Economic Effects of Public Pensions

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Abstract

While there are various arguments about public pension system reform, this paper examines the conflicting views from the perspective of economic theory. The paper first explains the raison d’etre of public pension systems and then discusses financing systems of public pensions (funding system and pay-as-you-go system). Under the pay as you go system, the benefits to the elderly at the introduction of the pension system are equal to the sum of the present discounted value of net burden of all generations born thereafter. Also, the pay as you go pension system has a huge amount of net pension debt, which must be borne by future generations. The paper points out that shifting to a funding system is an issue of how many years the repayment of such net liabilities should desirably be spread over. Finally, although consumption tax is generally thought to be a desirable revenue source of the pension system, we point out this idea is incorrect from a theoretical view point.

Keywords: public pension reform, pension debt, transition to funded system
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I. Introduction

There are various arguments about public pension reform. While many people have suspicion about the sustainability of the public pension system, some argue that financial problems were alleviated by “macroeconomic sliding” of benefit and by “balanced budget over finite horizon” (Yugen Kinko Houshiki) which were introduced in the 2004 public pension reform. While many people feel that the generational gap of burden and benefit is one of the most serious problems of the Japanese public pension system, others argue that mere comparison of pecuniary benefit and burden is not a whole story. As for pension system reform from the long run perspective, there is an argument that transition to a fully funded system is desirable, while others argue that such a plan is very hard because transitional generations will bear the “double burden”. There is also a controversy about the source of revenue. Some argue that consumption tax should be important, while others believe that the pension system should be based on insurance contribution, not on tax. Of course, there are many arguments among them not based on economic theory. In this paper, we will consider which is desirable between opposing views from the perspective of economics.

In section II, we will show why the public pension system is necessary and discuss the role of it. In section III, the macro economic effects of the pension system will be discussed,
which depends on financing method (funded system vs. pay as you go system). Issues on the transition to a funded system will also be discussed in this section. In this regard, we will pay much attention to the characteristics of intergenerational transfer caused by pension system, or to the intertemporal budget constraint of the pension system, because we believe much confusion about pension reform arises from a lack of understanding on these topics. In section IV, we will discuss the problems related to revenue source of pension system and other issues.

II. The role of public pension system

This section discusses the role of the public pension system. First, we summarize arguments about why the public pension system is necessary. The most important reason is a market failure due to the asymmetric information about the longevity of a member of annuity insurance. If there is no annuity market, it is the role of the government to provide annuity insurance publicly. If annuity insurance is available, people are freed from the risk of uncertain lifetime. By using a simple model, we can show the gains from annuity insurance in this section.

II-1. The role of public pension

There are various arguments why public pension is necessary: (1) failure of private annuity market due to the asymmetry of information, (2) insufficient saving for retirement due to myopic behavior of people, (3) necessity of redistribution between generations, (4) necessity of intergenerational risk sharing (what kind of risk is often not clear), and so on.

As for the market failure due to asymmetric information, adverse selection is most important. Since people don’t know how long they will live in advance, they must save sufficiently for fear of running out of money to live on when they live longer than expected if annuity insurance is not available. And almost all individuals in such an economy do not spend all the money they have. If annuity insurance is available, people will be free from the risk of leaving money unspent. However, if the asymmetric information problems are serious, the private insurance market may not work well and in the worst case may not exist at all.

Generally, members of insurance are thought to be informationally superior at their own longevity than insurance companies. On the other hand, insurance companies can know the average longevity of members, but they cannot know the longevity of each member. In this case, if the insurance company provides the combination of contribution and benefit based on the average longevity of members, those who are likely to die youngest may get out because they think it does not pay. If this happens, because of the increases in the average longevity of members, the contribution must be revised upward (or the benefit must be revised downward) in order to break even. But, it causes further withdrawal of those who are likely to die young. If such a vicious cycle continues, no company may provide annuity insurance
in the worst case.

There is no clear evidence about how serious the adverse selection problem is. It is because almost all advanced countries have well maintained public pension systems, so private annuity insurance has only been a complement. However, if the adverse selection is important, it is necessary that people are obliged to purchase insurance, which is thought to be one of the reasons for the public pension system. Notice that this argument requires compulsory participation but does not require that government must provide annuity insurance. Moreover, the annuity insurance we mentioned here is actuarially fair insurance and not the one which causes intergenerational transfer.

The second reason for the public pension system comes from the fact that people may not save enough for their life after retirement, because people might be myopic or might expect public assistance. But what is derived from this reason is the pension system as a mean of forced savings. It is important to notice that it does not justify the pay-as-you-go system.

As for (3) and (4), even when these objectives are required, it is not necessary to use the pension system, since this kind of redistribution can be feasible by using the combination of the tax and debt issue. Rather, it makes the primary role of the pension system ambiguous, which leads to an obstacle for rational institutional design. Moreover, it is doubtful whether pension system can really achieve a desirable redistribution or risk sharing between generations. The income transfer caused by a pay as you go pension system is basically the one which benefits the initial recipients at the cost of the subsequent generations (which will be explained in section III). It is important to notice that the arguments for (3) and (4) often seem to pay attention only to the within-period redistribution. But the redistribution caused by the pay as you go pension is not completed at a point of time. We must also pay attention to the lifetime transfer or the intertemporal budget constraints of the pension system.

II-2. Gains from annuity insurance

In this section, we will show the gains of annuity insurance by using the 2 period model. Assume that each individual lives with certainty in period 1, but it is uncertain whether he will survive to period 2. Let \( p \) be the probability of survival to period 2 \((0 \leq p \leq 1)\). The path of consumption and utility of each individual will be calculated in the economy with and without annuity insurance, which tells us the “gains” from annuity insurance.

