\[
D_t = \int_0^1 \left( Y_t(f) - W_t(f) - R_t^\gamma u_t(f) K_{t-1}(f) \right) df
= (1 - mc_t) \left( Y_t \Delta_t + \Phi Z_t \right) - \Phi Z_t,
\]

where \( \Delta_t = \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-1} \frac{1}{\xi_t^p} df. \)

### II-3. Policy Rules

Monetary policy is expressed as the weighted average of the lag term and Taylor rule thus:

\[
\log R_t^n = \phi^n \log R_{t-1}^n + (1 - \phi^n) \left\{ \log R^n + \phi_e^z \left( \frac{1}{4} \sum_{j=0}^{3} \log \frac{\pi_{t+j}}{\pi_t} \right) + \phi_e^z \log \frac{Y_t}{Y_t^*} \right\} + z_t^r
\]

where \( R_t^n \) and \( R^n \) represent the nominal gross interest rate and its steady-state value, and \( z_t^r \) is a monetary policy shock. \( Y_t^* \) is potential output, defined as
\[ Y_t^* = Z_t^{-\alpha \gamma} (u k Z_{t-1})^{\alpha l - \alpha} (k^\rho Z_{t-1})^\gamma - \Phi Z_t, \]  

(29)

where \( u \) and \( l \) express the steady-state values of capital utilization rate and labor, respectively, and \( k \) and \( k^\rho \) are the steady-state values of detrended private capital \( K_t/Z_t \) and public capital \( K^\rho_t/Z_t \).

As for government expenditure, we consider two kinds of government consumption (merit and public goods), government investment, and net income transfer.\(^6\) These expenditures are financed by the issuance of government bonds, consumption tax, labor income tax, and capital income tax, and the government's budget constraint is given by

\[ B_t = \frac{R_t^{i-1}}{\pi_t} B_{i-1} + G^m_i + G^p_i + G^i_i + T^*-\tau_i^* C_t - \tau_i^* w_l - \tau_i^* (R^i_t u, K_{t-1} + D_t), \]  

(30)

where \( B_t \) is the aggregate government bonds, \( G^i_i \) is government investment, and \( C_t \) is total private consumption. Public capital is accumulated through government investment as follows (\( \delta^s \in [0,1] \) is the social capital depreciation rate):

\[ K^s_t = (1-\delta^s) K^s_{t-1} + G^i_t. \]  

(31)

The government expenditure rules are formulated as follows:

\[ \log G^m_i = \phi^m \left( \log G^m_{i-1} + \log z \right) + (1 - \phi^m) \left( \log Z_t g^m + \phi^m \log \frac{Y_{i-1}}{Y^*_{i-1}} + \phi^m \log \frac{B_{i-1}}{b_{tar}} \right) + \omega^m_t, \]  

(32)

\[ \log G^p_i = \phi^p \left( \log G^p_{i-1} + \log z \right) + (1 - \phi^p) \left( \log Z_t g^p + \phi^p \log \frac{Y_{i-1}}{Y^*_{i-1}} + \phi^p \log \frac{B_{i-1}}{b_{tar}} \right) + \omega^p_t, \]  

(33)

\[ \log G^i_i = \phi^i \left( \log G^i_{i-1} + \log z \right) + (1 - \phi^i) \left( \log Z_t g^i + \phi^i \log \frac{Y_{i-1}}{Y^*_{i-1}} + \phi^i \log \frac{B_{i-1}}{b_{tar}} \right) + \omega^i_t, \]  

(34)

\[ \log T^* = \phi^* \left( \log Y_{i-1} + \log z \right) + (1 - \phi^*) \left( \log Z_t \tau + \phi^* \log \frac{Y_{i-1}}{Y^*_{i-1}} + \phi^* \log \frac{B_{i-1}}{b_{tar}} \right) + \omega^T_t, \]  

(35)

where \( g^j (j \in \{m, p, i\}) \) and \( \tau \) are the steady-state values of \( G^j_t/Z_t \) and \( T^*_t/Z_t \), respectively, \( b_{tar} \) is the target ratio of government debt to output, and \( \omega^j_t (j \in \{gm, gp, gi\}) \) represent shocks to each expenditure. The government expenditure rules of this study include a smoothing term and respond to output gap and the deviation of the debt-to-output ratio from its target in the

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\(^6\) \( T^* \) is interpreted as a lump-sum income transfer where positive and a lump-sum fixed tax where negative. Since \( T^*/Y_t \) is positive in the steady state under the parameter values used in the subsequent analysis, it is an income transfer in this study. Therefore, it should be noted that the income transfer in this study is somewhat different from the income transfer in reality, such as a pension. Based on this point, we do not employ observation data on income transfers in the estimation.
previous period. Here, if the sign of $\phi_j (j \in \{gm, gp, gi, T\})$ is positive (negative), it implies that government expenditure is procyclical (countercyclical). As pointed out by Fève, Matheron, and Sahuc (2013), when government expenditure rules are estimated without including countercyclical terms, the Edgeworth complementarity between government expenditure and private consumption can be underestimated. Moreover, if the sign of $\phi_j (j \in \{gm, gp, gi, T\})$ is negative, this implies that government expenditure decreases when the ratio of government debt to total output exceeds the target. Corsetti, Meier, and Müller (2012) show that this kind of debt stabilization rule suppresses increases in fiscal policy by suppressing increases in the future inflation and interest rates through monetary policy rules.

Similar to the government expenditure rules, the taxation rules are composed of a lag term, output gap term, and debt-to-output term. Specifically, the rules on the consumption tax rate, labor income tax rate, and capital income tax rate are respectively given by the following:

$$
\tau^c_t = \phi^c_t \tau^c_{t-1} - (1 - \phi^c_t) \left( \phi^c_t \log \frac{Y_{t-1}}{Y^*_t} + \phi^b_t \log \frac{B_{t-1}/Y_{t-1}}{b^{tar}} \right) + \epsilon^c_t, 
$$

$$
\tau^w_t = \phi^w_t \tau^w_{t-1} - (1 - \phi^w_t) \left( \phi^w_t \log \frac{Y_{t-1}}{Y^*_t} + \phi^b_t \log \frac{B_{t-1}/Y_{t-1}}{b^{tar}} \right) + \epsilon^w_t, 
$$

$$
\tau^k_t = \phi^k_t \tau^k_{t-1} - (1 - \phi^k_t) \left( \phi^k_t \log \frac{Y_{t-1}}{Y^*_t} + \phi^b_t \log \frac{B_{t-1}/Y_{t-1}}{b^{tar}} \right) + \epsilon^k_t, 
$$

$$
\epsilon^j_t \sim N(0, \sigma^2_j),
$$

where $j \in \{tc, tw, tk\}$.

