Public Debt Overhang and Economic Growth *

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Abstract

Reinhart, Reinhart and Rogoff (2012) call the deterioration of the economy due to an increase in public debts a public debt overhang. This paper provides an overview of empirical studies on public debt overhang using data concerning economic crises that have recently been accumulated. Moreover, it takes a look at related theoretical models and shows that it is difficult to explain public debt overhang based on existing theories. Under existing models, which assume that fiscal deterioration brings low growth, an interest rate rise arising from fiscal deterioration causes economic deterioration. Essentially, this phenomenon is similar to crowding-out. However, in many cases of public debt overhang, including the situation of Japan, low growth and low interest rates coexist, a phenomenon which cannot be explained by the crowding-out effect.

The latter half of this paper examines a new theoretical model and provides a theoretical basis for a possible formation of public debt overhang that simultaneously brings low growth and low interest rates. Under this theoretical model, as public debts (government bonds) function as liquid assets, their increase itself has the effect of promoting economic growth (liquidity supply effect). Meanwhile, if productive economic entities are taxed and subsidies are paid to the labor sector as a result of fiscal policy, workers whose income has increased reduce the supply of labor (income effect). As a result, the wage rate rises, prompting highly productive companies to reduce production. When the income effect prevails over the liquidity supply effect, fiscal deterioration reduces production. In addition, market interest rates fall because highly productive companies’ demand for borrowings declines. In this way, fiscal deterioration (increases in public debts and subsidies) leads to decreases in both productivity and interest rates.

This theoretical model brings results consistent with the public debt overhang hypothesis. If there is a cause-and-effect relationship between fiscal deterioration and the deterioration of real economic conditions as indicated by Reinhart et al. and this paper, it will render useless the strategy that Japan has pursued for many years - stimulating the economy by continuing expansionary fiscal policy. That is because such fiscal policy works to restrain economic growth by aggravating the fiscal position. According to the theoretical model, in order to increase overall production, it will be necessary to reduce the fiscal burden on productive companies while increasing the fiscal burden on the labor sector. The labor sector in the

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model corresponds to the household sector in the real world. Therefore, the policy implication for the real world is that it will be desirable to increase the ratio of the fiscal burden on the household sector by further raising the consumption tax rate and reducing social security benefits.

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I. Introduction

It goes without saying that restoring fiscal sustainability is one of the greatest economic policy challenges for Japan.

Although the consumption tax rate has been raised twice (1997 and 2014), the amount of public debt in Japan has consistently increased for around 20 years since the bubble economy era, reflecting a lack of sufficient fiscal consolidation efforts. Presumably, one major reason why Japanese policy authorities have been unable to launch a full-fledged fiscal consolidation initiative for many years is their recognition of the cause-and-effect relationship between the fiscal position and economic growth. It has generally been presumed that there is a cause-and-effect relationship of “low growth leading to fiscal deterioration” — stagnant economic growth and economic pump-priming intended to stimulate growth are aggravating the fiscal position — but that the opposite of this relationship, “fiscal deterioration leading to low growth,” does not exist.

If a country’s economic growth rate slows down, its fiscal position will deteriorate due to a tax revenue fall unless there are changes in institutional systems and policies. There is no room for doubting the causal relationship of “low growth leading to fiscal deterioration.” On the other hand, it is intuitively difficult to understand the argument that fiscal deterioration lowers economic growth through some sort of mechanism. Moreover, such an argument has seldom been made until recently. Until now, there have not been prominent proponents for the argument, in either economic academic circles or among economic commentators.

From the viewpoint of the general public, fiscal consolidation is not a goal in itself. The ultimate goal is sound development of the Japanese economy and the Japanese people’s everyday lives, and fiscal consolidation is a means to that end. Therefore, given the goal of improving the people’s everyday lives, it is rational to conclude that priority should be given to rebuilding the economy, which has been stagnant due to the collapse of economic bubbles as well as the declining birthrate and the aging of society, and that fiscal consolidation should be put off until later. This thinking has until now supported the policy judgment to implement economic pump-priming measures over and over again and refrain from reducing the increasing social security expenditures while reluctantly accepting fiscal deterioration.

However, if this thinking is to have legitimacy, it is essential that a cause-and-effect relationship of “fiscal deterioration leading to low growth” does not exist. If fiscal
deterioration could become a cause of low growth, it is possible that expansionary fiscal policy, including economic pump-priming, has undermined economic growth, at least in the medium to long term. A fall in the economic growth rate impedes the realization of the ultimate economic policy goal of improving the people’s everyday lives. In other words, if there is a cause-and-effect relationship of “fiscal deterioration leading to low growth,” economic management based on the policy of aiming to raise the economic growth rate while putting off fiscal consolidation until later will have quite the opposite effect — lowering economic growth — from the one intended. If that is true, it could mean the core strategy of Japan’s economic policy for the past 20 years — promoting economic growth through economic pump-priming — has been misguided.

As is shown above, a study that examines a cause-and-effect relationship between fiscal deterioration and economic growth will have very significant policy implications. The relationship between the fiscal position and growth has come to be actively debated among U.S. economic commentators, as empirical studies by Reinhart, Reinhart and Rogoff (2012) and other experts have recently argued for the existence of a cause-and-effect relationship of “fiscal deterioration leading to low growth.” In this paper, we examine preceding studies concerning the relationship between fiscal deterioration and economic growth, including the study by Reinhart et al. and sort out the points of debate. At the same time, I propose a new theoretical model and explore policy implications by analyzing the characteristics through simulations.

This paper is structured as follows. Section II provides an overview of empirical studies concerning the phenomenon of fiscal deterioration undermining economic growth, and Section III provides an overview of theoretical studies. Section IV provides an overview of theoretical studies concerning a public debt increase’s effect of promoting economic growth. Section V establishes a theoretical model for fiscal deterioration lowering the economic growth rate and analyzes the characteristics of the model through simulations while making references to its relationship with preceding studies. Section VI offers our conclusion.

II. Empirical Studies concerning Fiscal Position and Economic Growth

II-1. Finding Public Debt Overhang

While how the fiscal position affects economic growth has long been a matter of interest, data on countries thrown into fiscal crises have been collected and databases concerning the relationship between the fiscal position and economic growth have been built up in recent years. Such data are starting to provide new insights.

Among studies using fiscal-position-related data that have recently been accumulated are those which argue for the existence of a cause-and-effect relationship of “fiscal deterioration undermining economic growth through some sort of mechanism,” which is the opposite of the cause-and-effect relationship of “low growth leading to fiscal deterioration.” If a cause-and-effect relationship of “fiscal deterioration leading to low growth” exists, the basic
strategy of the Abenomics policy — putting off fiscal consolidation until later and raising the
economic growth rate through other policy measures — could prove useless. That is because
in that case, even if policy measures to raise the economic growth rate are implemented,
fiscal deterioration would curb economic growth, making it difficult to achieve the original
goal of raising the growth rate. This is an important issue that concerns the core of Japan’s
current economic policy.

The argument that accumulation of public debt (fiscal deterioration) has a negative
impact on economic growth was made in the studies by Reinhart et al. that concern public
debt overhang (Reinhart, Reinhart and Rogoff, 2012, and Reinhart and Rogoff, 2010). Reinhart, Reinhart and Rogoff (2012) reviewed 26 cases of high accumulation of public debt
in advanced countries and reported that in 23 of those cases, economic growth remained
stagnant for more than a decade. What is notable about their findings is the presence of a
non-linear relationship between public debts and economic growth. It was shown that when
the ratio of public debts to GDP is higher than 90%, the annual economic growth rate is as
much as 1.2% lower than when the public debt ratio is less than 90%. When the public debt
ratio is small, an increase in debts is not observed to have an impact on economic growth.
However, when the public debt ratio is higher than 90%, there is a tendency that an increase
in debts lowers economic growth. In light of this non-linear relationship, Reinhart, Reinhart
and Rogoff (2012) argues for the existence of a cause-and-effect relationship of “cumulative
increase in public debts undermining economic growth.”