II-2-1. Consumption under uncertain lifetime

Consider the economy without annuity insurance. Let \( c_1 \) and \( c_2 \) be consumption of period 1 and 2, respectively. And let \( \beta \) be subjective discount factor \((0 < \beta < 1)\), and assume that expected utility function is given by

\[
EU = u(c_1) + p\beta u(c_2)
\]
where \( u(\cdot) \) is within period utility function and \( \gamma \) is a degree of relative risk aversion.

Assume that initial wealth of an individual, \( A_1 \), and interest rate, \( r \), are given. And also assume that there is no labor income. Then the asset is evolved according to

\[ A_{t+1} = (1 + r)[A_t - c_t], \]

where \( A_{t+1} \) is assets at the end of period \( t \) (or asset at the beginning of period \( t+1 \)). From the assumption of no bequest motive, \( A_3 \), the assets at the end of period 2, must be 0. Then, we can get

\[ c_1 + \frac{c_2}{1+r} = A_1 \]  

(3)

This is the lifetime budget constraint of each individual\(^1\). Each individual chooses a consumption path so as to maximize equation (1) subject to equation (3). The first order condition of this problem is given by

\[ u'(c_1) = \beta u'(1 + r)u'(c_2) \]  

(4)

Using equation (2), the first order condition can be rewritten as

\[ c_2 = (\beta (1 + r))^{1/\gamma}c_1 \]  

(5)

Using equation (5) and equation (3), we can get a closed form solution of the consumption path, which is substituted into equation (1) to get expected lifetime utility as a function of initial wealth.

\[ V(A_1) = \frac{v^\gamma}{1-\gamma}A_1^{1-\gamma} \]  

\[ v = 1 + \frac{(\beta (1 + r))^{1/\gamma}}{1+r} \]  

(6)

II-2-2. Consumption in the economy with annuity insurance

Consider the economy with annuity insurance. Let \( r \) be the interest rate of ordinary asset. If the annuity insurance is actuarially fair, the expected discounted value of benefit is equal to contribution. If an insurance company invests one unit of contribution into ordinary asset (with interest \( r \)) and benefit is paid to survived beneficiaries, then benefit per survived individual is equal to \( (1 + r)/p \). Since \( 0 < p < 1 \), gross return of annuity, \( (1 + r)/p \), is higher than (gross) interest rate of ordinal asset, \( 1 + r \). Therefore, if the individual has no bequest

\(^1\) \( c_2 \) in equation (3) is the consumption conditional on survival in period 2. If he does not survive to period 2, \( A_2 \) is left for bequest and of course his consumption is 0. If he survives, all amount of \( A_2 \) is used for \( c_2 \).
motive, he will invest his all the wealth into annuity insurance. Therefore the budget constraint is as follows.

\[ c_1 + p \cdot \frac{c_2}{1+r} = A_1 \]  

(7)

The first order condition in this case is given by

\[ u'(c_1) = \beta (1 + r)u'(c_2) \]  

(8)

And the consumption path must obey the following equation.

\[ c_2 = (\beta(1 + r))^{1/\gamma} c_1 \]  

(9)

By the same calculation as before, we can get indirect utility function in the economy with annuity insurance.

\[ W(A_1) = \frac{w^{\gamma}}{1-\gamma} A_1^{1-\gamma} \]

\[ w = 1 + (1 + r)^{-1}[\beta(1 + r)]^{1/\gamma} \]

(10)

II-2-3. Gains from annuity insurance

The expected utility of the individual in the economy with and without annuity insurance is given by \( W(A) \) (equation (10)) and \( V(A) \) (equation (9)), respectively. The gains from annuity insurance can be expressed by increase in utility, or by equivalent variation: what amount of increase in initial wealth is equivalent. Suppose that utility gains from annuity insurance are equivalent to \((m/100)\%\) increase of initial wealth. Since \( V[(1+m)A] \) (utility without insurance) is equal to \( W(A) \) (utility with insurance), \( m \) must satisfy the following equation.

\[ 1 + m = \left(\frac{w}{\bar{w}}\right)^{\gamma/(1-\gamma)} \]  

(11)

Equation (11) shows that gains from annuity insurance depend on survival probability \( p \), degree of relative risk aversion \( \gamma \), interest rate \( r \), and subjective discount factor \( \beta \). Table 1 shows the value of \( 1 + m \) (equation (11)) for different value of \( p \) and \( \gamma \). It is assumed that \( 1 + r = 2.427 \) and \( \beta(1 + r) = 1 \). Since 1 period in this model is about 30 years, \( 1 + r = 2.427 \) corresponds to 3% of the annual interest rate.

In order to get realistic value of \( 1 + m \), a multi period model would be necessary. However, only the result of Table 1 is useful enough. First, the gains from insurance are increasing function of risk aversion. Secondly, decrease in \( p \) generally increases gains for given \( \gamma \). For example, in the case of \( \gamma = 2.0, 3.0, 5.0 \), decrease in \( p \) increases \( 1 + m \). However, when \( \gamma \) is a
little lower (1.01 and 0.5), there are cases that decrease in \( p \) reduces \( 1 + m \) (the case where \( p \) falls from 0.4 to 0.2).

The gains from annuity insurance are derived from two channels. The one is related to relative price of consumption today and future (or price of risk). The other is related to income effect: return from annuity is greater than interest rate of ordinal assets.

The former channel is easily understood when we consider the case \( \beta(1 + r) = 1 \). If there exists no insurance, \( c_1 > c_2 \) must be held as equation (5) shows. That is, the consumer gives greater weight on consumption when survival is more certain. But if annuity insurance is available, equation (9) tells us that \( c_1 = c_2 \) must be held. That is, the consumer chooses a consumption path as if there is no uncertainty.

