Flexible of Deficit Ceiling and Income Fluctuation

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Abstract

To avoid fiscal crisis, a government can impose a long-term deficit ceiling. A key issue that has arisen with respect to this is how long-term objectives should be modified to accommodate economic fluctuations and to maintain some flexibility with respect to fiscal stabilization policy in a political economy. By incorporating the political efforts of private agents into a simple model, we explore how income fluctuations affect the deficit ceiling in a political economy. If the government can optimally impose a deficit ceiling, normally, the deficit ceiling should rise in a recession as a first-best case; however, interestingly, a recession does not necessarily prompt an increase in the deficit ceiling in the second-best case. The response of the optimal deficit ceiling to income fluctuations mainly depends on the effect of fiscal privileges on the marginal benefit of useful public spending and on the extent of political effort.

Keywords: deficit ceiling, fiscal privilege, political effort, income fluctuation
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I. Introduction

To avoid fiscal crisis, a government can impose a long-term deficit ceiling. This study investigates the impact of income fluctuations on the deficit ceiling by using a simple political economy model. In the face of fiscal instability, it becomes important to determine the target levels of the deficit ceiling. For example, in Europe, the Stability and Growth Pact (SGP) is an agreement among the 27 member-states of the European Union (EU), executed to facilitate and maintain the stability of the Economic and Monetary Union. As is well known, the actual budget criteria that each member-state must respect are: (1) the annual budget deficit can be no higher than 3% of gross domestic product (GDP), and (2) the national debt must be lower than 60% of GDP. In March 2005, the EU Council relaxed the rules to address criticisms of insufficient flexibility and to make the SGP more enforceable.

To be specific, the ceilings of 3% for budget deficit and 60% for public debt were maintained, but the decision to declare a country in excessive deficit would be made through the use of certain parameters: the behavior of the cyclically adjusted budget, the level of debt, the duration of the slow growth period, and the possibility that the deficit is related to productivity-enhancing procedures. In Japan, a similar Fiscal Structural Reform Act was implemented in early 1997 to achieve consolidation targets similar to those of the EU;
however, facing the severe recession of late 1997, the Japanese government implemented expansionary fiscal measures because of political pressure from various interest groups in recessions. Thereafter, the Act was no longer considered a legal constraint. See Doi and Ihori (2009) among others for Japan’s fiscal policy in the 1990s.

These developments suggest the importance of income fluctuations in imposing a deficit ceiling in a political economy. A key issue that has arisen is how long-term objectives should be modified to accommodate economic recessions and to maintain some flexibility with respect to fiscal stabilization policy. The conventional wisdom is that counter-cyclical fiscal policy is needed in a recession as in the first best solution; nonetheless, it is quite rare to see sufficient budget surplus in a boom even in developed economies. This suggests the existence of interest groups which pursue their revenue even if—or, all the more because—their economic environment is preferable. This study explores how flexible the ceiling should be in response to recessions in a political economy. Since the size of public deficit is endogenously determined within the political process and affected by the community’s privilege and consolidation activities, the government cannot necessarily employ the first-best solution in a political economy even if it can impose the ceiling. We show that pro-cyclical policy might be desirable in such a second-best case.

In the course of this study, we explicitly consider political efforts made to secure fiscal privileges. Analytically, the fiscal consolidation efforts made by private agents have a public-good nature of improving the overall fiscal situation. Hence, as shown by Ihori and Itaya (2002, 2004) and Ihori (2011, 2014), the analytical framework of the private provision of public goods, as summarized by Cornes and Sandler (1996), is useful in examining the outcome of fiscal consolidation. See also Auerbach (2006), Velasco (2000), and Woo (2005) among others.

In section II of this paper, we develop a basic analytical framework within a political economy. We assume that a government is politically strong in terms of setting macro-level policy—at least sufficiently so, to impose a deficit ceiling—but politically weak in terms of setting micro-level policy in the sense that it cannot control fiscal privileges directly. Interest groups can seek personal fiscal privileges by making political efforts. We consider two types of fiscal privileges, agent-specific transfer and local public spending, which is useful for the agent but wasteful for others. We also assume that taxes are exogenously given and cannot easily be controlled by the government.

In section III, we investigate the first-best benchmark, where political activities are absent and the government may attain the first-best solution. We show that the government should conduct counter-cyclical fiscal policy in response to income fluctuations, in order to smooth private consumption over time.

In section IV, we investigate the second-best outcome, where the government cannot directly control two fiscal privileges in a political economy. First of all, we show that a weaker deficit ceiling results in a greater extent of political effort being made in the present by interest groups. A larger amount of levied tax results in a greater extent of political effort for two privileges. It is interesting to note that a larger amount of income results in a greater
extent of political effort for agent-specific wasteful spending but a lesser extent of political effort for agent-specific transfers.

Secondly, the optimal level of deficit ceiling from the long-term perspective is determined by the smoothing rule of the intergenerational allocation of public goods at the marginal utility level. The optimal ceiling at an interior solution lowers with the interest rate, but increases with the discount rate. We then investigate how the optimal ceiling responds to income fluctuations. It is often argued that a good macroeconomic situation is needed to attain successful outcomes from fiscal consolidation. However, this does not necessarily mean that the government should raise the deficit ceiling during a recession.