If there is a cause-and-effect relationship of a decline in economic growth increasing
public debts but not of a public debt increase lowering economic growth, the correlation
between a public debt increase and a decline in economic growth would be observed
regardless of the size of the public debt ratio. However, according to the available data, such
a correlation is not observed when the public debt ratio is small. Such a correlation is
observed only when the public debt ratio is large (roughly higher than 90%). This suggests
the existence of a cause-and-effect relationship of a public debt increase above a certain
threshold undermining economic growth through some sort of mechanism. This is the
argument made by Reinhart, Reinhart and Rogoff (2012).

Moreover, Reinhart et al reported that in 10 of the 26 high-debt cases they reviewed,
including Japan’s situation during the past 20 years, interest rates were either declining or
flat. Indeed, in Japan, real interest rates remained stable at a low level for the past two decades
since the collapse of economic bubbles compared with the rates in the prior period (an
international comparison of real lending interest rates based on GDP deflator shows that real
interest rates in Japan have remained stable at around 3% for the past two decades, a level
similar to or slightly lower than the rates in such countries as the United States and France).
The absence of an interest rate rise during the period of low growth suggests that the usual
crowding-out mechanism is not functioning. In other words, it is possible that fiscal
deterioration reduces demand through some type of indirect mechanism and makes private
economic activity inefficient (this point will be discussed again in Section II-2).

Regarding Reinhart and Rogoff (2010), which forms the basis of this paper, Herndon,
Ash and Pollin (2013) pointed out data processing errors, raising much controversy over the reliability of the results. However, the errors have been addressed by Reinhart, Reinhart and Rogoff (2012). Moreover, the revised results provided by Herndon, Ash and Pollin (2013) were not very different qualitatively from the results provided by Reinhart, Reinhart and Rogoff (2012) (Reinhart and Rogoff, “Full Response from Reinhart and Rogoff,” New York Times, April 17, 2013).

The empirical finding that fiscal deterioration lowers the economic growth rate was confirmed by Checherita-Westphal and Rother (2012) and Baum, Checherita-Westphal and Rother (2013) as well. Checherita-Westphal and Rother (2012) examined the relationship between public debts and per-capita GDP through various methods based on data concerning 12 euro-area countries for the past 40 years. As a result, it was confirmed that when the ratio of public debts to GDP was higher than the 90-100% range, an increase in the public debt ratio had the effect of reducing per-capita GDP. It was also shown that this effect worked through three channels — a decrease in private savings, a decline in public investments and a decrease in total factor productivity. Baum, Checherita-Westphal and Rother (2013) examined the relationship between public debts and per-capita GDP based on data concerning 12 euro-area countries for the period since 1990. As a result, it was observed that when the ratio of public debts to GDP was lower than 67%, a public debt increase had a positive correlation with GDP, meaning it had the effect of increasing GDP. However, it was also shown that when the debt ratio was higher than 95%, a public debt increase had the effect of reducing GDP. Moreover, it was empirically verified that a rise in the debt ratio affected interest rates. It was confirmed that although a rise in the debt ratio had the effect of lowering interest rates when the debt ratio was lower than 70%, it puts upward pressure on interest rates when the debt ratio was higher than 70%.

II-2. Difference with Preceding Empirical Studies

Public debt overhang has been found as a result of the development of a database concerning fiscal crises in recent years. Before the development of data by Reinhart et al., it was not known that the balance of public debt affects economic growth. For example, Barro and Sala-i-Martin (1995) empirically showed that the ratio of government consumption to GDP has a negative impact on per-capita GDP. However, it was not confirmed whether the amount of public debt has a significant impact. Meanwhile, Fischer (1991) empirically showed that a fiscal deficit has a negative impact on per-capita GDP but did not confirm whether or not the amount of public debt affects per-capita GDP.

The differences between the studies by Barro and et al. and by Reinhart et al. have a notable significance. According to the studies by Barro et al. and by Fischer, an increase in government expansion and an expansion of the fiscal deficit represent a waste of resources on inefficient government activities. Therefore, their findings indicate that government activities led directly to an increase in inefficiency, resulting in a decline in economic growth. This is consistent with the theory of crowding-out, which refers to private activities being
crowded out by government activities. In other words, while the findings of Barro et al. and Fischer indicate that inefficient government activities crowd out private activities, it cannot be said that private economic activities themselves become inefficient as a result of fiscal deterioration. However, the recent findings of Reinhart et al. indicate that fiscal deterioration somehow distorts decision-making by private economic entities, thereby making private activities themselves inefficient, rather than causing economic growth to slow down because private activities are crowded out by inefficient government activities.

While Barro and Sala-i-Martin (1995) and Fischer (1991) also indicate that an increase in fiscal spending and a deterioration of the fiscal balance have a negative impact on economic growth, the correlation is linear. On the other hand, according to the findings of Reinhart et al., there is a threshold line (public debt ratio of around 90%), above which public debts have the effect of lowering the economic growth rate but below which they do not affect the growth rate.

The theory of public debt overhang, which maintains that public debts have a negative impact on economic growth when the public debt ratio is above a threshold line, is different from the findings of Barro et al. in that the debt-growth relationship under this theory is non-linear, so we may say that a new insight is provided by the recent studies by Reinhart et al.

III. Overview of Theoretical Studies—Can public debt overhang be explained by existing theories?

In this section, I will discuss whether public debt overhang can be explained by existing economic theories. My investigation of several relevant theories has led me to the conclusion that it would be difficult to explain the phenomenon of public debt overhang using existing economic theories.

III-1. Crowding-Out

What first comes to mind as a textbook explanation for the relationship between an increase in public debt and a decline in economic growth is the crowding-out mechanism. If a government continues to waste resources through loose public expenditures, the economy as a whole will face a resource shortage, preventing sufficient private-sector investment. If the crowding-out mechanism is triggered, private-sector capital accumulation will consequently become insufficient, leading to economic stagnation. Given that a cumulative increase in public debt can be viewed as a barometer of loose fiscal policy, this argument sounds persuasive. However, the data on public debt overhang used by Reinhart et al. show that the crowding-out theory did not apply in many cases. A crowding-out would surely have caused a rise in real interest rates. However, in 10 of the 26 high-debt cases reviewed by Reinhart et al., interest rates were either declining or flat, as was mentioned earlier. It is
difficult to explain these cases on the basis of the crowding-out mechanism.  

**III-2. Non-Keynesian Effects**

The theory of non-Keynesian effects is another candidate theory to explain the negative impact of public debt on economic growth. Non-Keynesian effects refer to the phenomenon of consumption demand rising as a result of fiscal contraction (consumption demand falling as a result of fiscal expansion). The effects are said to have been observed in the cases of fiscal consolidation implemented in Denmark and Ireland in the 1980s (Giavazzi and Pagano 1990). Non-Keynesian effects have attracted attention from time to time in relation to the deterioration of Japan’s fiscal position (Kameda 2008 and 2010; Nakazato 2002, etc.).