II-3. Reservations

The gains from annuity insurance in the preceding section might be overestimated. One reason is a possibility that risk sharing within family can substitute for annuity insurance. The other is that annuity insurance may have an adverse effect on capital accumulation.

First, Kotlikoff and Spivak (1981) argued that family can substitute (imperfectly) for annuity insurance. According to their calculation, the gains from joint consumption of couples are about 40% of that of perfect annuity insurance. Members of family are of course not limited to husband and wife. If we consider joint consumption including their children or parents, the gains of risk sharing within family will be larger. In many countries, until recently, small group consisting of families or kin actually dealt with various risks they faced. Now, in many advanced countries, we have experienced the break-up of family, which may probably be attributable to the development of the social-security system or private insurance market. If this conjecture is correct, the gains from insurance must be discounted.

Another point to be added is the adverse effect on capital accumulation. The existence of annuity insurance may reduce savings, which decreases capital and output and therefore weakens the gains from annuity insurance in some degree.

III. Pay as you go system and funded system

It is useful to classify the public pension system into two types; the funded system and
pay as you go system (unfunded system). The pay as you go system is the one which has no accumulation of funds because all contributions are used to pay the benefits for the elderly at the same point of time, while the funded system requires an accumulation of funds before the retirement of the workers in order to pay future benefits of their own. The funded system does not bring about intergenerational transfer, while the pay as you go system does, which is important for understanding the effects of the pension system. In addition, it is important that the pay as you go pension system always has positive net pension debt, which must be the burden for the subsequent generations as the intertemporal budget constraint of the pension system shows.

III-1. Intergenerational transfer under the pay as you go system

To simplify the discussion, consider the following 2 period OLG model. People work in the first period, when they are young, retire in the second period, when they are old. All people are assumed to be alive till the end of the second period, that is, there is no uncertainty of longevity. Generation $t$, people born at period $t$, works and receives wage $w_t$ and pays contribution when young (at period $t$), and receive benefits when old (at period $t+1$). Let $g$, $n$ and $r$ be growth rate of wage, growth rate of population and interest rate, respectively, which are assumed to be constant. And let $L_t$ be the population of generation $t$. From the assumption above, $w_t$ and $L_t$ grow according to $w_{t+1} = (1+g)w_t$, $L_{t+1} = (1+n)L_t$.

In order to compare the pay as you go system with the funded system, the pension benefit is assumed to be a constant fraction of wage per worker so that benefit is equal under both systems. Let $b$ be the benefit wage ratio, then the benefit received per generation $t$ when he is old (at period $t+1$) is equal to $bw_{t+1}$. Since the benefit per elderly is exogenously given, contribution per worker is determined endogenously. Let $\tau^F$ and $\tau^P$ be the contribution rate (the ratio of contribution to wage) of the funded system and pay as you go system, respectively. Since benefit per person under the funded system is equal to the principal and interest of his contribution, $\tau^F w_t (1+r) = bw_{t+1}$ must be satisfied. From this condition and $w_{t+1} = (1+g)w_t$, the contribution rate of the funded system is determined as follows.

$$\tau^F = b(1+g)/(1+r)$$ (12)

Next, we will derive the contribution rate of the pay as you go system, under which the total contribution paid by the worker is transferred to the elderly. Since the total contribution at period $t$ is $\tau^P w_t L_t$, and total benefit at the same period is $bw_t L_{t-1}$ (the total population of the elderly at period $t$ is $L_{t-1}$), $\tau^P w_t L_t = bw_t L_{t-1}$ must be satisfied. Therefore we can get

$$\tau^P = b/(1+n)$$ (13)

From equation (12) and (13), the gap between $\tau^P$ and $\tau^F$ is derived
This equation shows that the contribution rate of a pay as you go system is higher than that of a funded system, when the interest rate exceeds the economic growth rate, that is, \(1 + r > (1 + g)(1 + n)\) is satisfied. While \(\tau^p\) is lower than \(\tau^f\) when interest rate is lower than economic growth rate².

From this result, some argue that the pay as you go system has an advantage when the economic growth rate is high (as in the 1960s or 70s), but when the economy is aging and high growth rate cannot be expected anymore, the funded system has an advantage. There are two problems in this argument. One is that the interest rate is not determined independently from the economic growth rate. Another is that equation (14) does not reflect the complete picture of the intergenerational transfer caused by the pay as you go system.

First, it is important that whether the interest rate is larger than the economic growth rate or not depends on the relative scarcity of capita to labor. If \(1 + r > (1 + g)(1 + n)\), then the capital in the economy is smaller than the golden rule level. If, on the other hand, \(1 + r < (1 + g)(1 + n)\), then the economy is overaccumulated. Such an economy is dynamically inefficient, since increase in consumption at some period does not decrease consumption at any other period. Although there is some possibility for such a case, at least theoretically, most countries including Japan are considered to be dynamically efficient. Therefore, it is appropriate to assume \(1 + r > (1 + g)(1 + n)\). Now, let us think about the following question. Does the funded system have an advantage over the pay as you go system in the economy where interest rate exceeds economic growth rate? The answer is not yes. Because equation (14) is a comparison of contribution rate only after the introduction of the pension system, in other words, the transfer to the initial recipients is not taken into consideration. It is also equivalent to say that the funded system has accumulated funds while the pay as you go system has not. In order to clarify these assertions, it is useful to investigate the interrogational transfer under each pension system.