The reason why the tax rates are influenced by not only the debt-to-output ratio but also the output gap is because these tax rates are generally interpreted as the effective tax rate in macroeconomics, especially in models assuming representative individuals. The tax system is extremely complicated in practice as it includes income deductions and tax credits, progressive tax rates, and tax-exempt items. Hence, when expressing this as a simplified proportional tax, the tax rate may also change depending on the economic conditions. Moreover, some previous studies adopt tax rules that respond to the output gap in the current period and debt. However, taking account of the lag associated with fiscal policy decision-making and implementation, we adopt a formulation that depends on past economic states. Meanwhile, shocks to the tax rate are not considered to be persistent due to the political difficulty of changing the tax rate. Therefore, in contrast to the other structural shocks defined below, they do not follow autoregressive processes but are independently and identically distributed. A negative sign of the coefficient parameters of the output gap and debt terms, similar to the government expenditure rules, imply countercyclical and debt stabil-

---

7 For details on the method of creating effective tax rate series from macro statistics and interpreting the effective tax rate in the macroeconomic model, see Mendoza, Razin, and Tesar (1994) and Forni, Monteforte, and Sessa (2009).
II-4. Market clearing, Aggregation, and Structural Shocks

The market clearing condition is given by

$$Y_t = C_t + I_t + G_t^m + G_t^p + G_t^i + xZ_t e^{it},$$

(39)

where $C_t$ and $I_t$ are aggregate private consumption and aggregate private investment satisfying

$$C_t = \omega C_{tR} + \int_0^1 C_t^R(h) \, dh,$$

(40)

$$I_t = \int_0^1 I_t^R(h) \, dh,$$

(41)

respectively. $x$ is the steady-state value of the other de-trended demand factors, and $z_t^i$ expresses an exogenous demand shock. Private capital, dividends, and government bonds are aggregated as follows:

$$K_t = \int_0^1 K_t^R(h) \, dh,$$

(42)

$$D_t = \int_0^1 D_t^R(h) \, dh,$$

(43)

$$B_t = \int_0^1 B_t^R(h) \, dh.$$  

(44)

Except for shocks to tax rates, each structural shock follows a first-order autoregressive process with an independent and identical normal shock:

$$z_t^j = \rho^j z_{t-1}^j + \varepsilon_t^j,$$

(45)

$$\varepsilon_t^j \sim N(0, \sigma_j^2),$$

where $j \in \{ b, l, z, i, x, r, gm, gp, gi, T \}$. Moreover, this study’s model has a balanced growth trend. Specifically, the variables $C_t^R$, $C_{tR}$, $I_t^R$, $K_t^R$, $D_t^R$, $K_t^p$, $G_t^m$, $G_t^p$, $G_t^i$, $Y_t$, $Y_t^*$, $B_t^R$, $B_t$, $G_t^m$, $G_t^p$, $G_t^i$, $T_t$, $W_t$, and $W_t^*$ grow at gross rate $z$ on the balanced growth path. In the solution procedure, these variables are divided by technology level $Z_t$ and log-linearized around the steady state. The log-linearized model is described in the appendix.

---

8 The formulation for the government expenditure and tax rate rules varies in previous research. While Coenen, Straub, and Trabandt (2013) do adopt government expenditure and tax rate rules including a lag term, output gap term, and debt response term, they differ in that they include responses to output and debt in the present period and preannouncement effects of shocks. Additionally, while tax rates in Forni, Monteforte, and Sessa (2009) respond to debt in the current period, those in Iwata (2011) respond to debt in the previous period. Both assume independently and identically distributed shocks to tax rates, rather than shocks following the first-order autoregressive process.
III. Estimation

In this section, the parameters are estimated with a standard Bayesian estimation based on the Markov chain Monte Carlo (MCMC) method. Specifically, we can use the solution equations of the log-linearized model and observation equations linking the model variables to data to evaluate the log likelihood function using a Kalman filter. Further, combining the log likelihood with the prior distribution of parameters, we perform MCMC sampling on the basis of a Metropolis-Hastings algorithm to obtain the posterior distribution. We generate two Markov chains with 700,000 draws and discard the first 280,000 draws as burn-in draws.

III-1. Data, Calibration, and Prior Distribution

In estimation, we employ thirteen quarterly data series from the first quarter of 1981 to the fourth quarter of 2012: real GDP, real private consumption, real private investment, real wages, real merit goods expenditure, real public goods expenditure, real government investment, working hours, the inflation rate, the nominal interest rate, the effective consumption tax rate, the effective labor income tax rate, and effective capital income tax.

Series except tax rates are constructed in the same manner as Hirose (2012) and Kotera and Sakai (2017). For tax rates, effective tax rate series are created by following Mendoza, Razin, and Tesar (1994), Forni, Monteforte, and Sessa (2009), and Hasumi (2014). These data series are related to the endogenous variables of the model through the following observation equation:

\[
\begin{align*}
\Delta \ln Y_t & = z^* + z_t^i + \tilde{y}_t - \tilde{y}_{t-1} \\
\Delta \ln C & = z^* + z_t^i + \tilde{c}_t - \tilde{c}_{t-1} \\
\Delta \ln I_t & = z^* + z_t^i + \tilde{i}_t - \tilde{i}_{t-1} \\
\Delta \ln W_t & = z^* + z_t^i + \tilde{w}_t - \tilde{w}_{t-1} \\
\Delta \ln G_m & = z^* + z_t^i + \tilde{g}_m - \tilde{g}_{m-1} \\
\Delta \ln G_p & = z^* + z_t^i + \tilde{g}_p - \tilde{g}_{p-1} \\
\Delta \ln G_i & = z^* + z_t^i + \tilde{g}_i - \tilde{g}_{i-1} \\
\ln l & = l \\
\Delta \ln P_t & = \pi^* + \pi_t \\
\ln R_t & = r^* + r_t \\
\tau^c_t & = 100\tau^c \\
\tau^w_t & = 100\tau^w \\
\tau^k_t & = 100\tau^k
\end{align*}
\]