Regarding the mechanism of how non-Keynesian effects arise, Perotti (1999) proposed the following relatively easy-to-understand theory. In a country with a sound fiscal position, fiscal expansion does not produce the effect of shrinking consumption but the Keynesian effect (fiscal expansion’s effect of increasing consumption demand, etc. through multiplier effects) dominates. However, in a country with a poor fiscal position, fiscal expansion leads the people to expect a tax increase. If it is assumed that a future tax increase will be distortionary, the people will expect to lose wealth (more than they would lose because of the effect of the tax increase alone) in the future because the economy will become less efficient and production will decline as a result of a higher tax burden. To prepare for a future loss of wealth, the people increase savings now. Consequently, their current consumption decreases. In other words, non-Keynesian effects arise: a current fiscal expansion (growth in public debt coupled with an increase in expenditures or tax reduction) leads households and companies to expect a deterioration of economic conditions due to a future tax increase, resulting in reduction of consumption demand, etc. at present.

Non-Keynesian effects are not observed while the amount of public debt is small, but they are presumed to arise when the amount increases cumulatively. In that respect, the effects are consistent with the results of the observation of public debt overhang by Reinhart et al.

However, the theory of non-Keynesian effects maintains that fiscal expansion reduces consumption demand but does not necessarily cause a decline in economic growth. Rather, because it brings about a reduction of consumption and an increase in savings, fiscal expansion is presumed to lead to an increase in capital accumulation and a rise in the economic growth rate (barring other problems such as price rigidity), according to the theory of non-Keynesian effects. Moreover, the theory of non-Keynesian effects assumes that fiscal expansion causes a short-term contraction in demand but not a long-term decline in economic growth, lasting as long as 10 years, as was shown by the results of verification by Reinhart et al.

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1 A study by Baun, Checherita-Westphal and Rother (2013) indicated that a cumulative increase in public debt results in upward pressure on interest rates. This suggests that the crowding-out mechanism was working when the fiscal position of the euro zone was deteriorating.
The reason why a short-term contraction in demand is not an adequate explanation of public debt overhang is that a contraction in demand does not necessarily cause a deterioration in economic productivity. Meanwhile, the findings of Reinhart et al. suggest that an increase in public debt and long-term stagnation in economic growth are linked. Unless some type of mechanism triggers a deterioration in economic productivity, it is difficult to explain a continuous stagnation in the economic growth rate. In this respect, the theory of non-Keynesian effects is not fully sufficient to explain the findings of Reinhart et al.

III-3. Soft Budget Constraint

The soft budget constraint is a concept widely debated as a mechanism whereby government finance undermines private economic activities (Kornai 1980, 1986, etc). Akai (2006) conducted a detailed survey on the soft budget constraint and proposed a theory that explains the deterioration of local governments’ fiscal position from the perspective of the soft budget constraint. The soft budget constraint refers collectively to the phenomena of companies and organizations on the brink of bankruptcy being bailed out with governmental fiscal support. In other words, it refers to a phenomenon of companies and organizations that would go bankrupt if the initial budget constraint was maintained being bailed out because of the ex-post softening of the constraint, from which the phrase “soft budget constraint” derives.

As companies and organizations with low productivity survive in a country where the soft budget constraint syndrome is endemic, the economic growth rate there is presumed to decline. In addition, as such companies and organizations are kept afloat with fiscal support from the government, the fiscal position is presumed to deteriorate due to an expansion of fiscal spending. Thus, the soft budget constraint syndrome is effective as a mechanism that explains a situation where the deterioration of the governmental fiscal position and economic growth co-exists. Even so, explaining the co-existence of a fiscal deterioration and a decline in economic growth on the basis of the soft budget constraint theory poses two problems. One is that the soft budget constraint is a theory that is typically applied to countries where the government constantly intervenes in private economic activities, such as socialist countries. Therefore, it is questionable whether the soft budget constraint theory should be used for the analysis of economic growth in advanced countries such as Japan, the United States and European countries. In addition, the soft budget constraint theory does not necessarily explain cases of interest rates declining when fiscal deterioration and a decline in economic growth are proceeding at the same time, a situation assumed under the theory of public debt overhang. Theoretical models of the soft budget constraint proposed by Berglof and Roland (1997) and other experts are invariably too abstract to be used for forecasting market interest rates. Moreover, Kornai (1980) pointed out that interest rates may be either

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2 The author expresses his appreciation to Ikuko Samikawa for pointing out this argument.
high or low when the soft budget constraint syndrome is observed, so it is not clear under what conditions interest rates fall. These points make it difficult for the soft budget constraint theory to explain public debt overhang.

III-4. Effects of Fiscal Deterioration under Economic Growth Models
— Interest Rate Rise

How public debt affects economic growth and social welfare has long been a matter of interest for economic growth theories. Diamond (1965) showed, based on the overlapping generations model, that in a dynamically inefficient economy in which real interest rates are lower than the economic growth rate, the issuance of public debt improves social welfare. When there is dynamic inefficiency, reducing capital investment through crowding-out due to an increase in public debt improves consumers’ welfare because capital investment would become excessive relative to the socially optimum level.

Diamond’s argument was discussed within the framework of the neo-classical model, in which technological advances are externally provided. Meanwhile, Saint-Paul (1992) showed that under an endogenous growth model, in which technological advances are endogenously determined by economic activities, the issuance of public debt always lowers the economic growth rate and causes social welfare deterioration.

Saint-Paul’s model assumes that the larger the capital stock amount is, the higher productivity is. That is because the model assumes that the externality of technology (which refers to direct diffusion of technological advances without going through market transactions) has the effect of raising productivity of the economy as a whole. Because of the assumption of this externality, the socially optimum level of capital stock is larger than under Diamond’s model, which means that the level of capital stock attained through market equilibrium never surpasses the socially optimum level (under Diamond’s model, the capital stock amount reached through market equilibrium was larger than the socially optimum level). Therefore, while reducing capital stock through the crowding-out effect caused by public debts was an appropriate policy under Diamond’s model, doing so always causes deterioration of consumers’ welfare in Saint-Paul’s model.

To sum up, under Saint-Paul’s model, an increase in public debts causes a decline in economic growth by further reducing capital stock through crowding-out. The level of capital stock is lower than the optimum level in the first place because the externality of technology is assumed. In other words, under Saint-Paul’s model, the crowding-out effect causes economic deterioration, while fiscal deterioration (an increase in public debts) undermines economic growth by causing an underinvestment through an interest rate rise. As was discussed in Section III-1, in many cases of public debt overhang, real interest rates were declining. Because the crowding-out theory does not explain such cases, it is difficult to recognize crowding-out as a cause of debt overhang. Saint-Paul also showed that reducing debts does not lead to a Pareto improvement under internal growth models. That is because if consumption by the current generation increases because of debt reduction, investment
decreases accordingly, resulting in a decrease in capital accumulation. Under Saint-Paul’s model, promoting capital investment through investment subsidies (namely, a policy of internalizing an external economy affected by capital stock) improves the social welfare of all generations, namely achieves a Pareto improvement.

In the context of growth theories, Brauninger (2005), like Saint-Paul, uses a model that combines an endogenous growth theory and an overlapping generation model for the analysis of the relationship between the ratio of fiscal deficit to GDP (rather than the ratio of debts to GDP) and economic growth. It was shown that when the deficit ratio is small, there are two balanced growth paths. In that case, if the deficit ratio rises, the economic growth rate declines. It was shown that if the deficit ratio surpasses a certain threshold line, balanced growth paths disappear, capital stock continues to decrease, and capital stock and production volume will be reduced to zero within a finite period of time.

