Risk and Return in Japanese Equity Market

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Abstract

The market portfolio is often used as a benchmark portfolio. Japanese equity market data however shows that the market portfolio is not efficient and furthermore not profitable. The empirical support for CAPM in the Japanese market is weak. Overall, Japanese investors experienced hard time, because they used the market portfolio as their benchmark without careful investigation and adopted many active managers against the benchmark. Not only because few active managers successfully added value but also because they surely increased the total risk and total cost, the portfolios of Japanese investors typically have shown disappointing performance. However, if portfolios are carefully constructed so that parameter estimation risk is not amplified in the process of portfolio formation, there is some evidence to suggest that past return data contains useful information to identify the risk and return trade-off in the Japanese equity market.

Key words: Japanese Equity Market, Capital Asset Pricing Model, Mean-variance portfolio, Minimum variance portfolio
JEL classification: G11, G12

1. Introduction

The trade-off between risk and return is often taken for granted. Japanese equity market data however arouses concern in believing the trade-off relation. If we have invested in the value-weighted portfolio for the last 30 years, then the cumulative return over risk free rate is about zero. Taking market risk in the Japanese equity market was not rewarded.

The value-weighted portfolio, that is, the market portfolio is supposed to be on the efficient frontier under CAPM. Many empirical studies in U.S. equity market data conclude that the market portfolio is not located near the efficient frontier. However, the investment in the market portfolio may make sense because its return in the long-run is positive.

Early empirical studies on CAPM found positive relation between market risk and return. Sharpe-Lintner CAPM was not strongly supported because the slope of regression of mean returns on betas is too flat. Black CAPM was not completely rejected because the market risk premium turned out to be positive.

Empirical performance of CAPM is much worse with Japanese data. As we will show

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later, high (low) beta equities tend to show low (high) average returns. Market risk and market return are negatively correlated. The market portfolio was not on the efficient frontier, and furthermore it was not profitable. Taking the market risk has not been rewarded. For investors who consider investing in the market portfolio as an investment in the equity market, they may conclude that equity market investment is not worth it in the Japanese equity market.

Empirical failure of CAPM does not necessarily mean that the efficient frontier is degenerated to a horizontal flat line. The zero beta portfolio, which was introduced by Black, Jensen, and Scholes [2] and is a portfolio having zero covariance with the market, has positive average return in the Japanese market. There are thus different sources of risks that reward.

The historical relation between risk and return can be different from the current relation between risk and return. To know today’s shape of the efficient frontier is not easy at all. It is well known that the mean-variance portfolio is so sensitive to its input parameters, that is, mean return vector and covariance matrix. Since we only know these parameters with large errors, estimated efficient frontier can be totally different from the true one. Because of the estimation error, the out-of-sample performance of the mean-variance portfolio problem is in general awful.

To improve out-of-sample performance of the mean-variance portfolio, many approaches are already suggested by researchers and practitioners. For example, Ledoit and Wolf [12] and Jorion [10] propose a Bayesian approach to improve the estimation of covariance matrix and mean return. Pástor [13] uses the asset pricing model to reduce estimation error. Jagannathan and Ma [9] points out constraints on portfolio positions such as short sales restrictions reducing the loss by parameter estimation error.

These solutions may not work out effectively. DeMiguel, Garlappi, and Uppal [5] apply these approaches to major data sets. Through their exhaustive study, they conclude that nothing consistently dominates the equally-weighted portfolio $1/N$, assuming that the number of assets is $N$. Some practitioners advocate that the global minimum variance portfolio (GMVP), which is the portfolio on the efficient frontier with the smallest variance, works better. For example, see Clarke, de Silva, and Thorley [3]. Since both $1/N$ and GMVP portfolios do not use any prediction on the expected returns, these results suggest that mean return prediction does not improve the out-of-sample performance of the mean-variance portfolio. It also suggests that to identify the current shape of the efficient frontier is difficult.

In this paper, we adduce evidence to show that the past return data contains some information on the efficient frontier. Previous studies typically investigate the out-of-sample performance of the tangent portfolio. Parameters are estimated with errors, and the out-of-sample performance is not stable because the tangent portfolio is so sensitive to input parameters. The parameter estimation error is amplified in the process of constructing the tangent portfolio.