(46)

\[\begin{bmatrix}
\Delta \ln Y_t \\
\Delta \ln C \\
\Delta \ln I_t \\
\Delta \ln W_t \\
\Delta \ln G_m \\
\Delta \ln G_p \\
\Delta \ln G_i \\
\ln l \\
\Delta \ln P_t \\
\ln R_t \\
\tau^c_t \\
\tau^w_t \\
\tau^k_t
\end{bmatrix} + \begin{bmatrix}
z^* + z_t^i \\
z^* + z_t^i \\
z^* + z_t^i \\
z^* + z_t^i \\
z^* + z_t^i \\
z^* + z_t^i \\
z^* + z_t^i \\
l \\
\pi^* + \pi_t \\
r^* + r_t \\
100\tau^c \\
100\tau^w \\
100\tau^k
\end{bmatrix} \leq \begin{bmatrix}
\tilde{y}_t - \tilde{y}_{t-1} \\
\tilde{c}_t - \tilde{c}_{t-1} \\
\tilde{i}_t - \tilde{i}_{t-1} \\
\tilde{w}_t - \tilde{w}_{t-1} \\
\tilde{g}_m - \tilde{g}_{m-1} \\
\tilde{g}_p - \tilde{g}_{p-1} \\
\tilde{g}_i - \tilde{g}_{i-1} \\
\tilde{l}_t \\
\tilde{\pi}_t \\
\tilde{r}_t \\
\tilde{\tau}^c_t \\
\tilde{\tau}^w_t \\
\tilde{\tau}^k_t
\end{bmatrix}\]

where the small-letter notation with the tilde expresses the log-deviation of the de-trended

\[\text{The effective tax rate series used in this study is calculated as the ratio of tax revenue to the tax base, using national accounts data. For consumption tax, social insurance fees are included in both individual consumption tax and labor income tax.}\]
variables from the steady state. In addition, \( z^*, \pi^*, r^*, l, \tau^c, \tau^w, \) and \( \tau^k \) are net growth rate of technology, net inflation rate, net real interest rate, labor worked, consumption tax rate, labor income tax rate, and capital income tax rate at the steady-state, respectively.

Following Sugo and Ueda (2008), Hirose and Kurozumi (2012), and Kotera and Sakai (2017), we do not estimate certain parameters and steady-state values. Calibration of these parameters are shown in Table 1. Moreover, the period averages of the effective tax rate series are used as the steady-state values for each tax rate.

### Table 1. Calibration

<table>
<thead>
<tr>
<th>Parameter, Ratio</th>
<th>Value</th>
<th>Parameter, Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Capital</td>
<td>( \alpha )</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Depreciation Rate of Private Capital</td>
<td>( \delta )</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>Depreciation Rate of Public Capital</td>
<td>( \delta^p )</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Capital Utilization Rate</td>
<td>( \kappa )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Wage Markup</td>
<td>( \lambda^w )</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Merit Goods Expenditure/Output Ratio</td>
<td>( g^m/y )</td>
<td>0.083</td>
<td></td>
</tr>
<tr>
<td>Public Goods Expenditure/Output Ratio</td>
<td>( g^p/y )</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>Government Investment/Output Ratio</td>
<td>( g^r/y )</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Government Debt/Output Ratio</td>
<td>( b^{tar} )</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Other Demand/Output Ratio</td>
<td>( x/y )</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Effective Consumption Tax Rate</td>
<td>( \tau^c )</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>Effective Labor Income Tax Rate</td>
<td>( \tau^w )</td>
<td>0.273</td>
<td></td>
</tr>
<tr>
<td>Effective Capital Income Tax Rate</td>
<td>( \tau^k )</td>
<td>0.446</td>
<td></td>
</tr>
</tbody>
</table>

While most of the prior distributions of the parameters are same as the previous studies, we adopt the following priors regarding parameters of our interest. To neutrally evaluate the degree of Edgeworth complementarity or substitutability, the priors of \( \nu^g_m \) and \( \nu^g_p \) are normal distributions with mean 0 and standard deviation 1.5. For the expenditure and tax rate rules, to evaluate whether these are procyclical or countercyclical and whether they stabilize debt or not, the prior distributions of \( \phi^q_j \) \(( j \in \{g_m, g_p, g_i, T, tc, tw, tk\} \)) are normal distributions with mean 0 and standard deviation 0.5. The priors of tax rate lag factor \( \phi^j \) \(( j \in \{tc, tw, tk\} \)) are beta distribution with mean 0.8 and standard deviation 0.1.

### III-2. Estimation results and Multipliers

Table 2 shows the estimation results of parameters and standard deviations of shocks. For \( \nu^g_m \) and \( \nu^g_p \), which express the extent of the complementarity or substitutability between private and public consumption, the posterior mean is respectively negative and positive, and neither include zero in the 90% credible interval. This finding suggestes that merit goods expenditure is complementary to private consumption and that public goods expenditure is substitute; this is consistent with the result is by Kotera and Sakai (2017). \( \omega \), representing the proportion of non-Ricardian households, is about 0.06, which is small compared with the results of Iwata (2011). The posterior mean of productivity of public capital \( v \) is es-