The mechanism whereby economic equilibrium paths are undermined significantly by a fiscal deficit can be explained by two factors. One is that the model is based on an endogenous economic growth theory that assumes the externality of technology, and the other is that fiscal deterioration causes crowding-out. While the behavior of economic systems is complex, this mechanism is the same as the one observed in Saint-Paul’s model in that fiscal deterioration causes an interest rate rise. In other words, this mechanism is the same as the crowding-out mechanism, so it is not consistent with the finding of Reinhart et al. that interest rates decline in some cases.

Under a framework different from endogenous economic growth theories, Arai, Kunieda and Nishida (2012) propose a model that succeeds, to some degree, in explaining the finding of Reinhart, Reinhart and Rogoff (2012). Arai et al. introduced constraints on financial transactions under a neo-classical growth model. They assumed a condition where private entities (companies) are subject to borrowing constraints and showed that when the ratio of public debt to GDP is small, an increase in public debt promotes economic growth, and that when the debt ratio is large, an increase in public debt lowers economic growth. Under their model, because an increase in public debt causes a rise in real interest rates, it has both the effect of encouraging depositors to increase deposits (crowding-in) in pursuit of high interest rates and the effect of prompting companies to reduce borrowings and investments (crowding-out) because they dislike high interest rates. When the debt ratio is small, the crowding-in effect prevails, resulting in higher economic growth, and when the debt ratio is large, the crowding-out effect prevails, resulting in lower economic growth. These results are consistent with the relationship between public debt and gross product that has been shown by Reinhart et al. in recent years. However, under the model proposed by Arai et al., an increase in public debt always results in an interest rate rise, thereby causing the crowding-in and crowding-out effects. This is different from the finding of Reinhart et al. that interest rates decline in some cases.

The theoretical studies taken up in this section found that when an increase in public debt causes economic deterioration, interest rates rise, so it can be said that an inability to explain cases of interest rate decline is a problem common to them. In order to explain public debt
overhang, it will be necessary to show that the fiscal position affects economic growth through a mechanism that does not entail an interest rate rise. In Section V, I discuss a new theoretical model that tackles this challenge.

IV. Overview of Theoretical Studies (Part 2): Liquidity Supply Effect of Public Debt

There are many theoretical studies, including Diamond’s abovementioned model, that argue that an increase in public debt raises the economic growth rate by improving the efficiency of the economy. To examine public debt overhang theoretically, it is important to first understand these theories. Here, I explain the mechanism whereby public debt promotes economic growth.

There is an increasing number of theoretical ideas arguing that if a public debt increase when the capital stock amount is insufficient at the point of equilibrium due to such factors as financial constraints, then the capital stock amount will increase, leading to growth in production, employment and consumption as well as expansion of economic activities and improvement of welfare. Below, I introduce some of these ideas. What is common to these ideas is the following argument: Households and companies face liquidity risk (e.g., risk of payments being concentrated in a short period of time unexpectedly), so they have no option but to engage in saving activity in preparation for the risk. If productive goods are stored as a means of saving, the efficiency of the economy as a whole declines and production volume decreases because the goods cannot be used for productive purposes. If public debts increase, households and companies facing liquidity risk will become able to choose to purchase public debts as a means of saving. As a result, the goods that have been stored as a means of saving are released, so they are used for productive purposes, leading to an improvement in the efficiency of the economy as a whole and an increase in production. In other words, the argument goes, public debts improve the economy by functioning as a means of liquid saving for households and companies.

IV-1. Woodford (1990): Smoothing of Consumption by Consumers Who Cannot Borrow

Under this model, two types of consumers are assumed to exist. The first type of consumers earn a large income on odd-number days and a small one on even-number days. The second type of consumers earn a small income on odd-number days and a large one on even-number days. They want to keep their consumption constant each day, but their only means to do so is either engaging in savings or purchasing government bonds (they cannot borrow money). In this case, if the supply of government bonds increases, households will have less need to save goods because they can engage in a large amount of savings through purchases of government bonds. Consequently, goods are consumed instead of being saved, and consumption at different points in time will be smoothed, leading to enhanced economic welfare.

Under this model, companies face liquidity risk midway through production (midway through production, it suddenly becomes necessary with a certain probability to input additional raw materials). If the liquidity risk is company-specific, it is possible to disperse the risk through financial intermediation by private-sector banks. However, if the liquidity risk occurs at companies across the country at the same time (if collective liquidity risk occurs), the risk cannot be dispersed through financial intermediation by banks. In this case, it will be important for the government to supply public debt as liquid assets. By holding government bonds in advance, companies can make necessary payments when they face liquidity risk. Under this model, government bonds are liquid assets (assets that can be used as a means of payment) and play the role of quasi-currency.

The government should mitigate the effect of liquidity risk by proactively adjusting the supply of government bonds in accordance with the scale of collective liquidity risk. In an economy in which liquidity is in short supply, increasing the issue amount of government bonds will lead to an increase in production by improving the efficiency of the economy. In other words, in an economy in which companies face liquidity risk, an increase in public debt has the effect of supplying liquidity.


Woodford (1990) and Holmstrom and Tirole (1998) emphasized that public debt has the liquidity supply effect, and this effect has attracted renewed attention in recent years in relation to financial crises. Among the causes of financial crises is an asset bubble, and Caballero and Krishnamurthy (2006) and Kocharlakota (2009) pay attention to the issuance of government bonds as a means to mitigate falls in production and employment due to the collapse of bubbles.

Under a model presented by Kocharlakota et al., bubble assets (assets priced in excess of their fundamental prices; for example, overvalued land and stocks) enhance economic welfare. Although economic welfare deteriorates when a bubble has collapsed, the deterioration can be halted by increasing the issuance of government bonds at the time of the bubble’s collapse.

Under their model, economic entities (households and companies) are subject to borrowing constraints and are unable to borrow a socially optimum amount of funds. Because of the inability to borrow sufficient funds, economic entities engage in savings in preparation for future payment needs. It is possible to store goods as a means of saving, but the stored goods cannot be consumed or used for productive purposes, so the storing of goods undermines the efficiency of the economy. In this case, bubble assets are useful as a means of saving. To make the explanation simple, let us assume that the fundamental value of the
bubble assets is zero. It may appear that if assets whose fundamental value is zero are traded at a positive price, the distribution of economic assets may become inefficient. However, that will not be the case in an economy in which there are borrowing constraints. Rather, if households and companies purchase bubble assets as a means of saving, it will become unnecessary for them to store goods, so the goods freed from the storage purpose can be used for more productive purposes. As a result, the circulation of bubble assets in the market will improve the efficiency of the economy. This point has been emphasized in various studies in recent years (refer to Martin and Ventura (2012), Hirano and Yanagawa (2013), and Aoki and Nikolov (2013), for example).

If the bubble has collapsed and the price of the bubble assets returns to the fundamental value (zero), the bubble assets cannot be used as a means of saving. As a result, an increasing number of people store goods that should be used for productive purposes, and the inefficiency of the economy as a whole grows due to declines in consumption and production. Thus, the bubble’s collapse causes the real economy to contract.

In this case, if the issue amount of government bonds is increased to coincide with the bubble’s collapse, the bonds serve as a substitute for the means of saving that has been lost due to the bubble’s collapse. Consequently, government bonds are used as a preliminary means of saving, making it possible to prevent goods from being used as a means of saving and remaining unused. As a result, the efficiency of the economy does not decline, nor do employment and production decrease. In other words, increasing the issue amount of government bonds flexibly at the time of a bubble’s collapse will become an effective policy measure to prevent economic contraction.