We study the out-of-sample performance of portfolios that invest all wealth to the GMVP and add zero-investment portfolio on it with the constraint on the tracking error from GMVP. The zero-investment portfolio short sales GMVP and holds long the tangent portfolio. The
motivation of this approach is to distinguish the parameter-estimation error and the portfolio-construction error. Since GMVP does not need mean return prediction as its input, it has the smallest estimation error among the portfolios on the efficient frontier. Other portfolios are not independent of mean-return predictions.

The return data we use is monthly FF25 portfolio return data in the Japanese equity market. For the expected return vectors, we use sample mean, the mean return assuming CAPM and FF three factor model, that are estimated using the last 60-month return data as typically done in literature such as Jorion [10]. The out-of-sample returns of these portfolios improve the risk and return profile, which means the past return data have some predictive power on the efficient frontier, at least locally near GMVP.

Finally, we consider the implication of these findings to Japanese investors. Asset management is often delegated to professional portfolio managers. The value-weighted portfolio is often chosen as their benchmark. Portfolio managers are constrained by tracking errors from the benchmark portfolio and are requested to make excess profit above the benchmark. As Roll [14] pointed out, this benchmarking approach is problematic because the portfolio chosen by fund managers is irrelevant to the benchmark choice and tends to increase the total risk of the portfolio. Furthermore, Fama and French [7] pointed out the return performance by fund managers are in general not satisfiable.

Many Japanese investors have been using the value-weighted portfolio as their benchmark for a long time. However, it was not efficient and furthermore was not profitable. Thus typical Japanese investors have used unprofitable benchmarks, and may have suffered from adopting non-profitable fund managers. The latter increases the total risk of the whole portfolio with significant management fees but with negligible returns.

In the following, we see descriptive statistics of Japanese equity market data in section two. The data is analyzed using zero-beta portfolio and the efficient frontier in section three. We show that the past data has some information about the current efficient frontier in section four. Finally, we discuss difficulties in delegating asset management to portfolio managers.

2. Returns in Japanese stock market

Table 1, which is extracted from Fama and French [8] Table 2, shows the descriptive statistics in the U.S. equity market. From July 1963 to December 1991, investors earn 0.97% monthly

Table 1: Monthly Excess Returns of FF Three Factors in U.S. Equity Market (\%)
(1963.7-1991.12)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>S.D.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM</td>
<td>0.97</td>
<td>4.52</td>
<td>3.97</td>
</tr>
<tr>
<td>RM-RF</td>
<td>0.43</td>
<td>4.54</td>
<td>1.76</td>
</tr>
<tr>
<td>SMB</td>
<td>0.27</td>
<td>2.89</td>
<td>1.73</td>
</tr>
<tr>
<td>HML</td>
<td>0.40</td>
<td>2.54</td>
<td>2.91</td>
</tr>
</tbody>
</table>
returns by taking 4.52% risk by holding the market portfolio. The excess return against risk-free rate is 0.43%, although its statistical significance is weaker. It is worth taking the market risk. SMB portfolio also shows positive return. HML portfolio shows 0.40% return, which is significantly different from zero. These numbers, that are reported by two influential researchers, are convincing evidence to believe that there is a trade-off between risk and return in the U.S. market. It is also consistent with the way many finance textbooks explain about basic tools such as the mean-variance approach and asset pricing models.

If we look at Japanese equity market data, however, the situation is very different. Table 2 shows similar statistics for Japanese data. Monthly average return by holding the market portfolio is 0.17%, which is not significantly different from zero. Excess return against risk-free rate is -0.07%. Investors are not rewarded by taking the market risk. SMB factor shows 0.03% return, and the size effect is not found in the Japanese market. On the other hand, HML factor shows 0.67% return with statistical significance. The high return by HML factor suggests the existence of risks that reward in the Japanese market. However, if investors consider the market return as a return from the stock market, then they may find very weak evidence to justify investment in the equity market.

If CAPM holds, the market portfolio is located on the efficient frontier. Japanese equity market data, however, shows that investing in the market portfolio is inefficient but also unprofitable. Although CAPM is not strongly supported by U.S. empirical studies, to hold U.S. market portfolio makes sense because there is positive return. On the other hand, to hold the market portfolio is not justifiable in Japanese equity market data.