Let \(\Delta W_t\) be the change in lifetime income per generation \(t\) discounted at period \(t\). First, consider the case of the funded system. Since the benefit is equal to the sum of principal and interest of contribution, discounted benefit is equal to contribution, which means \(\Delta W_f = 0\) under the funded system, while the pay as you go system causes intergenerational transfer. Suppose that the pay as you go pension is introduced at period 0. Then the elderly at period 0 (generation \(-1\)) can receive benefit without contribution. Since the benefit per generation \(-1\) at time 0 is \(bw_0\), change in lifetime income discounted when they are young is given by

\[
\Delta W_{-1} = \frac{bw_0}{1+r} > 0
\]  

On the other hand, the change in lifetime income of generation born after period 0 is as

² Gross return of pension is given by \(bw_{t+1}/\tau w_t (i = F, P)\), which is equal to \(1 + r\) for the funded system and \((1 + g)(1 + n)\) for the pay as you go system.
follows ($t=0,1,2,..$).

$$\Delta W_t = \frac{bw_{t+1}}{1+r} - \tau^P w_t = -(\tau^P - \tau^F)w_t < 0$$

That is, while generation $-1$ gains, all the subsequent generations suffer a loss. Furthermore, the following relationship is derived by simple calculation.\(^4\)

$$\Delta W_{-1}L_{-1}(1 + r) + \sum_{t=0}^{\infty} \frac{\Delta W_tL_t}{(1+r)^t} = 0$$

Equation (17) shows that income transfer caused by the pay as you go pension system has a zero sum nature, that is, excess benefit of generation $-1$ is just equal to the sum of discounted excess burden of all the subsequent generations. In this sense, $(\tau^P - \tau^F)w_t$ of equation (16) is considered to be a (implicit) tax for financing the transfer to generation $-1$. And higher contribution rate of the pay as you go system is considered to be attributable to this “implicit tax”.

The discussion above is also confirmed by comparing net pension debt with the “implicit tax” of equation (16). Net pension debt is the gap between pension liability (the discounted value of benefits that the government is obliged to pay for eligible recipients) and accumulated funds. Net pension debt is equal to $\tau^P w_tL_t$, since gross pension debt is $bw_tL_{t-1} = \tau^P w_tL_t$ and the accumulated fund is zero (because a pure pay as you go system is assumed). Let $T$ be the total “implicit tax” at period $t$, and let $D$ be the net pension debt at the same period. Then the ratio of $T$ to $D$ is given by

$$\frac{T}{D} = \frac{(\tau^P - \tau^F)w_tL_t}{\tau^P w_tL_t} = 1 - \left(\frac{(1+g)(1+n)}{1+r}\right) = r - (n + g)$$

That is, the ratio of total implicit tax to net pension debt is equal to the gap between interest rate and economic growth rate. As is well known, if we want to keep the debt GDP ratio constant, required primary surplus is $(r - (n + g))$ times $D$, which is just equal to $T$ of equation (18). That is, implicit tax is just equal to the amounts which keep net pension debt relative to GDP constant. In other words, the excess burden of generation $t (=0,1,2,..)$ is the minimum burden for sustainability of the pension system.

### III-2. Intertemporal budget constraint of the pension system

Next, we will consider the transfer caused by the pension system from another viewpoint. Let $F_t$ be the accumulated funds of pension at the beginning of period $t$, and $T_t$ and $B_t$

\(^3\) The second equality in equation (16) is derived from $bw_{t+1} = \tau^F w_t(1 + r)$.

\(^4\) Equation (17) is derived by using equation (14), (15), (16) and $w_{t+i} = (1 + g)^i w_t$, $L_{t+i} = (1 + n)^i L_t$, and by using the formula for the sum of geometric series.

\(^5\) Net pension debt of the funded system is zero, since gross pension debt is equal to accumulated funds.
be the total revenue (contribution and tax) and total benefit at period $t$, respectively. For simplicity, the interest rate, $r$, is assumed to be constant. From accounting identity, we can get

$$\sum_{i=0}^{k-1} \frac{B_{t+i}}{(1+r)^i} + \frac{F_{t+k}}{(1+r)^k} = F_t + \sum_{i=0}^{k-1} \frac{T_{t+i}}{(1+r)^i}$$

(19)

If the second term of LHS of equation (19) satisfies the following condition, the pension system is sustainable\(^6\).

$$\lim_{k \to \infty} \frac{F_{t+k}}{(1+r)^k} = 0$$

(20)

Equation (20) is called the No-Ponzi Game (NPG) condition. If this condition is satisfied, equation (19) becomes

$$\sum_{i=0}^{\infty} \frac{B_{t+i}}{(1+r)^i} = F_t + \sum_{i=0}^{\infty} \frac{T_{t+i}}{(1+r)^i}$$

(21)

LHS of equation (21), the sum of the discounted value of total benefit from present to future can be broken down into two parts; (1) $B^P$: benefit obligation corresponding to past contribution, and (2) $B^F$: benefit obligation corresponding to future contribution. Then equation (21) can be rewritten as

$$B^P - F = T - B^F$$

(22)

where $F$ and $T$ are the first and the second term of RHS of equation (21). LHS of equation (22) represents net pension debt, the gap between benefit obligation (corresponding to past contribution) and accumulated funds. And RHS of equation (22) is the excess burden corresponding to future contributions.

If the pension system is a funded one, funds are accumulated to meet benefit obligations (that is, $B^P = F$), then there is no excess burden corresponding to future contributions (that is, $T = B^F$ must be satisfied). On the other hand, a pay as you go pension system has positive net pension debt, which must be financed by excess burden corresponding to future contribution ($T - B^F$). Net pension debt in Japan is in fact very large, amounting to at least 150% of GDP.