---

10 Among tilde-affixed variables, only the tax rate variables are defined as the difference from the steady state.
11 We also estimate the model where the tax rate shocks follow a first-order autoregressive process, but the posterior distribution and log data density change little.
Table 2. Prior and Posterior Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Prior Distribution</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Posterior Distribution</th>
<th>Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu^{\alpha m}$</td>
<td>Normal</td>
<td>0</td>
<td>1.5</td>
<td>-2.068</td>
<td>-2.690</td>
<td>-1.437</td>
</tr>
<tr>
<td>$\nu^{\alpha p}$</td>
<td>Normal</td>
<td>0</td>
<td>1.5</td>
<td>0.834</td>
<td>0.006</td>
<td>1.684</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gamma</td>
<td>0.1</td>
<td>0.025</td>
<td>0.101</td>
<td>0.058</td>
<td>0.142</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Beta</td>
<td>0.25</td>
<td>0.1</td>
<td>0.058</td>
<td>0.019</td>
<td>0.098</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Gamma</td>
<td>1</td>
<td>0.375</td>
<td>2.008</td>
<td>1.561</td>
<td>2.424</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.15</td>
<td>0.253</td>
<td>0.152</td>
<td>0.353</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Gamma</td>
<td>2</td>
<td>0.75</td>
<td>5.633</td>
<td>4.110</td>
<td>7.091</td>
</tr>
<tr>
<td>$1/\xi$</td>
<td>Gamma</td>
<td>4</td>
<td>1.5</td>
<td>5.107</td>
<td>2.986</td>
<td>7.094</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gamma</td>
<td>1</td>
<td>1</td>
<td>0.024</td>
<td>0.000</td>
<td>0.047</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Gamma</td>
<td>0.075</td>
<td>0.0125</td>
<td>0.074</td>
<td>0.054</td>
<td>0.094</td>
</tr>
<tr>
<td>$\gamma^w$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
<td>0.343</td>
<td>0.033</td>
<td>0.618</td>
</tr>
<tr>
<td>$\xi^w$</td>
<td>Beta</td>
<td>0.375</td>
<td>0.1</td>
<td>0.447</td>
<td>0.336</td>
<td>0.554</td>
</tr>
<tr>
<td>$\gamma^p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
<td>0.069</td>
<td>0.000</td>
<td>0.141</td>
</tr>
<tr>
<td>$\xi^p$</td>
<td>Beta</td>
<td>0.375</td>
<td>0.1</td>
<td>0.682</td>
<td>0.641</td>
<td>0.725</td>
</tr>
<tr>
<td>$\lambda^p$</td>
<td>Gamma</td>
<td>0.15</td>
<td>0.05</td>
<td>0.346</td>
<td>0.211</td>
<td>0.471</td>
</tr>
<tr>
<td>$z^*$</td>
<td>Gamma</td>
<td>0.19</td>
<td>0.05</td>
<td>0.133</td>
<td>0.080</td>
<td>0.185</td>
</tr>
<tr>
<td>$l^*$</td>
<td>Normal</td>
<td>0</td>
<td>0.05</td>
<td>0.000</td>
<td>-0.082</td>
<td>0.084</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>Gamma</td>
<td>0.175</td>
<td>0.05</td>
<td>0.198</td>
<td>0.117</td>
<td>0.280</td>
</tr>
<tr>
<td>$r^*$</td>
<td>Gamma</td>
<td>0.498</td>
<td>0.05</td>
<td>0.519</td>
<td>0.446</td>
<td>0.591</td>
</tr>
<tr>
<td>$\phi^{r}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.785</td>
<td>0.734</td>
<td>0.835</td>
</tr>
<tr>
<td>$\phi^{c}$</td>
<td>Gamma</td>
<td>1.7</td>
<td>0.1</td>
<td>1.821</td>
<td>1.667</td>
<td>1.972</td>
</tr>
<tr>
<td>$\phi^{c}$</td>
<td>Gamma</td>
<td>0.125</td>
<td>0.05</td>
<td>0.035</td>
<td>0.016</td>
<td>0.053</td>
</tr>
<tr>
<td>$\phi^{gm}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.975</td>
<td>0.964</td>
<td>0.985</td>
</tr>
<tr>
<td>$\phi^{gm}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.240</td>
<td>-1.019</td>
<td>0.518</td>
</tr>
<tr>
<td>$\phi^{gm}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.126</td>
<td>-0.198</td>
<td>-0.052</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.966</td>
<td>0.944</td>
<td>0.991</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>0.406</td>
<td>-0.521</td>
<td>1.339</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.064</td>
<td>-0.126</td>
<td>-0.012</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.973</td>
<td>0.952</td>
<td>0.996</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.164</td>
<td>-1.012</td>
<td>0.653</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>0.035</td>
<td>-0.146</td>
<td>0.209</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.882</td>
<td>0.786</td>
<td>0.976</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>0.584</td>
<td>-0.210</td>
<td>1.386</td>
</tr>
<tr>
<td>$\phi^{ap}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.111</td>
<td>-0.155</td>
<td>-0.065</td>
</tr>
<tr>
<td>$\phi^{ac}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.495</td>
<td>0.366</td>
<td>0.621</td>
</tr>
<tr>
<td>$\phi^{ac}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.036</td>
<td>-0.064</td>
<td>-0.008</td>
</tr>
<tr>
<td>$\phi^{ac}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>0.001</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>$\phi^{tw}$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.880</td>
<td>0.815</td>
<td>0.949</td>
</tr>
<tr>
<td>$\phi^{tw}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>-0.142</td>
<td>-0.381</td>
<td>0.076</td>
</tr>
<tr>
<td>$\phi^{tw}$</td>
<td>Normal</td>
<td>0</td>
<td>0.5</td>
<td>0.012</td>
<td>0.004</td>
<td>0.020</td>
</tr>
</tbody>
</table>
The sample period is from the first quarter of 1981 to the fourth quarter of 2012. The posterior distribution is based on two Markov chains with 700,000 draws, obtained using the Metropolis-Hastings algorithm. The first 280,000 draws are dropped as burn-in draws. In the first column, normal, beta, gamma, and inv. gamma represent the normal, beta, gamma, and inverse gamma distributions, respectively.

**Table 3. Policy Rules**

<table>
<thead>
<tr>
<th>Merit Goods</th>
<th>Public Goods</th>
<th>Government Investment</th>
<th>Income Transfer</th>
<th>Consumption Tax Rate</th>
<th>Labor Income Tax Rate</th>
<th>Capital Income Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
<td>0.975</td>
<td>0.966</td>
<td>0.973</td>
<td>0.882</td>
<td>0.495</td>
<td>0.880</td>
</tr>
<tr>
<td>Output</td>
<td>-0.006</td>
<td>0.014</td>
<td>-0.004</td>
<td>0.069</td>
<td>-0.018*</td>
<td>-0.017</td>
</tr>
<tr>
<td>Debt</td>
<td>-0.003*</td>
<td>-0.002*</td>
<td>0.001</td>
<td>-0.013*</td>
<td>0.001*</td>
<td>0.001*</td>
</tr>
</tbody>
</table>

(Note) The asterisk (*) means that zero is not included in the 90% credible interval of the posterior distribution of the response coefficient parameters for output or debt. Values for inertia is the posterior mean, and values for output and debt are calculated based on the posterior means.
estimated to be about 0.1, considerably larger than the 0.04 estimated by Iwata (2013); however, the government investment multiplier calculated based on the estimation results is smaller than that of Iwata (2013), as shown below.

Table 3 summarizes the posterior mean of the effective response coefficients of the output gap and debt-to-output ratio, taking account of the inertia of each expenditure and tax rules. The posterior mean of the parameters representing the inertia of government expenditure is large, suggesting that Japanese government expenditure is mostly explained by inertia and that responses to economic fluctuations and debt accumulation are small. The posterior means of lag coefficients of effective tax rate are fairly close to Iwata’s (2011) value, indicating that the response to output gap and debt accumulation is small. Previous studies such as Frankel, Végh, and Vuletin (2013) and Végh and Vuletin (2015) show that the cyclicity of government expenditures and tax rates are scarcely observed in developed economies. Hence, the estimation results of this study for Japan are consistent with this finding. Moreover, the our results show that in Japan, the response to the accumulation of debt through government expenditure and tax rates is limited, generally indicating that the automatic stabilization functions of Japan’s fiscal policy rules in response to economic fluctuations and debt accumulation are not large.

Table 4 presents the changes in the estimation results stemmed from the introduction of multiple tax systems into the model. Here, while Model (1) is a formulation including the multiple tax systems shown in Section 2, and the estimation results are repeated in Table 2, Model (2) only considers a lump-sum tax, and effective tax rate data are not used in the estimation.

As for $v_{gm}$ and $v_{gp}$, the complementarity of merit goods is estimated as larger in Model (1) than in Model (2). Conversely, the substitutability of public goods is small, and these influence the policy effect of government consumption expenditure. Regarding the parameters associated with real and nominal rigidity, large differences are observed in $\theta$, $1/\zeta$, $\mu$, $\gamma^W$, $\xi^W$, $\gamma^p$, and $\lambda^p$. These differences, resulting from the extension of the model, imply that since the rigidity of consumption, investment, and prices as well as intermediate goods producers’ markup rate are low, and the rigidity of the capital utilization rate is largely estimated. These changes generally reduce the costs accompanying the adjustment of macroeconomic variables and monopolistic competition. For government expenditure rules, few changes are observed.