This argument by Kocherlakota is a major basis for justifying a large-scale fiscal expansion after the failure of Lehman Brothers. According to Kocherlakota, the mechanism whereby an expansionary fiscal policy prevents the deepening of a financial crisis is different from the textbook Keynesian economic mechanism whereby total demand is expanded through multiplier effects. They argue that a massive amount of government bonds issued to finance public works projects and tax reduction function as liquid assets and mitigate a liquidity shortage, thereby preventing the financial crisis from worsening. What is significant is not implementing fiscal spending on public works projects, etc. Rather, government bonds issued to finance such fiscal spending are useful because of their liquidity supply effect.

IV-4. Liquidity Supply Effect and Public Debt Overhang

How are studies arguing that public debt promotes economic growth through their liquidity supply effect related to the public debt overhang found by Reinhart, Reinhart and Rogoff (2012)? Although this will be discussed in detail in Section V-2, one possibility is that although public debt itself, as a means of liquid saving, promotes economic growth, fiscal spending implemented through the issuance of public debt (e.g., income redistribution such as social security policy measures) constrains economic growth. In an economy in which there are borrowing constraints, if income is transferred from economic entities which
are suffering from strict borrowing constraints to those which are not, the constraints will become more severe for the economy as a whole, and as a result, economic growth will be undermined. If such an income transfer is implemented through a fiscal policy measure in accordance with an increase in public debt, economic growth may slow down. That is because although public debt has the growth promotion effect, an income transfer may have a greater growth promotion effect in some cases. In Section V, I build a theoretical model based on this hypothesis and consider the results of simulations.

Regarding the effects of governmental redistribution policies on economic growth, Benabou (2002), and Seshadri and Yuki (2004) conducted theoretical analysis. Under Benabou’s model, individual economic entities consume and invest but they cannot borrow funds for investment purposes. In a market equilibrium, highly productive entities can make only socially insufficient investments. However, if the government implements a policy of universally distributing funds collected through consumption tax as investment subsidies, investments will be promoted and dynamic social welfare will be enhanced. While consumption tax causes a tax distortion that causes the labor supply to become insufficient, the redistribution policy using investment subsidies promotes investments, thereby encouraging capital accumulation. In other words, the redistribution policy based on consumption tax has a tradeoff between a deterioration in production efficiency and an increase in capital accumulation. Under the framework proposed by Benabou et al., a redistribution policy has the effect of mitigating market failure (presence of borrowing constraints). However, depending on how borrowing constraints are introduced, a redistribution policy may have quite an opposite effect. The model presented in Section 5 represents exactly such a case.

V. A Theory of Public Debt Overhang

In this section we follow Kobayashi (2014) and present a theoretical model of public debt overhang, in which deterioration of public finance lowers economic growth. In this model, the public debt provides liquidity and thus enhances economic growth. On the other hand, income redistribution associated with public debt accumulation tightens the borrowing constraint of productive agents and thus lowers economic growth. These two effects of public debt and income redistribution makes accumulation of debt and low economic growth.

The model represents a closed economy that can be viewed as a simplified version of Buera and Nicolini (2013) and a modified version of Kiyotaki (1998). There are a continuum of workers with measure 1, a continuum of high-productivity entrepreneurs with measure \( n \) \((0 < n < 1)\), and a continuum of low-productivity entrepreneurs with measure \(1 - n\), and the government. The workers live forever. The entrepreneurs die with probability \(1 - \gamma\) in each period and \(1 - \gamma\) new entrepreneurs are born in each period. The productivity of an entrepreneur does not change until his death. The discount factor for workers is \(\beta\) \((< 1)\). The discount factor for entrepreneurs, conditional on survival, is \(\beta'\) \((> \beta)\). We assume that the unconditional discount factor for entrepreneurs is equal to that for workers, that is, \(\beta = \gamma\beta'\).
The entrepreneurs can produce the consumption good with Cobb–Douglas production technology; that is, \( y_t = A_t k_t^\alpha l_t^{1-\alpha} \), where \( A_t \) is the productivity parameter. We assume that entrepreneurs are subject to the borrowing constraint that will be specified shortly.

**Fiscal policy:** We focus on the following fiscal policy: \( \{ B_{t+1}, T_t, S_t \} \), where \( B_{t+1} \) is the one-period bond redeemable at \( t+1 \), \( T_t \) is the lump-sum tax, and \( S_t \) is the lump-sum subsidy to the workers. In period \( t \), the government issues bonds \( B_{t+1}/r_t \), where \( r_t \) is the gross market rate of interest, subject to the following budget constraint:

\[
\frac{B_{t+1}}{r_t} + T_t = B_t + S_t.
\]

The same amount of tax, \( T_t \), is imposed on all (high- and low-productivity) entrepreneurs. No tax is imposed on workers. The budget constraint implies that the value of the current bond \( B_t \) satisfies

\[
B_t = \sum_{j=0}^{\infty} \frac{T_{t+j} - S_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}},
\]

where we define \( \prod_{s=0}^{-1} r_{t+s} = 1 \).

V-1. Workers

There is a unit mass of workers who can save but cannot borrow. They choose consumption, \( c'_t \), labor supply, \( l_t \), and bond holdings, \( b'_{t+1}/r_t \), to maximize utility,

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c'_t + \omega \ln(1 - l_t) \right],
\]

subject to the budget constraint,

\[
c'_t + \frac{b'_{t+1}}{r_t} = w_t l_t + b'_t + S_t,
\]

and the non-negativity constraint, \( b'_{t+1} \geq 0 \). The labor supply is determined by

\[
w_t = \frac{\omega c'_t}{1 - l_t}.
\]

If the non-negativity constraint is binding, the workers become hand-to-mouth workers who consume all their income during the same period and do not save. The condition for the workers to be hand-to-mouth is

\[
\frac{c'_{t+1}}{c'_t} > \beta r_t
\]
which I assume is always satisfied. The parameter value is chosen such that the labor supply equals \( \frac{1}{3} \) when the workers are hand-to-mouth, that is, \( \omega = 2 \).

\[ \text{V-2. Entrepreneur} \]

There is a unit mass of entrepreneurs. Measure \( n \) of the entrepreneurs have productivity \( z \) and measure \( 1 - n \) have productivity 1, where \( z > 1 \). We consider entrepreneurs with productivity \( z \) as having high productivity and those with productivity 1 as having low productivity. The terms “entrepreneur” and “firm” are used interchangeably throughout the paper. At the end of every period, \( 1 - \gamma \) entrepreneurs are randomly chosen to die and \( 1 - \gamma \) new entrepreneurs are born at the beginning of the next period. Among the newborn entrepreneurs, \( (1 - \gamma)n \) have productivity \( z \) and \( (1 - \gamma)(1 - n) \) have productivity 1. The newborn entrepreneurs inherit the wealth of the dead entrepreneurs according to an exogenously given law that will be described later (see Assumption 2). The entrepreneur’s utility is as follows:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t,
\]

where \( c_t \) is the entrepreneur’s consumption. An entrepreneur with productivity \( A \in \{1, z\} \) can produce output \( y_t \) from labor \( l_t \) and capital \( k_t \) by using the following production technology:

\[
y_t = Ak_t^{\alpha}l_t^{1-\alpha}.
\]

We assume for simplicity that capital stock \( k_t \) fully depreciates after production of output. The budget constraint for an entrepreneur is as follows:

\[
c_t + k_{t+1} - \frac{b_{t+1}}{r_t} \leq Ak_t^{\alpha}l_t^{1-\alpha} - w_t l_t - b_t - T_t,
\]

where \( b_{t+1} \) is the bond issued in period \( t \) and redeemed in period \( t + 1 \). Note that in cases where an entrepreneur purchases bonds issued by other entrepreneurs, \( b_{t+1} \) can be a negative number. We consider the following assumption pertaining to lack of commitment.