### 3. CAPM and mean-variance frontier in Japanese equity market

By utilizing standard tools that are used in empirical studies of CAPM, we study Japanese equity market data in this section. We show CAPM is not supported in Japanese data by looking at the relation between market betas and mean returns and by constructing zero-beta portfolio returns.

Japanese equity market data is obtained from Financial Data Solutions, Inc. From their NPM database, we constructed a FF25 portfolio and measured its returns from September 1977 to September 2011. For each \( j = 1, 2, \ldots, 25 \), time series of excess return \( r_{j} - r_{f} \) is regressed on the excess return \( r_{M} - r_{f} \) of market portfolio to find estimates \( \hat{\beta}_{j} \) of market

<table>
<thead>
<tr>
<th>Factor</th>
<th>mean</th>
<th>S.D.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
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<td>RM</td>
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<td>0.52</td>
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<tr>
<td>RM-RF</td>
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<td>5.69</td>
<td>-0.23</td>
</tr>
<tr>
<td>SMB</td>
<td>0.03</td>
<td>3.71</td>
<td>0.15</td>
</tr>
<tr>
<td>HML</td>
<td>0.67</td>
<td>3.01</td>
<td>3.89</td>
</tr>
</tbody>
</table>

Table 2: Monthly Excess Returns of FF Three Factors in Japanese Equity Market (%) (1986.1-2011.9)
beta. Then sample means $\bar{r}_j - \bar{r}_f$ of each portfolio $j$ is cross-sectionally regressed on $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_{25})$. The result of the cross-sectional regression is shown in Table 3 and Figure 1.

The Sharpe-Lintner CAPM implies that the expected return $E[r_j]$ is determined by

$$E[r_j] - r_f = \beta_j(E[r_M] - r_f).$$

The risk of asset or portfolio is measured by market beta, and there is linear and positive relation between the expected return and beta. Figure 1 however clearly shows a negative relation between them. Higher (lower) beta portfolios have lower (higher) mean returns in Japanese equity data. Table 3 shows the regression of average returns of the FF25 portfolio on their betas. The slope is negative with statistical significance.

In early CAPM empirical studies using U.S. data, the linear and positive relation between return and beta was found, although the relation is not strong enough to support the Sharpe-Lintner CAPM. The regression slope is smaller than expected, and the regression line on the beta and mean return diagram is too flat. Positive market risk premium is found, and the

|                | Est.  | t-value | P(>|t|) |
|----------------|-------|---------|---------|
| $\gamma_0$    | 2.28  | 2.75    | 0.01    |
| $\gamma_1$    | -1.99 | -2.32   | 0.03    |
| Adj. $R^2$    |       | 0.16    |         |

Table 3: CAPM Beta and Return

Figure 1: CAPM Beta and Average Return
Black CAPM is not rejected in early U.S. studies. The situation for CAPM is worse in the Japanese data set. The positive market risk premium is suspicious, and the Black CAPM is not supported.

In order to study more, we apply the classical technique to study CAPM by Black, Jensen, and Scholes [2]. They introduced the following two factor model:

\[
R_j = R_z(1 - \beta_j) + R_M \beta_j + w_j.
\]  

(1)

Table 4: Zero Beta Portfolio

<table>
<thead>
<tr>
<th></th>
<th>(R^*_Z)</th>
<th>(\sigma(R^*_Z))</th>
<th>(t(R^*_Z))</th>
<th>(r(R^<em>_Z, R^</em>_{Z,t-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. 1/31-12/65</td>
<td>0.338</td>
<td>4.36</td>
<td>1.62</td>
<td>0.113</td>
</tr>
<tr>
<td>U.S. 7/48-3/57</td>
<td>0.782</td>
<td>1.99</td>
<td>4.03</td>
<td>-0.181</td>
</tr>
<tr>
<td>U.S. 4/57-12/65</td>
<td>0.997</td>
<td>2.28</td>
<td>4.49</td>
<td>0.414</td>
</tr>
<tr>
<td>Japan 9/77-9/12</td>
<td>2.97</td>
<td>17.36</td>
<td>3.46</td>
<td>0.14</td>
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</tbody>
</table>