Table 5 shows the multiplier for government expenditures in Models (1) and (2). The

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12 The inertia coefficient of labor income tax rule is estimated as larger in this study. This can be attributed because the difference in the estimation period and formulation of the tax rate rules.

13 Estimates may change depending on the estimation period and changes to the formulation, and therefore the estimated results of this study must be interpreted broadly. In Coenen, Straub, and Trabandt (2013), the coefficients of the response terms for output gap and debt in the Eurozone’s revenue and expenditure rules are also shown to be small.

14 Model (2) is basically the same model as that of Kotera and Sakai (2017). However, there are differences such as the presence of a shock to lump-sum tax and setting the prior distribution. Moreover, while $\phi^T$, $\phi^y_T$, $\phi^b_T$ are associated with lump-sum income transfer rules in Model (1), these pertain to the lump-sum tax rule in Model (2).

15 With respect to the rigidity of real wages, the decrease in $\gamma^r$ reduces rigidity, while the increase in $\xi^r$ has the opposite effect, and thus depends entirely upon which of the two is greater.
Table 4. Posterior Distribution Comparison

|                | Model (1) |  | Model (2) |  |
|----------------|-----------|---------------------------|---------------------------|
|                | Mean      | 90% Interval             | Mean                      | 90% Interval             |
| $\nu^{gm}$     | -2.068    | -2.690 -1.437            | -1.642                    | -2.128 -1.156            |
| $\nu^{gp}$     | 0.834     | 0.006 1.684              | 0.959                     | 0.144 1.765              |
| $\nu$          | 0.101     | 0.058 0.142              | 0.112                     | 0.068 0.154              |
| $\omega$       | 0.058     | 0.019 0.098              | 0.078                     | 0.024 0.130              |
| $\sigma$       | 2.008     | 1.561 2.424              | 2.342                     | 1.972 2.716              |
| $\theta$       | 0.253     | 0.152 0.353              | 0.388                     | 0.259 0.519              |
| $\chi$         | 5.633     | 4.110 7.091              | 5.008                     | 3.604 6.363              |
| $1/\zeta$      | 5.107     | 2.986 7.094              | 6.565                     | 3.645 9.328              |
| $\mu$          | 0.024     | 0.000 0.047              | 0.940                     | 0.419 1.438              |
| $\phi$         | 0.074     | 0.054 0.094              | 0.071                     | 0.051 0.090              |
| $\gamma^m$     | 0.343     | 0.033 0.618              | 0.492                     | 0.143 0.833              |
| $\xi^{m}$      | 0.447     | 0.336 0.554              | 0.327                     | 0.245 0.409              |
| $\gamma^p$     | 0.069     | 0.000 0.141              | 0.139                     | 0.005 0.269              |
| $\xi^p$        | 0.682     | 0.641 0.725              | 0.720                     | 0.684 0.757              |
| $\lambda$      | 0.346     | 0.211 0.471              | 0.476                     | 0.335 0.621              |
| $\zeta^*$      | 0.133     | 0.080 0.185              | 0.155                     | 0.098 0.210              |
| $l^*$          | 0.000     | -0.082 0.084             | 0.001                     | -0.080 0.081             |
| $\pi^*$        | 0.198     | 0.117 0.280              | 0.178                     | 0.099 0.255              |
| $r^*$          | 0.519     | 0.446 0.591              | 0.529                     | 0.457 0.602              |
| $\phi^r$       | 0.785     | 0.734 0.835              | 0.706                     | 0.643 0.772              |
| $\phi^<\zeta$  | 1.821     | 1.667 1.972              | 1.793                     | 1.643 1.942              |
| $\phi^{gm}$    | 0.035     | 0.016 0.053              | 0.030                     | 0.013 0.047              |
| $\phi^{gp}$    | 0.975     | 0.964 0.985              | 0.978                     | 0.967 0.990              |
| $\phi_{gm}$    | -0.240    | -1.019 0.518             | 0.410                     | -0.458 1.265             |
| $\phi_{gp}$    | -0.126    | -0.198 0.052             | -0.197                    | -0.294 -0.106            |
| $\phi_{m}^{pp}$| 0.966     | 0.944 0.991              | 0.969                     | 0.944 0.996              |
| $\phi_{m}^{p}$ | 0.406     | -0.521 1.339             | 0.367                     | -0.504 1.208             |
| $\phi_{m}^{p}$ | -0.064    | -0.126 -0.012            | -0.060                    | -0.138 0.018             |
| $\phi_{m}^{i}$ | 0.973     | 0.952 0.996              | 0.950                     | 0.925 0.972              |
| $\phi_{y}^{i}$ | -0.164    | -1.012 0.653             | -0.004                    | -0.799 0.758             |
| $\phi_{y}^{i}$ | 0.035     | -0.146 0.209             | 0.162                     | 0.058 0.260              |
| $\phi_{y}^{p}$ | 0.882     | 0.786 0.976              | 0.791                     | 0.659 0.930              |
| $\phi_{y}^{c}$ | 0.584     | -0.210 1.386             | 0.008                     | -0.514 0.530             |
| $\phi_{y}^{<}$ | -0.111    | -0.155 0.065             | 0.011                     | -0.017 0.041             |
| $\rho^s$       | 0.073     | 0.015 0.128              | 0.072                     | 0.013 0.124              |
| $\rho^b$       | 0.536     | 0.312 0.753              | 0.331                     | 0.110 0.527              |
| $\rho^i$       | 0.237     | 0.104 0.369              | 0.286                     | 0.162 0.412              |
| $\rho^{m}$     | 0.243     | 0.056 0.417              | 0.186                     | 0.048 0.313              |
| $\rho^{pp}$    | 0.960     | 0.933 0.987              | 0.974                     | 0.954 0.994              |
| $\rho^{p}$     | 0.991     | 0.985 0.998              | 0.932                     | 0.894 0.971              |
| $\rho^{r}$     | 0.536     | 0.418 0.661              | 0.661                     | 0.562 0.757              |
| $\rho^{gm}$    | 0.260     | 0.084 0.433              | 0.117                     | 0.020 0.206              |
| $\rho^{pp}$    | 0.060     | 0.007 0.110              | 0.056                     | 0.007 0.102              |
multiplier is defined as the percent change in de-trended output in the first period after shocks, when output ratio of each government expenditure increases by one percent in a single period at the steady state. The influence of the extension of the model on the multiplier is classified into the difference from the estimation results of the parameters presented in Table 4 and changes in the model itself. The latter is expected to decrease the effects of expansionary policy since it introduces distortionary tax systems.

The multiplier of merit goods expenditure is reduced from 1.92 to 1.43 through the extension of the model. From the estimate of the complementarity parameter, it is expected that complementarity with private consumption become strong and thus the multiplier would increase; however, here it is thought that the tax distortion pushed down the multiplier to a greater extent. For public goods expenditure, the decrease in substitution increase the multiplier, while distortionary tax systems reduce it, resulting in a decrease from 0.25 to 0.11.