In OECD countries, fiscal policy is generally counter-cyclical; we may justify fiscal policy in the case of the first-best solution. In addition, we rationalize this policy in the second-best case, in which the government may control fiscal privileges and/or tax revenues. However, Gavin et al. (1996), among others, point out that in many developing countries, fiscal policy is rather pro-cyclical. Alesina and Tabellini (2005) explain that voters do not trust a corrupt government with resources and, hence, demand tax cuts or an increase in productive government spending or transfers when positive shocks hit the economy during a boom; otherwise, they fear that the available resources will be “wasted” in revenue. Although these studies are interesting in the course of explaining pro-cyclical policy in the context of a boom, they do not effectively explain if a benevolent government should set pro-cyclical policy during a recession from the long-run perspective.

Section V of this paper provides a new and plausible explanation as to why some benevolent governments should control the deficit ceiling counter-cyclically, while others do not, even in times of recessions. When in times of a recession, the government cannot attain consumption smoothing intertemporally, private consumption declines and political efforts to obtain transfers are stimulated, while political efforts for wasteful spending are depressed, resulting in an ambiguous effect on nation-wide useful public spending. Suppose that political efforts for wasteful spending dominate political efforts for specific transfers, so that the total amount of fiscal privileges declines in a recession. If, in addition, wasteful spending and useful public spending are not strong substitutes, they will reduce the marginal benefit of useful public spending; at this point, it becomes desirable to reduce useful public spending during a recession by reducing the deficit ceiling, and the setting of pro-cyclical fiscal policy could be justified.

Finally, section VI concludes this paper. When people have selfish preferences and the government is politically strong enough to impose the deficit ceiling but weak with respect to control several fiscal privileges, our analytical framework may more closely resemble reality. We show that income fluctuations in the present and future periods could paradoxically affect the optimal deficit limit in a political economy. The flexibility of the deficit ceiling may depend on the evaluation on private consumption, two fiscal privileges, and useful public spending and the magnitude of political efforts.
II. Analytical Framework: Fiscal Consolidation and Deficit Ceiling

As stressed by Ihori and Itaya (2002, 2004) and Ihori (2011, 2014), interest groups may accept the overall objective of fiscal consolidation, although they may not agree with how the fiscal reconstruction process should be employed. We assume that interest groups seek fiscal privileges, and we incorporate two costs of political efforts. First, political efforts incur the private cost of reducing private consumption. Moreover, once the ceiling on fiscal deficits is imposed by the government, greater fiscal privilege leads to a smaller amount of useful public good, thus damaging welfare. This cost produces a negative spillover—that is, one group’s fiscal privilege has negative spillovers on other groups, because they hurt the overall fiscal situation. We also assume that the government cannot control tax revenues.

We suppose that there are two generations: the present generation and the future generation. Each agent lives for only one period and is selfish in the sense of failing to leave any bequests. We assume that \( n \) agents are born in period 1, and that \( n \) agents also live in period 2. They are identical. A benevolent government would discount the welfare of future generations by \( \rho \) (<1). Here for simplicity we assume \( n=1 \). In general, improving the fiscal situation has a public-good nature. However, in this paper we do not examine the public-good nature of voluntary fiscal consolidation, for simplicity.

The government budget constraint for each period is given as

\[
T_i - G_i - z_i = -\bar{D}, \quad i = 1, 2 \tag{1-1}
\]

\[
T_2 - G_2 - z_2 = (1 + r)\bar{D}, \quad (1-2)
\]

where \( T_i \) is the exogenously given total tax revenue in period \( i \), \( i = 1, 2 \), \( z_i \) is the sum of the agent-specific transfer \( e_i \) and agent-specific but wasteful public spending \( h_i \) (which may be politically determined), and it benefits only the interest group in period \( i \). \( h_i \) becomes wasteful spending when its size becomes too great due to political pressure.

\[
z_i = e_i + h_i \quad (i = 1, 2) \tag{1-3}
\]

Finally, let \( G_i \) be the amount of useful public good in period \( i \), which benefits all agents. We could say that \( h \) is a local public good, while \( G \) is a pure public good.

The ceiling on fiscal deficits is denoted by \( \bar{D} \), which is determined by the government in period 1. \( r \) is the exogenously given interest rate. The government can impose the deficit ceiling as a consolidation rule, but it cannot control the fiscal privileges directly at the micro-policy level. In other words, the government may impose a ceiling on the total public spending, \( G + z \), but cannot control the distribution between \( G \) and \( z \). In fact, many countries have recently imposed such a ceiling on total spending. As Nerlich and Reuter (2012) explain, many EU countries have introduced some kind of scale rule; in particular, expenditure rules and balanced budget rules are the most common scale rules in place among EU countries, while there are generally only very few revenue rules in place. The Japanese government has imposed a similar ceiling on the total fiscal spending. See Doi and Ihori (2009). Since taxes
are fixed in our second best model, expenditure rules are equivalent to deficit rules here.