### III-3. Implications

As explained in the previous section, the pay as you go pension system transfers income

\(^6\) These conditions must be satisfied even if $F_{t+k}$ is negative. In general, the pension system may be considered to be unsustainable, if funds are exhausted. But the condition here is weaker, because equation (20) only requires the growth rate of funds (in absolute value) to be smaller than the interest rate. A more realistic condition may be that the absolute value of $F_{t+k}$ must not exceed some certain fraction of GDP.
to the initial recipient from the later generations. Alternatively it is equivalent to say that the pay as you go pension system has net pension debt at all times, which must be financed by implicit tax of the subsequent generations. In this section, the discussion of the previous section is applied to the following problems: (1) whether intergenerational transfer can be a reason for the pay as you go system or not, (2) examination of the “double burden”, which are generally believed to make a shift to the funded system difficult, (3) how to evaluate the transition to the funded system, and (4) the effects of “Balanced Budget within Finite Horizon” (Yugen Kinko Houshiki) employed in the 2004 Pension Reform in Japan.

III-3-1. Can intergenerational transfer be a reason for the pay as you go system?

First, we will examine whether intergenerational transfer can be a reason for a pay as you go system. As already explained, the only generation who gains from the pay as you go system is the initial recipient. If this generation are in a difficult situation and need to be assisted, there may be a good reason for transfer. But, in general, we cannot expect that situation. Of course, we can think of redistribution between generations, which is different from a pure pay as you go pension system. For example, since later generations benefit from economic growth, we can think of transfer from later generations to earlier generations so that after tax (and transfer) income is completely equalized for all generations. But we will show that such transfer is infeasible.

Here, the same model in the previous section will be used. Let \( n, g \) and \( r \) be the growth rate of population, growth rate of wage and interest rate, respectively, all of which are assumed to be constant. Wage income per generation \( t \) is represented by \( w_t \), and population of generation \( t \) is represented by \( L_t \). First, we will consider what amount of transfer is required in order to achieve full equalization of after tax (and transfer) income. Since any income transfer has a zero sum nature, the following equation must be satisfied.

\[
\sigma_{t=0}^{\infty} \left( \frac{w_t L_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{\bar{w} L_t}{(1+r)^t} \right) = 0 \tag{23}
\]

Where \( \bar{w} \) is the equalized after tax (and transfer) income, from this equation we can solve for \( \bar{w} \) as follows.

\[
\bar{w} = \frac{1}{1 + \frac{1+n}{1+g}(1+r)} w_0 \tag{24}
\]

Let \( t_0 \) be the \( t \) that satisfies \( w_t - \bar{w} = 0 \). It is obvious that the generations born before \( t_0 \) get transfer and that the generations born after \( t_0 \) bear the burden. In order to implement such transfer, government bonds must be issued for a while after period 0. When \( t > t_0 \), workers pay tax but the government still issues bonds as long as interest payment is larger than tax revenue. Therefore, it is necessary to investigate the path of government debt in order to clarify this redistribution scheme.

Since income transfer in general has a zero sum nature, net debt at period \( t \) must be equal
to the sum of discounted burden of subsequent periods. Let $D_t$ be net government debt at period $t$. Then the following equation must be satisfied.

$$D_t = \sum_{i=0}^{\infty} \frac{(w_{t+i}-\bar{w})L_{t+i}}{(1+r)^i}$$  \hspace{1cm} (25)

By some calculation, we can get

$$\frac{D_t}{w_t L_t} = \frac{1}{\frac{1+(n+g)}{1+r}} \left[ 1 - \frac{1}{(1+g)^t} \right]$$  \hspace{1cm} (26)

Equation (26) shows the ratio of debt to total wage is an increasing function of $t$. When $t$ goes to infinity, debt relative to total wage approaches the following value.

$$\lim_{t \to \infty} \frac{D_t}{w_t L_t} = \frac{1}{1 - \frac{(1+n)(1+g)}{1+r}} \approx \frac{1+r}{r-(n+g)}$$  \hspace{1cm} (27)

The value of equation (27) is 3.98, if interest rate, population growth rate, and wage growth rate are at annual rate 3%, 1%, 1%, respectively. If the gap between interest rate and economic growth rate $(n+g)$ is somewhat larger, this value becomes smaller. For example, if the annual interest rate is 3% and annual economic growth rate is 1%, the debt to total wage ratio converges to 2.25. Notice that the total wage at period $t$ in this model $(w_t L_t)$ corresponds to the cumulative total wage for 30 years. Therefore, we must multiply $D_t/w_t L_t$ of equation (27) by about 20 to 30 in order to get the actual value of the debt to total wage ratio. That is, it is shown that debt will eventually be accumulated from about 50 to 100 times the annual total wage.

The corresponding ratio of a pure pay as you go pension system is smaller compared to this ratio. Since net pension debt of the pure pay as you go system at period $t$ is $\tau^P w_t L_t$, the debt to total wage ratio is $D_t/(w_t L_t) = \frac{\tau^P}{b/(1+n)}$. This ratio is 0.37 when $b = 0.5$ and annual population growth rate is 1% ($1 + n = (1.01)^{30}$). That is, the size of debt under the pure pay as you go system is only one-tenth of that of the transfer scheme considered here, but it is still sufficiently large, because 0.37 multiplied by 30 is 11.1 (corresponding to the actual debt to total wage ratio).

Moreover, it is also important that the burden of future generations must be very heavy. For the generations born after $t_0$, the gap between before tax wage $(w_t)$ and after tax wage $(\bar{w})$ increases at an exponential rate with $t$. The ratio of after tax wage to before tax wage approaches 0 as $t$ goes to infinity. This result is robust if a somewhat moderate transfer is considered. For example, instead of full equalization of after tax income, the program such as reducing growth rate of (after tax) wage to $g'$, which is smaller than that of before tax wage, will not change the qualitative results.