<table>
<thead>
<tr>
<th></th>
<th>Model (1)</th>
<th>Model (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merit Goods Expenditure</td>
<td>1.427</td>
<td>1.920</td>
</tr>
<tr>
<td>Public Goods Expenditure</td>
<td>0.217</td>
<td>0.246</td>
</tr>
<tr>
<td>Government Investment Expenditure</td>
<td>0.706</td>
<td>0.924</td>
</tr>
</tbody>
</table>

(Note) The multiplier is the percent change in de-trended output in the first period after shocks, when output ratio of each government expenditure increases by one percent in a single period at the steady state. They are calculated by using the posterior mean of the estimated parameters.
0.22. The government investment multiplier also falls from 0.92 to 0.71 as a result of the introduction of tax systems and reduction in public capital investment productivity.

IV. Simulation Analysis

IV-1. Simulation of merit goods spending when financed by different taxes

In this section, based on the posterior mean of the parameters estimated in the previous section, we perform a simulation analysis of merit goods expenditure financed by a single type of tax without adjusting for other taxes, expenditures, and the issuance of government bonds. This analysis allows us to compare the effects of increases in consumption tax, labor income tax, and capital income tax, because the path of government expenditure is given and covered by a tax increase of the equivalent size. Moreover, since merit goods expenditure pertains to so-called payments in kind in healthcare, long-term care, and education expenditures, this simulation can also be interpreted as an analysis examining how policy effects vary depending on the method of financing when financing these expenditures without the issuance of additional government bonds.

Figure 1 depicts the results of the simulation, and it is confirmed that the path of merit goods expenditure is the same regardless of the tax used for finance. Moreover, there are almost no differences between the cases of consumption tax and labor income tax because for infinitely lived representative individuals, the amount of tax collected is the same in both cases. Hence, these two taxes are essentially equivalent.

Ricardian households’ consumption increases most in the case of financing through capital income tax in the short term. This is because capital income decreases through an increase in the capital income tax rate and the substitution of future consumption for present consumption takes place. By contrast, consumption declines compared with the other tax cases in the long run through reductions in saving and investment and the resulting deterioration in the economy. In addition, also for non-Ricardian households’ consumption, the largest increase is in the capital income tax case; however, the reason is not intertemporal substitution as seen in Ricardian households, but rather that non-Ricardian households are not directly negatively affected by an increase in capital income tax since they do not possess any assets. Aggregate consumption fluctuates and almost equivalent to that of Ricardian households since the share of non-Ricardian households is small. Since capital income tax causes a decrease in capital return in the long term, private investment continues to decline compared with the other two tax instances, and consequently, capital falls sharply. In addition, the low level of capital stock raises the return on capital in the long run. Labor supply increases due to the negative wealth effect, little difference is observed between any of the taxes in the short term. However, the adjustment is slow in the case of capital income tax.

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16 Although omitted from Figure 1, because tax increases are the same size as expenditure, the responses in the output ratio of total tax revenues are also the same for each tax case.
because of a prolonged negative influence. Reflecting these patterns in labor supply, a greater long-term decline in real wages continues in the case of capital income tax. The fluctuation in price reflects these changes in marginal cost, and no large differences are observed between the taxes. In addition, the capital tax rate remains high in the longer term, because medium-to-long-term economic deterioration decreases tax revenues and the adjust-

(Note) The solid, dashed, and dotted lines in the figure respectively represent the impulse responses of each variable when financing merit goods expenditure by consumption tax, labor income tax, and capital income tax. In the tax rates panel, they correspond to the consumption tax rate, labor income tax rate, and capital income tax rate, respectively. In the simulation, the size of a merit goods shock is one percent of steady-state output, and the other expenditures and tax rates and government bond issuance are unchanged.
ment of the tax rate is slow.

To summarize the above, there are no significant differences between consumption tax and labor income tax as a means of financing government expenditures; however, capital income tax causes a large decline in investment and capital accumulation, which has mid-to-long-term effects on economy accompanying fluctuations in the relative price of production factors. More specifically, for consumption and labor income taxes, the deviation in output from the equilibrium path becomes negative around the 20th period and stands at about −0.5% by the 100th period. By contrast, for capital income tax, this becomes negative in the 15th period, falling to −1.3% in the 100th period. Meanwhile, since capital income tax causes the intertemporal substitution of consumption in the short term, growth in consumption is maximized until around the 30th period. These results are qualitatively similar to the conclusions of standard economic models, and the simulation results show that financing government expenditures by capital income tax leads to the most negative effects on the economy. Finally, although there are no significant differences between consumption and labor income taxes, the model in this study did not consider the retirement of households, unlike the overlapping generations model. Furthermore, since, in practice, labor income tax has a progressive structure, when considering these features, labor income tax may be less effective than consumption tax.

IV-2. Simulation of differing government expenditures through an increase in the consumption tax rate

Subsequently, when using higher tax revenues from an increase in the consumption tax rate for additional government expenditure, we examine which differences arise in the policy effects depending on expenditure types. In contrast to the previous analysis, while holding tax rates besides the consumption tax rate and expenditures other than the additional government expenditure, public debt and lump-sum income transfers can change in accordance with the government’s budget constraints and policy rules. Note that shocks to the consumption tax rate in this analysis are not permanent but temporary, although sufficiently persistent.

Figure 2 illustrates the simulation results when the consumption tax rate increases such that the consumption tax revenue is one percent of the steady-state output. As shown in the panel of tax rates and government expenditure, the rise in tax rates is equal in each case, and each government expenditure increases by one percent of steady-state output.

Owing to the Edgeworth complementarity, although Ricardian households’ consumption somewhat increases instantaneously in response to an increase in the consumption tax rate, it decreases by 0.6% in the mid-to-long run. Moreover, labor supply increases due to the

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17 When public debt fluctuates, since the Blanchard-Kahn condition is not satisfied unless there is a sufficient decrease in expenditure or increase in tax receipts in response to an increase in debt, we permit fluctuations in income transfers that have a strong debt stabilization effect.

18 In the simulation, the parameter regarding the persistency of consumption tax rate rule $\phi^t$ is set to 0.99.
negative wealth effects, and therefore, wage decreases. As a result, output increases in the short term, and falls by 0.17% in the mid-to-long run. Output ratio of tax revenues then increases more than one percent in the short term through increases in consumption and output, whereas they decrease with lowered tax rates, consumption, and output. Corresponding to these movements, the output ratio of public debt reduces by two percent or more at the maximum but starts to increase in the long run.