The (lifetime) utility function of representative agent of generation i (i=1,2) is given as 
\[ U^i = U^i(c_i, h_i, G_i). \]

The utility is dependent on private consumption \( c_i \), the quantity of (wasteful) agent-specific spending \( h_i \), and the quantity of useful (pure) public good, \( G_i \). \( c_i \), \( h_i \), and \( G_i \) would be an increasing function of \( U^i \), since private consumption, wasteful fiscal spending, and useful public goods are normal goods.

By substituting the budget constraints (1-1) and (1-2) into the utility function, the social welfare function of benevolent government may be written as
\[ W = U^1(c_1, h_1, T_1 - z_1 + \bar{D}) + \rho U^2(c_2, h_2, T_2 - z_2 - (1+r)\bar{D}). \]  

The utility is dependent on the amount of two fiscal privileges, \( h_i, e_i \), as well as the quantity of private goods consumed, \( c_i \). A utilitarian government would have discounted preferences on the future period. We may regard \( \delta \equiv (1 - \rho) / \rho \) as the discount rate.

Greater political effort by each agent leads to more privileges and more disposable income. Hence, it is plausible to formulate, for simplicity, that an agent’s production of overall disposable income is given by \( w_i - T_i + e_i \) and to some extent associated with a cost of obtaining privileges, denoted by \( \phi e_i + \Phi h_i \), which is increasing with political efforts. Thus, each agent’s (lifetime) budget constraint is given as
\[ c_i = w_i - T_i + e_i - \phi e_i - \Phi h_i. \]

Here, the before-tax income, \( w_i \), is exogenously given. The disposable income includes the cost of fiscal privileges, \( \phi e_i + \Phi h_i \). In order to obtain \( z_r \), some resources are needed. Given parameters \( \phi, \Phi \) summarize the degree of inefficiency of political efforts. In other words, the larger \( \phi, \Phi \) are, the greater the political efforts must be to produce a given amount of privilege and, hence, the less economically efficient the political effort technology is from the viewpoint of interest groups.

**III. First-Best: Control of Fiscal Privileges**

To explore the implications of political efforts with regard to fiscal policy, let us first investigate the first-best solution. Suppose that the government can control fiscal privileges as well as taxes, and that political efforts are thus absent in the first-best benchmark case. The political effort activities equal zero: \( \Phi = 0 \). \( e_i \) is just a transfer and \( h_i \) is not regarded as wasteful spending but as local public goods since the government now can control the size of \( h_i \). Since we assume \( n=1 \), \( h \) and \( G \) are just two different types of public goods. The objective of the government is to maximize
\[ W = U^1(w_1 - T_1 + e_1, h_1, G_1) + \rho U^2(w_2 - T_2 + e_2, h_2, G_2) \]  

by choosing \( e_1, e_2, h_1, h_2, T_1, T_2, G_1, G_2 \). Note that \( w_1, w_2 \) are exogenously given, as before. The budget constraint is given by
\[ T_1 + \frac{T_2}{1 + r} = G_1 + \frac{G_2}{1 + r} + z_1 + \frac{z_2}{1 + r}. \]  

(4)

Then, the optimality conditions are given by

\[ U^1_c = U^1_h = U^1_G = \rho(1 + r)U^2_c = \rho(1 + r)U^2_h = \rho(1 + r)U^2_G. \]  

(5)

These equations relate to the Samuelson condition of public goods and the consumption-smoothing condition over time, respectively.

Suppose that \( w_1 \) declines but that \( w_2 \) increases, to keep the present discounted value fixed. \( dw_1 = -\frac{1}{1 + r} dw_2 < 0 \). The present is in a recession, while the future is in a boom. In such an income fluctuation, it is optimal to have \( dT_1 = dw_1 < 0 \) and \( dT_2 = dw_2 > 0 \). In other words, the government should decrease tax \( T_1 \) in period 1 and raise tax \( T_2 \) in period 2, so that the disposable income including the subsidy may not change in either period. To do so, the government should increase the deficit ceiling in period 1 by an amount equal to the decline of income. \( d\bar{D} = -dw_1 > 0 \). Two types of public goods, \( h_1, h_2 G_1, G_2 \), remain fixed at the original values. This is a counter-cyclical fiscal policy that smooths private consumption and public goods between periods. In other words, the intertemporal smoothing condition normally implies the setting of conventional counter-cyclical fiscal policy at the first-best solution.

In the first-best case, taxes and transfers are perfect substitutes. So long as the government can control lump-sum transfer \( e_i \) and hence disposable income, it may optimize private consumption overtime even if taxes are exogenously fixed. Then, the government should increase subsidies whenever before-tax income declines in recessions, so as to maintain after-tax disposable income at the original level. This policy can attain the intertemporal smoothing allocation of private consumption and public goods at the original optimal values.

### IV. Second-Best Outcome

We now consider the second-best case where the government cannot control either taxes or fiscal privileges. The structure of the second-best game in a political economy is as follows. At the first stage, the government imposes a deficit ceiling. At the second stage, a representative agent conducts political efforts to obtain privileges in each period.