The transfer program considered here seems infeasible not only because it conflicts with the ordinal sense of fairness or equity, but also because future growth rate of wage is

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7 $r, n, g$ are calculated as cumulative rate for 30 years.
unpredictable. For example, growth rate of wage, $g$, may be a nonstationary random variable. In this case, there would be considerable risk if the transfer program is designed solely by expected value of $g$. If an unexpected decrease in $g$ happens in some period, the government may be in trouble to repay the interest rate of (huge) accumulated debt. The transfer program itself may have effects on wage, interest rate and growth rate. For example, increase in consumption of the generations whose benefits exceed burden may reduce capital accumulation, which will reduce wage rate and increase interest rate. It might be important that the transfer have adverse effects on human capital accumulation, which may depress growth rate.

Next, we will discuss whether the pension system can save the specific generations who suffered a loss because of war or natural disaster. If it is one that happens once a century, it is better to accumulate funds in peacetime against an emergency and to dissave (or issue bonds) at the time of the crisis, which leads to risk sharing between different generations and also equalize the tax burden of different generations. Since the desired average level of funds in the long run is zero from this viewpoint, we cannot justify consistently positive debt as a pay as you go system has. Moreover, it is not the regional shocks or individual specific shocks but the generation specific shocks that a pay as you go system can treat well. For example, there are many who did not suffer in some areas even in the disaster such as the Great East Japan Earthquake, while it affected almost all ages in the affected area. If the shocks of the disaster cannot be broken down by age group, the pay as you go pension system cannot treat them well.

III-3-2. Double Burden

Some argue that the transition to a funded system from a pay as you go system is difficult because of the double burden of the transitional generations. This is not correct, because even when the pay as you go system is maintained there is a net pension debt, which must be financed by the implicit tax of the subsequent generations. Another problem is related to the length of transition periods. Those who argue that transition is difficult seem to assume as if transition will be completed instantaneously. They argue that some generations must pay the contribution of the existing pay as you go system (for the retirees) and also save for retirement on their own, which is the origin of the term “double burden”. Clearly some specific generations need not pay the entire cost of transition. If periods of transition are taken longer, the transition cost will be spread over more than several generations.

Suppose that there exists a pay as you go pension system and that transition to a funded system is started at period $t$ and it is completed by the beginning of period $t + T$. In order to complete transition for $T$ periods, the burden imposed on transitory generations needs to be heavier than $(\tau^p - \tau^f)w_{t+i}$, implicit tax where the pay as you go system is maintained forever. Let $(1 + \eta_T)(\tau^p - \tau^f)w_{t+i}$ be the burden for transitional generations when transition takes $T$ periods. From the zero sum nature of income transfer, the following equation must
be satisfied.

\[ D_t = \sum_{i=0}^{T-1} \frac{(1+\eta_T)(r^p - r^f)w_{t+i}L_{t+i}}{(1+r)^i} \]  (28)

Where \( D_t = r^p w_tL_t \) is the net pension debt at period \( t \). From equation (28), we can get

\[ 1 + \eta_T = 1/ \left( 1 - \frac{(1+n)(1+g)}{1+r} \right)^T \]  (29)

Equation (29) shows that \( \eta_T \) becomes small as \( T \) becomes large. Notice that when \( T \) is infinite (the case when the pay as you go system is maintained forever), \( 1 + \eta_T \) is equal to 1, the same burden as the pay as you go system.

When the annual rate of \( r, n, g \) is 3%, 1%, 1%\(^8\), \( \eta_T = 2.975, 0.722, 0.307, 0.058 \) for \( T = 1, 3, 5, 10 \), respectively. If the annual rate of \( r, n, g \) is 3%, 0%, 1%, then \( \eta_T = 1.149, 0.207, 0.056, 0.0028 \) for \( T = 1, 3, 5, 10 \), respectively. This result shows that transition cost imposed on the transitional generations is not always heavy if the length of transition periods is sufficiently long. Of course 30 years of transition periods (which corresponds to 1 period in this model) are clearly unrealistic, but if 90 years of transition periods are taken (3 periods), the additional cost required is only 20% compared to the pay as you go system if the gap between (annual) interest rate and economic growth rate is 2%.

There is an important point to be noticed. True transition requires complete redemption of net pension debt. Suppose that at some time the existing pay as you go pension is abolished and at the same time a funded pension system is newly established. But the government has to pay pension benefits of the old system (pay as you go system). If the government issues bonds in order to finance this expenditure, the subsequent generation must pay the burden of debt. If the burden of debt is spread over entire generations, it is exactly the same as maintaining the pay as you go system except that implicit debt is replaced by explicit debt.

**III-3-3. Transition to a funded system**

As already mentioned, it is not correct to argue that transition to a funded system is difficult. If sufficient periods for transition are taken, the cost per person will not be so heavy because it is spread over several generations. But if any pension system reform has a zero sum nature, then where is the merit of the transition?

As to this point, Feldstein (1995) discussed that transition to a funded system has efficiency gains. But his discussion has two problems. First, he considered the total consumption at each period, not the consumption of each generation, which obscures the conflicts of interest between different generations. Second, he assumed that the discount rate is smaller than the market interest rate, by which he overlooked the zero sum nature of

\(^8\) Equation (29) is based on the 2 period OLG model. So, \( r, n, g \) in equation (29) are the cumulated values for 30 years.
pension system reform (evaluated at the initial interest rate). On the other hand, Geanakoplos and Mitchell and Zeldes (1998) argued that there is no gain of transition since there exists cost for net pension debt, which is the same argument as equation (17) or equation (22) shows. Orszag and Stiglitz (2001) also discussed the same.

But the zero sum nature of income transfer fails to hold once the effects on resource allocations are considered. Especially, the effect on capital accumulation is important. Capital stock will be increased as net pension debt is reduced, which increases wage rate and decreases interest rate. From some point of time, increase in wage will exceed the transition cost.