\[ \text{Assumption 1} \quad \text{An entrepreneur cannot commit to repayment of debt (} b_{t+1} \text{). The creditors (or the bondholders) can seize } \theta y_{t+1}, \text{ where } y_{t+1} \text{ is the output and } 0 < \theta < 1, \text{ if the entrepreneur repudiates his debt.} \]

Under this assumption, a part of the output (\( \theta y_{t+1} \)) works as collateral for the debt and the upper limit of the amount that can be borrowed. Thus, the entrepreneur faces the following borrowing constraint:

\[
b_{t+1} \leq \theta Ak_t^{\alpha}l_t^{1-\alpha}t_{t+1},
\]

where \( l_{t+1} \) is the labor input in \( t + 1 \) that is decided in period \( t + 1 \). As we will see later, \( l_{t+1} \) is a linear function of \( k_{t+1} \). The optimization problem for an entrepreneur with productivity
$A$ is to choose $\{c_t, k_{t+1}, b_{t+1}\}$ in period $t$ and $l_{t+1}$ in period $t+1$ to maximize his utility (1), subject to the budget constraint (2) and borrowing constraint (3).

**Low-productivity firms’ problem:** Following Kiyotaki (1998), we limit our attention to the equilibrium where the borrowing constraint is sufficiently tight, such that the high-productivity firms cannot use up all the capital stock in the economy. In this case, the low-productivity entrepreneurs buy both bonds and capital stock, implying that the labor (MPL) for the low firms equals the wage rate, that is, $w_t = (1 - \alpha)(k_t/l_t)^\alpha$. These equations imply

$$r_t = \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{1-\alpha}$$

(high-productivity firms’ problem: Given $r_t$, the reduced form of the high firms’ problem is written as follows.

\[(LP) \quad \max \sum_{t=0}^{\infty} \beta^t \ln c_t, \quad \text{s.t.} \quad a'_{t+1} = r_t (a'_t - c_t - T_t),\]

where $a'_t$ denotes the asset holdings of low firms. Because the borrowing constraint is not binding for the low firms, the marginal productivity of capital (MPK) for the low firms equals the market rate, that is, $r_t = \alpha (l_t/k_t)^{1-\alpha}$. Similarly, the marginal productivity of labor (MPL) for the low firms equals the wage rate, that is, $w_t = (1 - \alpha)(k_t/l_t)^\alpha$. These equations imply

$$r_t = \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{1-\alpha}$$

**High-productivity firms’ problem:** Given $r_t$, the reduced form of the high firms’ problem is written as follows.

\[(HP) \quad \max \sum_{t=0}^{\infty} \beta^t \ln c_t, \quad \text{s.t.} \quad a_{t+1} = R_t (a_t - c_t - T_t),\]

where $a_t = z k_t^\alpha l_t^{1-\alpha} - w_t l_t - b_t$ and $R_t$ is the gross rate of return for the high firms that is determined by the solution to the problem below. The problem is the maximization of the return on investment for the high firms, given the amount of remaining assets after consumption and tax payment ($a_t - c_t - T_t$), as follows:

$$\max_{k_{t+1}, b_{t+1}} a_{t+1} = \pi (k_{t+1}, w_{t+1}) - b_{t+1},$$

s. t. \[\begin{align*}
k_{t+1} - \frac{b_{t+1}}{r_t} &\leq a_t - c_t - T_t, \\
b_{t+1} &\leq \theta A k_{t+1}^\alpha l_{t+1}^{1-\alpha},
\end{align*}\]

where
Given that the market rate of interest is given by (4), the solutions are

\[
l_{t+1} = \arg \max_{l} z k_{t+1}^{\alpha} l^{1-\alpha} - w_{t+1} l = \left( \frac{(1 - \alpha)z}{w_{t+1}} \right)^{\frac{1}{\alpha}} k_{t+1},
\]

\[
\pi(k_{t+1}, w_{t+1}) = \max_{l} z k_{t+1}^{\alpha} l^{1-\alpha} - w_{t+1} l = \alpha z \frac{1}{w_{t+1}} \left( 1 - \alpha \right)^{\frac{1}{\alpha}} k_{t+1}.
\]

Given that the market rate of interest is given by (4), the solutions are

\[
k_{t+1} = \hat{k}(a_t - c_t - T_t),
\]

\[
a_{t+1} = R_t(a_t - c_t - T_t),
\]

where

\[
\hat{k} = \frac{1}{1 - \frac{\theta}{\alpha} z}, \quad \text{(5)}
\]

\[
R_t = \left( 1 - \frac{\theta}{\alpha} \right) z^{\frac{1}{\alpha}} r_t \hat{k}. \quad \text{(6)}
\]

**Solution to entrepreneurs’ problems**: The first order conditions (FOCs) for (HP) and (LP) imply that the consumption of a high entrepreneur who has wealth \( a_t \) is given by

\[
c_t = (1 - \beta) \left[ a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right],
\]

and the consumption of a low entrepreneur who has wealth \( a_t' \) is given by

\[
c_t' = (1 - \beta) \left[ a_t' - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right].
\]

In the next period, the wealth of a high entrepreneur is given by

\[
\frac{a_{t+1}}{R_t} = \beta \left[ a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}}, \quad \text{(7)}
\]

and that of a low entrepreneur is given by

\[
\frac{a_{t+1}'}{r_t} = \beta \left[ a_t' - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}}. \quad \text{(8)}
\]
V-3. Aggregate dynamics

Given the exogenous policy \( \{T_t, S_t\}_{t=0}^{\infty} \), the dynamics are described as the evolution of two state variables, namely, \((s_t, W_t)\), where \(W_t\) is the total wealth in period \(t\) \((W_t = \alpha Y_t + B_t)\) and \(s_t\) is the high firms’ share in the total wealth. The wealth of dead entrepreneurs is inherited by the newborn entrepreneurs in the same period by the following law:

**Assumption 2** *A newborn high-productivity entrepreneur inherits the wealth of a low-productivity entrepreneur who died in the same period.*

This is almost equivalent to assuming that in every period, \((1 - \gamma)n\) high firms change to low firms and the same measure of low firms become high firms, and that firm managers must exit when the productivities of their firms change. We define \(\hat{K}_{t+1}\) and \(\hat{L}_{t+1}\) as capital and labor used by the high firms, respectively, and \(K'_{t+1}\) and \(L'_{t+1}\) as those used by the low firms. Given \((s_t, W_t)\), the variables \((r_t, R_t, W_{t+1}, s_{t+1}, K_{t+1}, \hat{K}_{t+1}, K'_{t+1}, \hat{L}_{t+1}, L'_{t+1}, L_{t+1}, \text{and } w_{t+1})\) are calculated by the following system of 11 equations.

\[
\begin{align*}
r_t &= \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1}{\alpha}}, \quad (9) \\
R_t &= \frac{(1 - \frac{\beta}{\alpha})}{1 - \frac{\beta}{\alpha}} z \frac{1}{\alpha} r_t, \quad (10) \\
W_{t+1} &= R_t \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\} \\
&\quad + r_t \left\{ \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + (1 - n) \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\}, \quad (11) \\
s_{t+1} W_{t+1} &= \gamma R_t \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\} \\
&\quad + (1 - \gamma) \frac{n}{1 - n} r_t \left\{ \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + (1 - n) \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\}, \quad (12)
\end{align*}
\]