**IV-1. Political Efforts by Two Generations at the Second Stage**

**IV-1-1. Future Generation**

First of all, the first-order condition with respect to \( h_2 \) of the future generation in period 2 is given as
$$U^2_h = U^2_G + U^2_e \Phi,$$

(6-1)

where $U^2_e \equiv \frac{\partial U^2}{\partial c_e}, U^2_h \equiv \frac{\partial U^2}{\partial h_2}$ and $U^2_G \equiv \frac{\partial U^2}{\partial G_2}$. We assume an interior solution of $h_2 > 0$.

In (6-1), $h_2$ likely decreases with $\bar{D}$. Actually, the reaction function of $h_2$ is given as

$$h_2 = h^2 (\bar{D}, w_2, T_2),$$

(7-1)

where

$$h^2_D \equiv \frac{\partial h_2}{\partial \bar{D}} = \frac{-1}{\Delta} (1 + r)[U^2_{cG} \Phi - U^2_{hG} + U^2_{GG}],$$

$$h^2_w \equiv \frac{\partial h_2}{\partial w_2} = \frac{-1}{\Delta} [-U^2_{cc} \Phi + U^2_{ch} - U^2_{Gc}],$$

$$h^2_T \equiv \frac{\partial h_2}{\partial T_2} = \frac{-1}{\Delta} [-U^2_{cG} \Phi + U^2_{hG} - U^2_{GG}],$$

and $\Delta \equiv U^2_{hh} - U^2_{cc} \Phi^2 + U^2_{GG} - 2U^2_{hG}$, where $U^2_{cc} \equiv \frac{\partial U^2}{\partial c_c}, U^2_{cG} \equiv \frac{\partial U^2}{\partial c_G}, U^2_{GG} \equiv \frac{\partial U^2}{\partial G_G}$.

From the second-order condition, $\Delta < 0$. Suppose that private consumption is additively separable from two public goods. $U^2_{cG} = U^2_{ch} = 0$. Then, we know that $h^2_w > 0$. A smaller amount of disposable income $w_2$ results in a smaller amount of political efforts to obtain $h_2$ in the future period. This effort becomes a normal good. The related intuition is as follows. When $w_2$ is low, $c_2$ is low, so that an increase in $h_2$ greatly raises the cost of $h_2$, as the marginal utility of $c_2$ is already high.

In addition if $U^2_{hG} > 0$ then $h^2_D < 0, h^2_T > 0$. When the government imposes a deficit ceiling at a higher level in period 1, it will produce a smaller amount of fiscal privilege by the agent in period 2. In other words, a weaker deficit ceiling in period 1 results in a smaller amount of political effort in period 2. The related intuition is as follows. When the ceiling is weak and $\bar{D}$ is high, $G_2$ is small, so that the marginal cost of $h_2, U^2_{G2}$, is large. Hence, $h_2$ is depressed. In other words, when the fiscal situation worsens, the agent in period 2 does not have a stronger incentive to obtain more fiscal privileges.

Moreover, a smaller amount of tax revenue $T_2$ results in a smaller amount of political efforts in the future period. When $T_2$ is low, $G_2$ is small, so that an increase in $h_2$ greatly raises the cost of $h_2$ if $U^2_{hG} > 0$. Even if $U^2_{hG} > 0$ but $|U^2_{hG}|$ were not large, we still have similar responses since $U^2_{GG} < 0$: $h^2_D < 0, h^2_T > 0$.

Second, the first-order condition with respect to $e_2$ of the future generation in period 2 is given as

$$U^2_e m = U^2_G,$$

(6-2)

where $m = 1 - \phi$. We assume an interior solution of $e_2 > 0$. In (6-2), $e_2$ likely decreases with $\bar{D}$. Actually, the reaction function of $e_2$ is given as
\[ e_2 = e^2(\bar{D}, w_2, T_2), \]  

(7-2)

where

\[ e_D^2 \equiv \frac{\partial e_2}{\partial \bar{D}} = -\frac{1}{\Delta} (1+r)[-U_{cG}^2 m + U_{GG}^2], \]

\[ e_w^2 \equiv \frac{\partial e_2}{\partial w_2} = -\frac{1}{\Delta} [U_{cG}^2 m - U_{Gc}^2], \]

\[ e_t^2 \equiv \frac{\partial e_2}{\partial T_2} = -\frac{1}{\Delta} [U_{cG}^2 m - U_{GG}^2], \]

and \( \Delta \equiv U_{cc}^2 m + U_{GG}^2 - 2U_{cG}^2 m \), where \( U_{cc}^2 \equiv \frac{\partial U_c^2}{\partial c_2}, U_{cG}^2 \equiv \frac{\partial U_{cG}^2}{\partial c_2}, U_{GG}^2 \equiv \frac{\partial U_{GG}^2}{\partial G_2} \).