When the shift is completed, the workers are free from transition costs and also enjoy higher wages as a result of increase in capital stock. If the periods of transition taken are short, the burden of transitional generations will be heavy but the generations of the near future will enjoy utility increase. While long periods of transition will impose a somewhat lighter burden for transitional generations, those who will enjoy the higher income are generations of the distant future. Therefore, the problem is reduced to comparison of interest of different generations, the problems of value judgment.

Aso (2005) investigated this problem based on simulation analysis using the 2 period OLG model with endogenous capital stock and found that there is no serious problem with relatively short periods of transition. This is because the effect of increase in capital stock is sufficiently large and surpasses the transition costs on transitional generations; as pension debt is reduced, capital stock is increased, which increases wage rates therefore increases before tax lifetime income. Fig. 1 shows costs imposed on transitional generations are not so

Fig. 1 The length of transition periods and lifetime income of transitional generations

Source: Fig 6-3 in Aso (2005).
heavy even if the length of transition periods is 2 or 3 periods (60 years or 90 years). Of course, it is dependent on a value judgment; comparison of utility of different generations.

**III-3-4. Balanced Budget over Finite Horizon**

“Balanced Budget over Finite Horizon” (Yugen Kinko Houshiki) was introduced in the 2004 Pension Reform. Previously, required funds reserved were about 4 - 5 years of benefits, but now it is only about 1 year of benefits that is required at the end of the planning periods, which is about 100 years ahead. This means that funds which have been accumulated so far are used to pay the current and near future benefits, which will increase future net pension debt, and hence further increase the burden of the subsequent generations.

**IV. Tax or contribution**

There are many who expect consumption tax to be the stable source of revenues for pension benefits, while some argue that revenues should be based on contribution. Although there are various opinions, it is not a difficult problem, at least theoretically. Since beneficiaries are easily identified, there is no reason why we must rely on consumption tax instead of contribution. The problem is that the terms generally used are ambiguous, which confuse the discussion. As mentioned before, contribution of the pay as you go pension consists of two parts; the fraction which corresponds to \((\tau^P - \tau^F) w_t\) and the discounted value of benefits of his own \((\tau^F w_t)\). The former is considered to have the same effects as wage tax and the latter is considered to be the same as savings. Notice that some proponents for “social insurance system” often support the “social insurance contribution” which is in fact equivalent to the tax used for specific expenditure (pension benefits). It is important to distinguish what part of the contribution is equivalent to tax and what part of it corresponds to savings in order to discuss the economic effects.

**IV-1. Correspondence of benefit with contribution**

In section III, we argued that the fraction \((\tau^P - \tau^F) w_t\) of the pay as you go contribution is equivalent to tax and that the fraction \(\tau^F w_t\) is equivalent to savings. When labor supply is endogeneous, the pay as you go pension contribution has distortional effects on labor supply, since some fraction of it is equivalent to wage tax. Although it does not distort intertemporal consumption choice, it reduces capital accumulation because the fraction \(\tau^F w_t\) is not funded. On the other hand, contribution of the funded system is equivalent to savings and it does not affect resource allocation including the effects on capital accumulation except that some portions of private savings are replaced by government savings.

So far, we have considered a pension system with contributions and benefits completely proportional to wage. But the actual benefits are often a two tier system; fixed amount component and earnings-related component. Let \(\beta\) be the ratio of the earnings-related benefit
to wage, and let \( \tau \) be the contribution rate (which is assumed to be proportional to wage). Then it is appropriate to assume \( \tau - \beta \) as tax rate on wage\(^9\). If pension system is designed to redistribute (within generation) more, that is if \( \beta \) is small, then the fraction of contribution equivalent to tax becomes large. On the other hand, if pension system redistributes less, contribution becomes more like savings.

The fraction of contribution equivalent to tax also depends on \( \tau \). The contribution rate of the Japanese pension system (Employees’ pension program: Kosei nenkin) has been gradually increased. Since it was very low for the current elderly when they were young, there are some generations whose \( \tau - \beta \) are negative, a subsidy to wage; while \( \tau - \beta \) of current and future generations is positive. There is also a case for \( \beta = 0 \) and \( \tau = 0 \): fixed amount of benefit and contribution. The benefit and contribution of the National Pension program (Kokumin nenkin: pension program for self-employed or freelance) are a fixed amount.

We can also imagine the funded system which has a fixed amount of benefit and contribution proportional to earnings. Since \( \beta = 0 \) in this case, all the contribution is equivalent to tax on wage. On the other hand, there is a pay as you go system whose tax rate on wage (\( \tau - \beta \)) is as small as possible. NDC (notional defined contribution) of Sweden is an example of such a system. It has individual accounts, on which all contributions paid by workers are recorded and the benefit of each worker is calculated based on notional rate of return. Thus benefits of NDC are proportional to wage. However, since the rate of rerun of the pay as you go system (including NDC) is lower than the interest rate, NDC also has a wage tax component (that is, \( \tau - \beta > 0 \)).

**IV-2. Consumption tax**

Many argue that an increase in consumption tax must be necessary to finance pension benefits since the contribution rate cannot be increased any more. Others argue that consumption tax is desirable because the elderly must bear some burden of government expenditure in an aging society. But, there are two problems in these arguments. First, these arguments are not based on the proposition that consumption tax and wage tax are equivalent. Second, shifting to consumption tax weakens correspondence of benefits with contribution, which has a distorting effect on labor supply more than before.

First, it is a famous proposition that proportional wage tax and consumption tax are equivalent. This proposition is easily derived from lifetime budget constraints, which says that the sum of discounted value of consumption is equal to (or less than) the sum of discounted value of wage, if bequests and inheritance are ignored. The difference of consumption tax and wage tax is only the differences in the timing of tax imposition. The argument that the elderly should pay sufficient tax in an aging society also has a problem; it lacks the understanding of lifetime budget constraints. There is no problem if the elderly paid sufficient wage tax when they were young even if they do not pay tax when they are old.