When using the increase in consumption tax receipts for public goods expenditure, Ricardian households’ consumption decreases by two percent in the short term because of the Edgeworth substitutability, and similar to when spending on merit goods, leading to an approximate 0.6% decrease in the mid-to-long term. Then, the increase in public goods and
decrease in consumption are cancelled out, and Ricardian households’ effective consumption is hardly changed. As a result, the other economic variables are relatively unchanged and public debt increases slightly from the effects of reduced consumption and output. When carrying out government investment, decreases in consumption and output are in the middle of those in the case of merit goods and public goods; however, in the long term, the increase in productivity through the public capital accumulation increases consumption and output by about 0.36% and 0.67%, respectively. Consequently, tax revenues increase, and public debt decreases. 

V. Conclusion

This study constructs a New-Keynesian DSGE model incorporating multiple government expenditures and tax systems, estimates each structural parameter by using Bayesian estimation, and reveals the characteristics of Japanese government expenditure and tax rate rules. Specifically, with respect to government expenditure and taxation, the response to output fluctuations and debt accumulation is quantitatively small. Furthermore, the estimated impact multipliers for merit goods expenditure, public goods expenditure, and government investment are 1.43, 0.22, and 0.71, respectively.

Additionally, we conduct two simulation analyses based on the estimation results. In the first simulation, we examine how the policy effects of merit goods expenditure change depending on the type of tax used for financing. The analysis indicates that financing by capital income tax, compared with doing so with other taxes, increases consumption in the short term through the intertemporal substitution of consumption and has a negative effect on the economy in the mid-to-long term by impeding capital accumulation.

The second simulation demonstrates that when using increased tax revenues from a rise in the consumption tax rate for additional government expenditure, how the policy effects change depending on the item of expenditure. Then, we show that the merit goods expenditure through tax receipt increases does not decrease private consumption in the short term because of the Edgeworth complementarity between merit goods and private consumption, thus yielding a positive economic effect. However, public goods expenditure largely reduces private consumption due to the substitution effect. For government investment, the short-term effects are in the middle of those in the case of merit goods expenditure and public goods expenditure. However, an increase in productivity through public capital accumulation leads to a positive effect on the economy in the long run.

To analyze the effects of more various fiscal policies, this study can be extended in several directions. Firstly, the model is extended to include a frictional labor market, heterogeneity among households, and the global market structure. These extensions respectively allow us to analyze unemployment, redistribution, and the effects on trade and exchange rate. Secondly, in the simulation analysis, we only focus on temporary tax increases. However, analyzing more realistic policy topics such as permanent structural changes in taxation accompanying expenditure increases and their effects on fiscal reform is considered to be ex-
tremely important in policy analysis. We wish to address these topics in the future.

References


Appendix

This appendix presents a log-linearized version of our model. The non-stationary variables in period $t$ are de-trended by technology level $Z_t$ and represented by lowercase letters with subscript $t$. Their steady-state levels are presented without subscripts. Alternatively, the log-deviations from steady-state levels are denoted in lowercase letters with a tilde and subscript $t$. For example, $y_t = Y_t / Z_t$ and $\tilde{y}_t = \log y_t - \log y$. Note that only tax rate variables with tilde are defined as the difference from steady-state levels, rather than log-deviations ($\tilde{\tau}_t^j = \tau_t^j - \tau^j, j \in \{c, w, k\}$).

Ricardian household’s effective consumption:

$$\frac{c^e}{y} \tilde{c}_t = \frac{c^R}{y} \tilde{c}_t^R + \frac{\nu^m g^m}{y} \tilde{g}_t^m + \frac{\nu^p g^p}{y} \tilde{g}_t^p$$

19 The de-trended version of the Lagrange multiplier is defined as $\lambda_t = \Lambda_t Z_t$. Moreover, the shocks $z_w^w$ and $z_w^p$ are defined as $z_w^w = (1 - \xi_w)(1 - \xi_w z^w) \lambda_t^w \lambda_t^w \chi (1 + \lambda^w)$ and $z_w^p = (1 - \xi_w)(1 - \beta \xi_w z^p) \lambda_t^p \lambda_t^p$. 
Ricardian households’ marginal utility of consumption:

\[
\left(1 - \frac{\theta}{z}\right) \left(1 - \frac{\beta \theta}{z}\right) \left(\tilde{\lambda}_t + \tilde{\tau}_t^c \right) - z \theta \sigma \left(\beta \theta - \lambda_t^t + 1\right) \left(1 - \frac{\theta}{z}\right) z_t^h + \frac{\beta \theta}{z} \left\{ \sigma \left(\mathbf{E}_t \tilde{c}_{t+1}^c + \mathbf{E}_t z_{t+1}^c - \frac{\theta}{z} \tilde{c}_t^c\right) - \left(1 - \frac{\theta}{z}\right) \mathbf{E}_t z_{t+1}^h \right\}
\]

Lagrange multiplier (Euler equation):

\[
\tilde{\lambda}_t = \mathbf{E}_t \tilde{\lambda}_{t+1} - \sigma \mathbf{E}_t z_{t+1}^c + \tilde{R}_t^n - \mathbf{E}_t \tilde{\pi}_{t+1}
\]

Wages:

\[
\tilde{w}_t = \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma \tilde{\pi}_{t-1} + z_t^i
\]

\[
= \beta z^{1-\sigma} \left(\mathbf{E}_t \tilde{w}_{t+1} - \tilde{w}_t + \mathbf{E}_t \tilde{\pi}_{t+1} - \gamma \tilde{\pi}_t + \mathbf{E}_t z_{t+1}^i\right)\]

\[
+ \frac{1 - \zeta^w}{\zeta^w} \left(1 - \frac{\zeta^w}{\zeta^w} \frac{\beta z^{1-\sigma}}{\lambda^w} \lambda^w \left(\chi \tilde{t}_t - \tilde{\lambda}_t - \tilde{w}_t + \frac{\tilde{\tau}_t^w}{1 - \tau^w} + z_t^i\right) + z_t^w\right)
\]

Private capital accumulation:

\[
\tilde{k}_t = \frac{1 - \delta}{z} \left(\tilde{k}_{t-1} - z_t^i\right) = \frac{(1 - \tau^t) R_k^t}{z} \tilde{u}_t + \left(1 - \frac{1 - \delta}{z}\right) \tilde{t}_t
\]

Capital utilization rate:

\[
\tilde{u}_t = \mu \left(\tilde{R}_t^k - \frac{\tilde{\tau}_t^k}{1 - \tau^k - \tilde{q}_t}\right)
\]

Investment:

\[
\tilde{r}_t - \tilde{r}_{t-1} + z_t^i + z_t^j = \frac{\beta z^{1-\sigma} \left(\mathbf{E}_t \tilde{r}_{t+1} - \tilde{r}_t + \mathbf{E}_t z_{t+1}^r + \mathbf{E}_t z_{t+1}^r\right)}{\zeta}
\]