From the second-order condition, \( \Delta < 0 \). If \( U_{cG} = 0 \), this condition is easily satisfied. Then, we know that \( e_D^2 < 0, e_w^2 < 0, e_t^2 > 0 \). Similar to (7-1), when the government imposes a deficit ceiling at a higher level in period 1, it will produce a smaller amount of fiscal privilege by the agent in period 2. This effort becomes an inferior good. In other words, a weaker deficit ceiling in period 1 results in a smaller amount of political effort in period 2.

Now, contrary to (7-1), a larger amount of disposable income \( w_2 \) results in a smaller amount of political efforts to obtain agent-specific transfer \( e_2 \) in the future period. The related intuition is as follows. When \( w_2 \) is high, \( c_2 \) is large, so that an increase in \( e_2 \) does not increase the benefit of \( e_2 \) much, as the marginal utility of \( c_2 \) is already low.

**IV-1-2. Present Generation**

Now, we consider the optimizing behavior of the representative agent of the present generation in period 1. Similar to (6-1), the optimality condition with respect to \( h_1 \) reduces to

\[ U_h^1 = U_G^1 + U_c^1 \Phi, \]

(8-1)

where \( U_c^1 \equiv \frac{\partial U^1}{\partial c_1} \) and \( U_G^1 \equiv \frac{\partial U^1}{\partial G_1} \). The reaction function of \( h_1 \) is given as

\[ h_1 = h^1(\bar{D}, w_1, T_1), \]

(9-1)

where

\[ h_D^1 \equiv \frac{\partial h_1}{\partial \bar{D}} = -\frac{1}{\Lambda} [U_{cG}^1 - \Phi U_{cG}^1 - U_{GG}^1], \]

\[ h_w^1 \equiv \frac{\partial h_1}{\partial w_1} = -\frac{1}{\Lambda} [-U_{cG}^1 \Phi + U_{cG}^1 - U_{gh}^1], \]
\[ h_T = \frac{\partial h}{\partial T_1} = -\frac{1}{\Lambda} \left( -U_{cG}^1 \Phi + U_{Gh}^1 - U_{GG}^1 \right), \]

and \( \Lambda \equiv U_{hh}^1 - U_{cc}^1 \Phi^2 + U_{GG}^1 - 2U_{hG}^1. \)

From the second-order condition, \( \Lambda < 0. \) As before, suppose that private consumption is additively separable from public goods. \( U_{cG} = U_{ch} = 0. \) Then, we know that \( h_T^1 > 0. \) In addition if \( U_{hG} > 0 \) then \( h_D^1 > 0, \) \( h_T^1 > 0. \) Now, the sign of \( h_D^1 \) is the opposite of the sign of \( h_T^1. \)

In (9-1), \( h_t \) is likely increasing with \( \bar{D}. \) When the government imposes a deficit ceiling at a higher level in period 1, it produces a larger amount of fiscal privileges in period 1—namely, a weaker deficit ceiling likely results in a greater extent of political effort being expended in the present period. As in the second period, either a larger amount of income \( w_1 \) or a larger amount of tax \( T_1 \) results in a larger amount of political effort in the present period, \( h_T \)

Second, we consider the optimizing behavior of the representative agent of the present generation in period 1. Similar to (6-2), the optimality condition with respect to \( e_1 \) reduces to

\[ U_1 m = U_G^1, \]  

(8-2)

The reaction function of \( e_1 \) is given as

\[ e_1 = e_1(\bar{D}, w_1, T_1), \]  

(9-2)

where

\[
\begin{align*}
e_D^1 & = \frac{\partial e_1}{\partial \bar{D}} = -\frac{1}{\Lambda} \left[ U_{cG}^1 m - U_{GG}^1 \right], \\
e_w^1 & = \frac{\partial e_1}{\partial w_1} = -\frac{1}{\Lambda} \left[ U_{cc}^1 m - U_{Gc}^1 \right], \\
e_T^1 & = \frac{\partial e_1}{\partial T_1} = -\frac{1}{\Lambda} \left[ U_{cG}^1 m - U_{GG}^1 \right],
\end{align*}
\]

and \( \Lambda \equiv U_{cc}^1 mm + U_{GG}^1 - 2U_{cc}^1 m. \)

From the second-order condition, \( \Lambda < 0. \) Then, if \( U_{cG} = 0, \) we have \( e_D^1 > 0, e_w^1 < 0, e_T^1 > 0. \)

Now, the sign of \( e_D^1 \) is the opposite of the sign of \( e_D^2. \) In (9-2), \( e_1 \) is likely increasing with \( \bar{D}. \) As in the second period, either a larger amount of income \( w_1 \) or a smaller amount of tax \( T_1 \) results in a smaller amount of political effort to obtain agent-specific transfer in the present period, \( e_1. \)

\textbf{IV-2. Debt Ceiling by the Government at the First Stage}

It is important to know how the objective of the government at the first stage of the game is specified. Since the agent is identical, it is plausible to assume that the government is benevolent—namely, the government determines the deficit ceiling by considering the
response of agents in both periods—given by reaction functions (7-1,2) and (9-1,2), respectively—so as to maximize the weighted sum of welfare of two generations, (2). An increase in $D$ corresponds to an increase in $G_1$ and a decrease in $G_2$, benefiting the present welfare, while hurting the future welfare. The present-value government budget constraint is given as (4).