---

3 Here, we implicitly assume that \(0 < n < 0.5\).

4 This system of equations can be solved by the backward shooting method on the premise that the economy converges to a deterministic steady state.
\[ K_{t+1} = \sum_{j=1}^{\infty} \frac{S_{t+j}}{\Pi_{s=0}^{j-1} r_{t+s}} + \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} R_{t+s}} \]
\[ + \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} r_{t+s}} \right] - n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} r_{t+s}}, \]
\[ \hat{K}_{t+1} = \hat{k} \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} r_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\Pi_{s=0}^{j-1} R_{t+s}} \right\}, \]
\[ K'_{t+1} = K_{t+1} - \hat{K}_{t+1}, \]
\[ \hat{L}_{t+1} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1}{\alpha}} \hat{K}_{t+1}, \]
\[ L'_{t+1} = \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1}{\alpha}} K'_{t+1}, \]
\[ w_{t+1} = \omega \left[ w_{t+1} L_{t+1} + S_{t+1} \right] \frac{1}{1 - L_{t+1}}, \]
\[ L_{t+1} = \hat{L}_{t+1} + L'_{t+1}. \]

**V-4. Steady state**

Because we are interested in analyzing the qualitative nature of the model, we focus on the steady-state equilibrium of this economy. Given the level of tax \( T_t = T \) and subsidy \( S \), the steady state variables \( (B, r, R, W, s, K, \hat{K}, K', \hat{L}, L', L, w) \) can be determined as the solution to the following system of 12 equations.

\[ B = \frac{r}{r - 1} (T - S), \]
\[ r = \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}}, \]
\[ R = \left( \frac{1 - \frac{\alpha}{\omega}}{\alpha} \right) \frac{z^{\frac{1}{\alpha}}}{1 - \frac{\alpha}{\omega} z^{\frac{1}{\alpha}}} r, \]
\[ W = R \left\{ \beta \left[ s W - n \frac{R T}{R - 1} \right] + n \frac{T}{R - 1} \right\} + r \left\{ \beta \left[ (1 - s) W - (1 - n) \frac{r T}{r - 1} \right] + (1 - n) \frac{T}{r - 1} \right\}. \]
This figure shows that as public debt and tax increase, output increases and the interest rates. Figure 2 shows the steady-state equilibrium in the case where the redistribution of resources $sW$ can be determined as

$$sW = \gamma R \left\{ \beta \left[ sW - n \frac{RT}{R - 1} \right] + n \frac{T}{R - 1} \right\} + (1 - \gamma) \frac{n}{1 - n} r \left\{ \beta \left[ (1 - s)W - (1 - n) \frac{rT}{r - 1} \right] + (1 - n) \frac{T}{r - 1} \right\}, \quad (24)$$

$$K = \frac{S}{r - 1} + \beta \left[ sW - n \frac{RT}{R - 1} \right] + n \frac{T}{R - 1} + \beta \left[ (1 - s)W - (1 - n) \frac{rT}{r - 1} \right] - n \frac{T}{r - 1}, \quad (25)$$

$$\hat{K} = \hat{k} \left\{ \beta \left[ sW - n \frac{RT}{R - 1} \right] + n \frac{T}{R - 1} \right\}, \quad (26)$$

$$K' = K - \hat{K}, \quad (27)$$

$$\hat{L} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} \hat{K}, \quad (28)$$

$$L' = \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} K', \quad (29)$$

$$w = \frac{\omega [wL + S]}{1 - L}, \quad (30)$$

$$L = \hat{L} + L'. \quad (31)$$

The method to arrive at the solution is described in the Appendix.

VI. Numerical simulations and discussions

Figure 1 shows the steady-state equilibrium in the case where $S = 0$. The parameter values are $\alpha = 0.8$, $\beta = 0.95$, $\gamma = 0.95$, $\theta = 0.1$, $\omega = 1$, $z = 1.05$, and $n = 0.01$. This figure shows that as public debt and tax increase, output increases, and the interest rate decreases. This result is qualitatively robust against perturbation of the parameter values. This result is consistent with the models in which public debt provides valuable liquidity (see, for example, Woodford 1990 and Holmstrom and Tirole 1998). The entrepreneurs are taxed and the tax revenue is transferred back to the bondholders, that is, the low entrepreneurs. In this economy, there is a shortage of privately provided bonds, because firms are subject to a lack of commitment. The government can mitigate this shortage by issuing government bonds, as its coercive power to impose tax can work as a source of trust that complements the limited commitment of the private agents. Larger tax $T$ and bonds $B$ imply more abundant liquidity. This leads to lower interest rates $r$ that enable the high-productivity firms to borrow more and produce more, leading to a larger output $Y$.

The redistribution of resources $S$ from entrepreneurs to workers reduces output and interest rates. Figure 2 shows the steady-state equilibrium in the case where $S = \frac{1}{2}T \ (> 0)$. The parameter values for the experiment shown in Figure 2 are the same as those in Figure 1.
Figure 1. The steady-state equilibrium: $S = 0$

- $r$
- $B$
- $R$
- $K$
- $L$
- $s$
- $w$
- $Y$

Figure 2. The steady-state equilibrium: $S = \frac{1}{2}T$ (> 0)

- $r$
- $B$
- $R$
- $K$
- $L$
- $s$
- $w$
- $Y$
In this case, the total output and interest rate can both decrease as $B$ and $T$ increase. This result can be explained as follows. As the supply of liquidity increases with an increase in $B$, interest rate $r$ decreases. Meanwhile, the increase in $S$ decreases labor supply, owing to the income effect of $S$ on the workers, and raises the wage rate $w$. The total effect of the increase in $B$ and $S$ is to reduce the net worth of the high firms and decrease $\tilde{K}$ and $\tilde{L}$. As production by the high firms decreases, the total output $Y$ decreases.\(^5\) We should note that the result is sensitive to the value of $n$. For a large value of $n$, say, $n = 0.1$, the output increases as $T$ and $S$ increase, because the liquidity effect is dominant.

These experiments show that the income effect of the lump-sum subsidy on labor supply is one of the crucial factors that generate the negative effect of debt and subsidy on output and interest rate. In short, the public debt per se can enhance efficiency in our economy, whereas the redistribution from entrepreneurs to workers may depress output through the income effect on labor supply. Figure 3 reinforces this observation. I fix the amount of the lump-sum tax and observe the steady-state equilibria corresponding to various values of the lump-sum subsidy. Figure 3 shows that the output and interest rate decrease as $S$ increases. The result is robust against perturbations of the parameter values for a small $T$.\(^6\) This result implies that as the wage rate $w$ increases with an increase in $S$ through the income effect on labor supply, the net worth of the high firms is depressed and output decreases. The interest rate also decreases, because the decrease in the high firms’ net worth reduces the demand for borrowing.

These results imply that persistent stagnation may not be caused by a large debt per se, but possibly by a large redistribution from the entrepreneurial sector to workers, such as through social security spending. Thus, a reduction of the outstanding amount of public debt may not be necessary to restore economic growth. Instead, it may be necessary to decrease the extent of redistribution through social security and/or impose a larger tax burden on the beneficiaries of social security spending.

\(^5\) In the case where the workers’ utility is a Greenwood–Hercowitz–Huffman (GHH) type function, or the labor supply does not depend on workers’ incomes, the accumulation of public debt increases the output, even if $S > 0$. This result, however, is not reported in this paper. In the case of GHH utility, the lump-sum subsidy $S$ has no income effect on the labor supply. Thus, the liquidity effect of government bonds becomes dominant such that total output increases as $B$ and $T$ increase.