Tobin’s \(q\):

\[
\tilde{q}_t = \mathbf{E}_t \tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma \mathbf{E}_t z_{t+1}^c + \frac{\beta}{z^c} \left(1 - \tau^t\right) R_k^t \mathbf{E}_t \tilde{R}_t^{t+1} - R_k^t \mathbf{E}_t \tilde{\tau}_{t+1}^k + \left(1 - \delta\right) \mathbf{E}_t \tilde{q}_{t+1}
\]
Non-Ricardian households’ consumption:
\[
\frac{c^{NR}}{\bar{y}} \mid (1 + \tau') \tilde{c}^{NR} + \tilde{r}_{t} = \frac{wL}{\bar{y}} \mid (1 - \tau' ) \left( \tilde{w}_{t} + \tilde{l}_{t} \right) - \tilde{c}^{NR} + \frac{\tau}{\bar{y}} \tilde{r}_{t}
\]

Intermediate goods firms’ production function:
\[
y_{t} = (1 + \phi) \left( (1 - \alpha) \tilde{l}_{t} + \alpha (u_{t} + \tilde{k}_{t-1} - z_{t}^{*}) + v (\tilde{k}_{t-1} - z_{t}^{*}) \right)
\]

Cost minimization:
\[
\tilde{w}_{t} - \tilde{R}^{k}_{t} = u_{t} + \tilde{k}_{t-1} - \tilde{l}_{t} - z_{t}^{*}
\]

Marginal cost:
\[
\tilde{mc}_{t} = (1 - \alpha) \tilde{w}_{t} + \alpha \tilde{R}^{k}_{t} - v (\tilde{k}_{t-1} - z_{t}^{*})
\]

Intermediate goods price (New-Keynesian Phillips curve):
\[
\tilde{p}_{t} = \gamma^{p} \tilde{p}_{t-1} = \beta \tilde{e}^{1-p} (E_{t} \tilde{p}_{t+1} - \gamma^{p} \tilde{p}_{t}) + \frac{(1 - \xi^{p}) (1 - \bar{\xi}^{p} \beta \tilde{e}^{1-p})}{\xi^{p}} \tilde{mc}_{t} + z_{t}^{p}
\]

Dividend:
\[
(\lambda^{p} - \phi) \tilde{d}_{t} = \lambda^{p} \tilde{y}_{t} - (1 + \phi ) \tilde{mc}_{t}
\]

Monetary policy rule:
\[
\tilde{R}_{t} = \phi \tilde{R}^{n}_{t-1} + (1 - \phi') \left\{ \phi^{n} \left( \frac{1}{4} \sum_{j=0}^{3} \tilde{p}_{t-j} \right) + \phi^{l} (\tilde{y}_{t} - \tilde{y}^{*}) \right\} + z_{t}^{l}
\]

Potential output:
\[
\tilde{y}_{t}^{*} = -(1 + \phi) (\alpha + v) z_{t}^{*}
\]

Government budget constraint:
\[
b^{tar} \tilde{b}_{t} = \frac{b^{tar}}{\beta \tilde{e}^{1-p}} (\tilde{R}^{n}_{t-1} - \tilde{p}_{t} - z_{t}^{*} + \tilde{b}_{t-1}) + \frac{g^{u}}{y} \tilde{g}^{u} + \frac{g^{p}}{y} \tilde{g}^{p} + \frac{g^{i}}{y} \tilde{g}^{i} + \frac{\tau}{y} \tilde{r}_{t} - \frac{c}{y} (\tilde{r}_{t} + \tilde{e} \tilde{c}_{t})
\]
\[
- \frac{wL}{y} \mid \tilde{c}^{w} + \tau^{w} (\tilde{w}_{t} + \tilde{l}_{t}) \mid - \frac{d}{y} (\tilde{k}_{t} + \tilde{d}_{t}) - \frac{\tilde{R}^{k}_{t-1}}{z} (\tilde{k}_{t} + \tilde{c}^{k} \tilde{R}^{k}_{t} + \tilde{u}_{t} + \tilde{k}_{t-1} - z_{t}) \mid
\]
Public capital accumulation:
\[ \ddot{k}_t^g = \frac{1 - \delta^g}{z} (\ddot{k}_{t-1}^g - z_t^* + \left(1 - \frac{1 - \delta^g}{z}\right) \ddot{g}_t^i) \]

Government expenditure rules:
\[ \ddot{g}_t^m = \phi^{gm} (\ddot{g}_{t-1}^m - z_t^*) + (1 - \phi^{gm}) (\phi^{gm}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{gm}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + z_t^{gm} \]
\[ \ddot{g}_t^p = \phi^{gp} (\ddot{g}_{t-1}^p - z_t^*) + (1 - \phi^{gp}) (\phi^{gp}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{gp}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + z_t^{gp} \]
\[ \ddot{g}_t^i = \phi^{gi} (\ddot{g}_{t-1}^i - z_t^*) + (1 - \phi^{gi}) (\phi^{gi}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{gi}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + z_t^{gi} \]
\[ \ddot{t}_t = \phi^T (\ddot{t}_{t-1} - z_t^*) + (1 - \phi^T) (\phi^T_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^T_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + z_t^T \]

Tax rate rules:
\[ \ddot{\tau}_t^c = \phi^{tc} \ddot{\tau}_{t-1}^c - (1 - \phi^{tc}) (\phi^{tc}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{tc}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + \varepsilon_t^{tc} \]
\[ \ddot{\tau}_t^w = \phi^{tw} \ddot{\tau}_{t-1}^w - (1 - \phi^{tw}) (\phi^{tw}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{tw}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + \varepsilon_t^{tw} \]
\[ \ddot{\tau}_t^k = \phi^{tk} \ddot{\tau}_{t-1}^k - (1 - \phi^{tk}) (\phi^{tk}_y (\ddot{y}_{t-1} - \ddot{y}_{t-1}^s) + \phi^{tk}_b (\ddot{b}_{t-1} - \ddot{y}_{t-1})) + \varepsilon_t^{tk} \]

Aggregate consumption:
\[ \frac{c}{y} \ddot{c}_t = \frac{(1 - \omega)}{y} \ddot{c}_t^R + \frac{\omega c^{NR}}{y} - \ddot{c}_t^{NR} \]

Market clearing condition:
\[ \ddot{y}_t = \frac{c}{y} \ddot{c}_t + \frac{i}{y} \ddot{r}_t + \frac{g^m}{y} \ddot{g}_t^m + \frac{g^p}{y} \ddot{g}_t^p + \frac{g^i}{y} \ddot{g}_t^i + \frac{x}{y} z_t^x \]

Structural shocks:
\[ z_t^j = \rho_t z_{t-1}^j + \varepsilon_t^j, \quad j \in \{b, w, p, z, i, x, r, gm, gp, gi, T\} \]
\[ \varepsilon_t^j \sim N(0, \sigma_j^2) \]