Unlike with the first-best case, the government cannot directly choose $z_i$ in the second-best case. $z_i$ is determined by the political efforts of private agents. The benevolent government maximizes $W$ by choosing $D$ (and hence effectively $G_1, G_2$) subject to (7-1,2), (9-1,2), and (4) and exogenously given $T_1, T_2$. Hence, the optimal level of $D$ is determined to satisfy the following equation to be zero.

$$\frac{dW}{dD} = U_G^1 - (1+r)\rho U_G^2. \quad (10)$$

Note that the effects of $z_1, z_2$ through changes in $D$ on the optimality condition (10) vanish from the envelope theorem. We have an interior solution of $D > 0$, where (10) equals zero. From now on, we will focus on an interior solution. The first term in (10) is the marginal benefit of increasing $D$ (or the marginal gain for the present generation), while the second term is the marginal cost of increasing $D$ (or the marginal gain for the future generation). Either an increase in $r$ or $\rho$ or a decrease in $\delta$ will increase the cost of $D$, so that the optimal level of $D$ declines. The optimality condition is a condition by which useful public spending is smoothed over time at the marginal utility level.

V. Income Fluctuations and Deficit Ceiling

V-1. Wage Fluctuation

Suppose $w_1$ declines and $w_2$ increases. Note that the after-tax income, $\hat{w} \equiv w - T$, and the before-tax income, $w$, change in the same way, since taxes are fixed. We have

$$\frac{d\hat{D}}{dw_1} = -\frac{U_{Gc}^1 (1 + m_e w_1 - \Phi h_1^1) - U_{GG}^1 \rho (e_w^1 + h_1^1) + h_1^1 U_{Gh}^1}{(1+r)^2 \rho U_{GG}^2 + U_{GG}^1}, \quad (11-1)$$

$$\frac{d\hat{D}}{dw_2} = -\frac{(1+r)\rho [U_{Gc}^2 (1 + m_e w_2 - \Phi h_2^2) - U_{GG}^2 (e_w^2 + h_2^2) + h_2^2 U_{Gh}^2]}{(1+r)^2 \rho U_{GG}^2 + U_{GG}^1}, \quad (11-2)$$

where $U_{GG} = \frac{G^2 U}{G^2 G}$ and so on. These two equations may capture the effect of disposable income.

The sign of (11-1) (or of (11-2)) is generally ambiguous. We can show that if private consumption $c$ is separable from two public goods $h, G \ (U_{CG} = U_{Ch} = 0)$, the term
\[ [U^1_{Gc}(1 + me^1_w - \Phi h^1_w) - U^1_{GG}e^1_w] = -U^1_{GG}e^1_w \]

becomes negative, while if, in addition, \( h \) and \( G \) are not strong substitutes \((U_{hG} > U_{GG})\), the term

\[ -U^1_{GG}h^1_w + h^1_wU^1_{Gh} \]

becomes positive. If the latter effect is greater than the former effect, such a change in disposable income reduces \( \bar{D} \). Namely, the sign of (11-1) (or (11-2)) becomes positive (or negative) under these conditions. When the political effort technology for transfer is small but for wasteful local spending is large, the second term would likely dominate the first term and hence pro-cyclical policy can more likely be justified. In other words, the combination of a worse macroeconomic situation in the present period and a better macroeconomic situation in the future period does not necessarily justify the counter-cyclical policy. It might be desirable to reduce \( \bar{D} \) in times of recession. Under such a case the government should conduct restrictive fiscal policy in times of recession.

The related intuition is as follows. Firstly, a decrease in \( w_1 \) directly raises \( e_1 \). An increase in \( e \) reduces \( G \) at the given ceiling level, increasing the marginal utility of public goods. However, if the absolute value of \( e^1_w \) is small, an increase in \( w_1 \) does not increase \( e_1 \) much, making this effect small. Secondly, if \( h^1_w > 0 \) is large, it reduces \( h \), raising \( G \) to a great extent. This increase in \( G \) greatly reduces the marginal utility of public good \( G \) in the present period. At the same time, as \( h_1 \) and \( G_1 \) are not strong substitutes \((U_{hG} > U_{GG})\), a decrease in \( h_1 \) would reduce the marginal utility of public good \( G_1 \). Hence, combining both of the effects, the marginal utility of public spending declines in period 1 under these conditions. Thus, it becomes optimal to reduce \( G_1 \) and to increase \( G_2 \), so as to maximize the expected sum of welfare in each of the two generations.