\(^6\) It is shown that for a large $T$, the output increases as $S$ increases.
VII. Conclusion

In this paper, I examined the possibility of fiscal deterioration producing a negative impact on economic growth, using the theory of public debt overhang presented by Reinhart, Reinhart and Rogoff (2012) as a starting point. Empirical studies using recently accumulated data concerning economic crises indicate the presence of a cause-and-effect relationship of fiscal deterioration leading to low economic growth. In particular, what is notable about several cases of public debt overhang, including the situation of the Japanese economy for the past two decades, is that fiscal deterioration brought low growth and low interest rates at the same time. This paper pointed out the difficulty of smoothly explaining such cases based on existing theoretical models. Under existing models, which assume that fiscal deterioration brings low growth, economic deterioration is caused by an interest rate rise triggered by fiscal deterioration. Essentially, this is a phenomenon similar to crowding-out. However, the Japanese economy for the past two decades has been characterized by low growth and low interest rates, so Japan’s case cannot be explained by the crowding-out effect.

Of course, regarding the Japanese economy, it is generally presumed that fiscal deterioration is not the cause of the long period of low growth. However, there is the possibility that fiscal deterioration led to low growth through a mechanism other than the crowding-out effect. In the latter half of this paper, we examined a new theoretical model and theoretically showed that fiscal deterioration could bring low growth and low interest rates at
the same time. The theoretical model examined in this paper indicated the following points:

- Public debt (government bonds) functions as liquid assets, so an increase in public debt itself has the effect of promoting economic growth (liquidity supply effect).
- If productive economic entities are taxed and subsidies are paid to the labor sector through a fiscal policy measure, workers whose income has increased reduce the supply of labor (income effect). As a result, the wage rate rises and highly productive companies reduce production. In addition, highly productive companies’ needs for borrowing also decline, so market interest rates drop.
- At a certain parameter value, the negative effect of the abovementioned two effects becomes dominant, and fiscal deterioration (increases in public debts and subsidies) lowers both productivity and market interest rates.

This theoretical model produced simulation results consistent with the hypothesis that fiscal deterioration is a cause of the prolonged economic stagnation. Meanwhile, Reinhart reported the presence of a non-linear relationship between fiscal deterioration and economic growth — in other words, while the fiscal position does not affect growth when the ratio of public debts to gross domestic product is lower than a threshold line, it curbs growth when the ratio rises above this line. Our theoretical model has not succeeded in reproducing the non-linear relationship. This is a great challenge that must be overcome if our theoretical model is to be developed further.

The theory of public debt overhang, which argues that fiscal deterioration causes deterioration of the real economy, is not widely accepted in economic academic circles, either empirically or theoretically. However, given the importance of the following policy implications, it is necessary to verify this theory through further study.

If there is a cause-and-effect relationship between fiscal deterioration and the deterioration of real economic conditions as indicated by the models proposed by Reinhart et al. and this paper, it will render useless the strategy that Japan has pursued for many years: stimulating the economy by continuing expansionary fiscal policy. That is because such fiscal policy works to restrain economic growth by aggravating the fiscal position through an increase in public debt. After taking a look at recent studies and examining this new theoretical model, I have come to the conclusion that it is possible that such a cause-and-effect relationship exists. How realistic this possibility is and how much its qualitative impact would be are challenges that should be tackled in the future.

Under the theoretical model proposed by this paper, income redistribution through fiscal expansion causes productivity deterioration. From this, we see that income redistribution from productive sectors to the household (labor) sector makes the economy inefficient. Therefore, correcting income redistribution through reduction of social security benefits and tax hikes will be an effective way to increase gross product. Such policy implications may be very important for Japan’s future tax and fiscal policies.
Appendix: Solution for the steady state

We first describe the method to arrive at the solution for the system of equations (20)–(31). T and S are given. First, we take \( \tilde{S} = S/w \) as given. Then the variables are described as functions of \((K, \hat{K})\). \( K' \) is given by (27); \( w \) is determined by (31), that is,

\[
\frac{1 - \omega \tilde{S}}{1 + \omega} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} \hat{K} + \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} K',
\]

implying that \( w \) is given by

\[
w(K, \hat{K}) = (1 - \alpha) \left( z^{\frac{1}{\alpha}} \hat{K} + K' \right)^{\alpha} \left( \frac{1 + \omega \tilde{S}}{1 - \omega \tilde{S}} \right)^{\alpha};
\]

\( r = r(K, \hat{K}) \) is given by (21); \( B = B(K, \hat{K}) \) is given by (20); \( R = R(K, \hat{K}) \) is given by (22); \( W = W(K, \hat{K}) \) is given by solving (25) as follows:

\[
W = \frac{1}{\beta} \left[ K - \frac{w \tilde{S}}{r - 1} + \frac{\beta R - 1}{R - 1} nT + \left\{ (1 - n) \beta r + n \right\} T \right];
\]

and \( s \) is given by solving (26) as follows:

\[
s = \frac{\hat{K} + \frac{\beta R - 1}{R - 1} n \beta T}{\hat{k} \beta W}.
\]

Then, the variables \((K, \hat{K})\) are determined as the solution to (23) and (24), given \( \tilde{S} \). Finally, \( \tilde{S} \) is determined by

\[
w \tilde{S} = S.
\]

**Steady state with** \( T = S = B = 0 \)

The solution is different for the steady state with \( T = S = B = 0 \), which is determined by the following system of equations:

\[
r = \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} \quad (32)
\]

\[
R = \left( \frac{1 - \theta}{\alpha} \right) \frac{z^{\frac{1}{\alpha}}}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}} r, \quad (33)
\]

\[
W = [Rs + r(1 - s)] \beta W, \quad (34)
\]

\[
sW = \gamma R \beta sW + (1 - \gamma) \frac{n}{1 - n} r \beta (1 - s) W, \quad (35)
\]

\[
K = \beta W, \quad (36)
\]

\[
\hat{K} = \hat{k} \beta sW, \quad (37)
\]
From (34) and (35), we have

\[ K' = K - \hat{K}, \quad (38) \]

\[ \hat{L} = z^{1/\alpha} \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} \hat{K}, \quad (39) \]

\[ L' = \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} K', \quad (40) \]

\[ L = \frac{1}{1 + \omega}, \quad (41) \]

\[ L = \hat{L} + L' \quad (42) \]

**Solution:** We denote \( x \) as

\[ x = \frac{(1 - \frac{\theta}{\alpha}) z^{1/\alpha}}{1 - \frac{\theta}{\alpha} z^{1/\alpha}}. \]

We take \( r \) as given. Then \( R = xr \).

From (34) and (35), we have

\[ s = \frac{\gamma xs + (1 - \gamma) \frac{n}{1 - n} (1 - s)}{sx + 1 - s}. \quad (43) \]

\( s \) is decided by (43). From (34), we have

\[ r = \frac{1}{\beta(sx + 1 - s)}. \quad (44) \]

\( r \) is given as the solution to (44). \( R \) is given by (33). Then \( w \) is given by (32). From (36)–(41), the last equation (42) can be written as

\[ \frac{1}{1 + \omega} = \left[ z^{1/\alpha} \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} \hat{k} \beta s + \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} (1 - \hat{k}s) \beta \right] W \quad (45) \]

which determines \( W \).

**References**


Johansson, Åsa, Yvan Guillemette, Fabrice Murtin, David Turner, Giuseppe Nicoletti, Christine de la Maisonneuve, Philip Bagnoli, Guillaume Bousquet, and Francesca