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\(^9\) The model in section III is the case of \( \tau = \tau^p \) and \( \beta = \tau^p \).
Consumption tax and (proportional) wage tax also have an equivalent effect on labor supply. Wage tax reduces labor supply because wage tax imposes a wedge between the wage employers pay and the wage workers receive. Consumption tax also imposes a wedge. Let $p$ be the price of consumption goods, $w$ be the nominal wage paid by employers, and $\theta$ be the consumption tax rate. Suppose that employers pay $w$ to workers to produce one unit of consumption goods. The cost of labor relative to consumption goods which employers face is $w/p$. On the other hand, real wage workers face is $w/(p(1+\theta))$ because workers pay $p(1+\theta)$ for one unit of output. Since $w/p > w/(p(1+\theta))$, consumption tax also imposes the tax wedge and has distortional effects on labor supply. Therefore, shifting to consumption tax from contribution does not ease distortion on labor supply\(^{10}\).

The second point is related to the discussion in section IV-1. The shift to consumption tax from a contribution (proportional to wage) has another important effect and is not the same as a switch to consumption tax from wage tax. Since consumption tax is indirect tax, it is impossible to record the amount of tax paid by each individual, which makes correspondence of benefits and contribution weaker than before. That is, the distortion on labor supply becomes heavier by shifting to consumption tax: in other words, $\tau-\beta$ becomes large since $\beta$ becomes small. Moreover, after much of the revenue is dependent on consumption tax, it become very difficult to reform the system toward the one with individual account\(^{11}\).

**IV-3. Switching to consumption tax**

Many studies using OLG simulation model insist that that a switch to consumption tax as a source of pension benefits promotes capital accumulation and increases social welfare. But, these results depend on several assumptions, which need to be paid attention to when we apply it to the actual economy.

First, switching to consumption tax from social insurance contribution or wage tax often causes intergenerational transfer. The assumption often employed is the revenue neutrality constraint, which states revenue at each period is held constant before and after the pension reform. We may call the constraint a neutrality constraint at each point of time. It is possible to employ another constraint; neutrality constraint for each generation, that is, lifetime burden of each generation is held constant before and after the pension reform. Much of the simulation studies employ the former constraint, which makes the lifetime burden of the current elderly

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\(^{10}\) The lifetime tax burden of the elderly may be increased and at the same time that of younger generations may be decreased if the shift to consumption tax is the revenue neutral one (which does not change total revenue at the time of pension reform). Such a transfer makes the elderly poorer and the younger worker richer, which affects labor supply through income effect. Moreover, since the members of the National Pension Plan (Kokumin Nenkin) such as self-employed workers also pay the consumption tax, the burden for members of the Employees’ Pension Plan (Kosei Nenkin) may be decreased to some extent, which might have some effects on total labor supply.

\(^{11}\) This argument does not necessarily deny the role of consumption tax. It has no problem if consumption tax is used to finance the net pension debt and contribution is used to accumulate funds. Of course, consumption tax is not the only solution for financing the current net pension debt.
heavier than before. Since income transfer basically has a zero sum nature, heavier tax burden for the elderly means a lighter tax burden for the younger generations. As a result, consumption of the elderly is decreased, while the consumption of the younger generations is increased. But the former effect exceeds the latter one for several periods after pension reform, because gains for the younger generations are spread over the generations who have not yet entered the economy. Therefore, for several periods after the pension reform, total consumption will be decreased, which helps capital accumulation. So far, we have mentioned two sources of welfare changes: first is the increased output due to increased capital stock, and second is the shift of tax burden (future generation gains at the expense of current elderly). In relation to the effects from intergenerational transfer, there is another channel of the efficiency gains: consumption tax on the current elderly works like lump sum tax on accumulated wealth. Notice that behind the total welfare gains there exists utility decrease of the current elderly, which is often overlooked by research.

Second, there is the effect that increases the distortion on labor supply. This effect emerges because switching to consumption tax weakens the relation between benefit and contribution, which has an effect that increases the tax component of the pension system, as we explained in section IV-1. Simulation studies with fixed labor supply cannot capture this effect. Moreover, if the simulation studies use the model in which labor supply decision does not depend on the earning related component of pension benefit, efficiency loss of consumption tax cannot be caught. In fact, many studies overlooked this effect.

Third, we can employ another revenue constraint, which causes no intergenerational transfer; the revenue constraint which does not alter the lifetime burden of all generations. Consider the case of switching to consumption tax from wage tax with this revenue constraint. In order not to change the lifetime tax burden of the elderly, government bonds need to be issued to finance the transfer for them which exactly offset the increase of consumption tax imposed on them. Since the lifetime burden of all generations do not change, consumption of each generation and thus total consumption does not change. But this increases the total private savings, because after tax wage income (of the workers) is increased because of the reduction of wage income tax. But since increase in private savings is exactly the same as government bond issues, there is no effect on national savings, that is, no effect on capital accumulation. In this case, the switching to consumption tax from wage tax does not bring about any efficiency gains.

V. Conclusion

We investigated the economic effects of pension systems while focusing on the intergenerational income transfer and the intertemporal budget constraint. We emphasized that the pay as you go pension system is the transfer system such that only the initial recipient gains and all subsequent generations bear the burden. We also emphasized that the pay as you go pension system always has positive net pension debt, which must be the burden of the subsequent generations. Thus, the essential problem is how to treat existing net pension debt;
how to share the burden between different generations in order to achieve economic efficiency and also generational equity.

References