We do not argue that the counter-cyclical case is unlikely to occur or is undesirable. If the government may control fiscal privileges optimally, this case becomes desirable and it is true that many governments actually do conduct counter-cyclical fiscal policy. Even in the second best case, if \( U_{hG} < U_{GG} \) (i.e., \( h \) and \( G \) are strong substitutes) and the political effort for transfer is relatively large, (11-1) could become negative and (11-2) positive. Fluctuations in disposable income \( w_1, w_2 \) (or macroeconomic situation) could affect deficit ceiling \( \bar{D} \) counter-cyclically. Conventional wisdom is justified in this case. Rather, our analysis suggests that the pro-cyclical policy is likely desirable if the sum of fiscal privileges, \( z=e+h \), increases with wages. In other words, this pro-cyclical policy may well be justified when \( h \) and \( G \) are not strong substitutes within a political economy where the political effort for wasteful local spending is relatively large.

\section*{V-2. Tax Revenue Fluctuation}

Next, we suppose that after-tax incomes \( w_1 - T_1, w_2 - T_2 \) remain fixed, but that tax revenues \( T_1, T_2 \) change exogenously. Then, we may normally expect that either a decrease in
\( T_i \) or an increase in \( T_2 \) would increase the optimal level of \( \bar{D} \), since the present fiscal situation becomes worse, while that of the future is better. However, either a decrease in \( T_i \) or an increase in \( T_2 \) could paradoxically reduce the optimal level of \( \bar{D} \).

We have

\[
\frac{d\bar{D}}{dT_i} = \frac{-U_{GG}^i (1-e^i_T - h^i_T) + U_{Ge}^i (-1 + me^i_T - \Phi h^i_T) + U_{Gh}^i h^i_T}{(1 + r)^2 \rho U_{GG}^i + U_{GG}^i},
\]

\[
\frac{d\bar{D}}{dT_2} = \frac{(1 + r) [U_{GG}^2 (1-e^2_T - h^2_T) + U_{Ge}^2 (-1 + me^2_T - \Phi h^2_T) + U_{Gh}^2 h^2_T]}{(1 + r)^2 \rho U_{GG}^2 + U_{GG}^2}.
\]

As before suppose \( U_{cG} = U_{ch} = 0 \). The term

\[ U_{GG}^1 < 0 \]

is negative. The term

\[ -U_{GG}^1 (e^1_T + h^1_T) > 0 \]

is positive. The term

\[ h^1_T U_{Gh}^1 > 0 \]

is positive if \( U_{hG} > 0 \). If \( e_T \) and \( h_T \) were large, (11-3) could become positive and (11-4) negative. In such a case, when tax revenue declines in the present period and increases in the future period, it would be optimal to reduce the initial debt limit in the present period. This debt management policy corresponds to pro-cyclical policy.

The related intuition is as follows. On the one hand, a decrease in \( T_i \) at a given level of \( w_i - T_i \) directly reduces \( G_i \), raising the marginal utility of \( G_i \). On the other hand, it directly depresses \( e_T \) and \( h_T \), increasing useful public spending \( G_i \) at a given ceiling level, which decreases the marginal utility of public spending. It indirectly reduces the marginal utility of public spending as well, if \( U_{Gh} > 0 \). If the former effect were dominated by the latter effect, then the marginal utility of public spending would decline in period 1, at which point it would become optimal to reduce \( G_i \).

V-3. **Cobb–Douglas Functional Form**

Suppose the utility function is additively separable between \( c \) and \( hG \), and is specified by the Cobb–Douglas functional form with respect to \( h \) and \( G \),

\[ U = c + h^\lambda G^{1-\lambda} \quad (0 < \lambda < 1) \]

Then, we know that in the special case

\[ U_{cc} = 0, U_{Gh} > 0, \frac{U_{Gh}}{U_{hh}} = -\theta, \frac{U_{GG}}{U_{hh}} = \theta^2, \frac{U_{Gh}}{U_{GG}} = -\frac{1}{\theta} \]
where \( \theta \equiv -\frac{c}{G} \). In this formulation, we have

\[
h_w^1 = h_w^2 = 0, \quad e_w^1 = e_w^2 = 0, \quad e_T^1 = e_T^2 = 1, \quad e_D^1 = e_D^2 = -1, \quad h_T^1 = \frac{\theta_1}{1 + \theta_1}, \quad h_T^2 = \frac{\theta_2}{1 + \theta_2},
\]

\[
h_D^2 = -\frac{\theta_2}{1 + \theta_2}.
\]

Substituting these values into (11-1) and (11-2), we find that (11-1), (11-2) become zero. It is optimal to retain the initial ceiling level when \( w_i \) or \( w_2 \) changes. This is the rigid balanced budget policy that is set in response to income fluctuations. We find that (11-3) becomes positive and (11-4) becomes negative.

\[
U_{GG}^1 (1 - e_T^1 - h_T^1) + U_{Gb}^1 h_T^1 = U_{GG}^1 [1 - (1 + \frac{\theta}{1 + \theta}) - \frac{1}{\theta} (1 + \theta)] = -U_{GG}^1 > 0
\]

Using a constant elasticity of substitution (CES) utility function, Coenen, Straub, and Trabandi (2012) calculated a simulation model of fiscal policy while assuming that the elasticity of substitution between private and government consumption goods is 0.29, so that the two goods enter the utility function as rather strong complements, \( U_{cg} > 0 \). We have not considered the effect of \( U_{cg} > 0 \) yet. If this is the case, the setting of pro-cyclical fiscal policy would likely be justified.

V-4. Disposable Income Fluctuation

Suppose now that \( T = tw_i \), where \( t \) is the tax rate; hence, the after-tax income \( \hat{w}_i \) is given as \( \hat{w}_i = (1 - t)w_i \). When the economy is in recession and before-tax income declines, both after-tax income and tax revenue decline. Thus, it is necessary to simultaneously consider both the effect of disposable income on \( D \) and the effect of tax revenue on \( D \). Assume that \( U_{cg} = U_{cg} = 0 \), as before. Then, under \( T_i = nt w_i \), we have

\[
\frac{dD}{dw_1} = -\left\{ -U_{GG}^1 (e_w^1 + h_w^1) + h_w^1 U_{Gb}^1 \right\} (1 - t) + \left\{ U_{GG}^1 (1 - e_T^1 - h_T^1) + h_T^1 U_{Gb}^1 \right\} t
\]

\[= \frac{(1 + r)^2 \rho U_{GG}^2 + U_{GG}^1}{(1 + \rho) (1 + r)^2 \rho U_{GG}^2 + U_{GG}^1}(12-1)\]

\[
\frac{dD}{dw_2} = \frac{1}{1 + \rho} \frac{1 + r}{(1 + r)^2 \rho U_{GG}^2 + U_{GG}^1} [U_{GG}^2 (1 - e_T^1 - h_T^1) + h_T^1 U_{Gb}^1] t
\]

\[= (12-2)\]

Then, if

\[
F = \left\{ -U_{GG}^1 (e_w^1 + h_w^1) + h_w^1 U_{Gb}^1 \right\} (1 - t) + \left\{ U_{GG}^1 (1 - e_T^1 - h_T^1) + h_T^1 U_{Gb}^1 \right\} t > 0
\]

the sign of (12-1) would be positive, and vice versa. In other words, if the sum of the effect of after-tax income on \( D \), \( \{-U_{GG}^1 (e_w^1 + h_w^1) + h_w^1 U_{Gb}^1 \}(1 - t) \), and the effect of revenue on \( D \), \( \{U_{GG}^1 (1 - e_T^1 - h_T^1) + h_T^1 U_{Gb}^1 \} t \), were positive, then the sign of (12-1) would become positive, and (12-2) becomes negative. It is thus optimal for the government to reduce the deficit ceiling in a recession. When the sum of fiscal privileges increases with disposable income, pro-cyclical
policy is likely desirable. On the other hand, if h and G were strong substitutes, we could justify the counter-cyclical policy as before.

This fluctuation may correspond to the case where tax T changes but before-tax wage w remains fixed. The effect of tax revenue fluctuation is examined in 5.2, while the effect of disposable income fluctuation due to tax changes is examined in 5.1. Hence, if the sum of fiscal privileges increases with taxes, pro-cyclical policy is likely desirable.

As shown above, in the Cobb–Douglas formulation with $U_{cc} = 0$, the sign of the first term of (12-3) is zero and the second term becomes positive. Hence, (12-1) would become positive. As before, the pro-cyclical policy becomes desirable in this case.

VI. Conclusion

To preclude fiscal crises, deficit growth must be in tune with fiscal consolidation. An important and powerful tool in doing so is to impose a deficit ceiling. This study incorporated the political behavior of private agents into analyses of the deficit ceiling when the government imposes a deficit ceiling and agents are selfish. When the ceiling is imposed, an increase in fiscal privileges results in a decrease in useful public goods. Hence, private agents may recognize the cost of political efforts to some extent, and not seek too many fiscal privileges.

The intertemporal smoothing condition normally suggests conventional counter-cyclical fiscal policy at the first-best solution. If we were to consider weak governments as the second-best solution, a weaker deficit ceiling results in a larger amount of political effort by the interest group. Such a reaction can be observed in the real economy. A smaller amount of disposable income results in a smaller amount of political effort made for wasteful public spending, while it results in a larger amount of political effort for agent-specific transfers. A smaller amount of tax levied results in a lesser extent of political effort for both types of privileges. These types of behavior are relevant in the real economy.

It is important to investigate how the macroeconomic situation affects the ceiling. Interestingly, the combination of a worse macroeconomic situation in the present and a better macroeconomic situation in the future does not necessarily prompt an increase in the deficit limit. The government should conduct counter-cyclical fiscal policy whenever fiscal privileges are optimally chosen or fiscal privileges and useful public spending are strong substitutes; however, we cannot exclude the opposite case where fiscal privileges are not optimally chosen, wasteful fiscal privileges and useful public spending are not strong substitutes, and political efforts for wasteful spending are relatively large, in which case, the sum of fiscal privileges increases with income and we might justify a relatively pro-cyclical fiscal policy.

Overall, we have shown that preferences in private consumption, fiscal privileges, and useful public spending, and the extent of political effort, in addition to the macroeconomic effect of public spending, are key factors that determine the optimal deficit ceiling in a political economy.
References