Table 5: Portfolio Return Statistics (1982.9 -2011.9)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>S.D.</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_M)</td>
<td>4.44</td>
<td>19.11</td>
<td>0.06</td>
</tr>
<tr>
<td>(R_f)</td>
<td>3.38</td>
<td>0.64</td>
<td>0.00</td>
</tr>
<tr>
<td>SMB</td>
<td>0.82</td>
<td>12.61</td>
<td>-0.20</td>
</tr>
<tr>
<td>HML</td>
<td>7.73</td>
<td>10.68</td>
<td>0.41</td>
</tr>
<tr>
<td>1/N</td>
<td>8.03</td>
<td>21.68</td>
<td>0.21</td>
</tr>
<tr>
<td>GMVP</td>
<td>12.64</td>
<td>20.37</td>
<td>0.45</td>
</tr>
<tr>
<td>GMVP+ sample mean tangent</td>
<td>16.43</td>
<td>21.37</td>
<td>0.61</td>
</tr>
<tr>
<td>GMVP+ CAPM mean tangent</td>
<td>11.54</td>
<td>21.43</td>
<td>0.38</td>
</tr>
<tr>
<td>GMVP+ FF mean tangent</td>
<td>15.16</td>
<td>21.06</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Black CAPM is not rejected in early U.S. studies. The situation for CAPM is worse in the Japanese data set. The positive market risk premium is suspicious, and the Black CAPM is not supported.

In order to study more, we apply the classical technique to study CAPM by Black, Jensen, and Scholes [2]. They introduced the following two factor model:

\[
R_j = R_z(1 - \beta_j) + R_M \beta_j + w_j.
\]  

(1)

The return \(r_j\) is determined by two factors \(r_M\) and \(r_z\), where the latter factor \(r_z\) is the return of zero-beta portfolio against the market portfolio. The coefficient on \(r_z\) is determined as 1 - \(\beta_j\). The theoretical justification for this two factor model is given by Black [1].

Table 4 extracts some results from Black, Jensen, and Scholes [2] Table 5. The excess return \(R^*_Z\) is defined as the excess return of \(r_z\) on the risk-free rate. Mean, standard deviation, and t-value, and autocorrelation of \(R^*_Z\) are reported. If the Sharpe-Lintner CAPM holds, the zero-beta portfolio excess return is zero. In U.S. data, in particular subsample data for 1948-1957 and 19571965, zero-beta portfolio has statistically significant positive returns. Black, Jensen, and Scholes [2] consider it strong evidence to reject the Sharpe-Lintner CAPM, and emphasize advantages of the two factor model (1).
In Table 4, the same analysis for Japanese data is also reported. Zero-beta portfolio return is 2.97% and is surprisingly large. Although its variance is large, the mean return is significantly different from zero. Not only is the Sharpe-Lintner CAPM rejected, but also the Black CAPM is rejected since positive relation between expected return and beta does not hold.

We then look how the efficient frontier looks like using Japanese equity market data. Figure 2 shows sample mean and standard deviation of FF25 portfolio returns. Equal weight portfolio (1/N), global minimum-variance portfolio (GMVP), the value weighted portfolio (VW), the tangent portfolio, and zero-beta portfolio are also shown. We can see from Figure 2 that the market portfolio (VW) shows less attractive performance than 1/N and GMVP. Naive strategies using no mean return predictions beat the market. As is well known, the tangent portfolio typically has unusually high return with large risk. Zero-beta portfolio also shows high positive return but with very large risk. Although Figure 2 suggests the existence of risk that rewards return in the Japanese equity market, this efficient frontier is drawn using all sample data. The efficient frontier in Figure 2 cannot be found until all data is observed.

In the next section, we discuss if we can identify the current efficient frontier using only past data.

4. Identification of efficient frontier

The out-of-sample performance of the sample-based mean-variance model is not successful as DeMiguel, Garlappi, and Uppal [5] pointed out. The result implies that the efficient frontier
is difficult to identify from past data. If no information is contained in return data about the current shape of the efficient frontier, we have to be pessimistic to invest in the Japanese equity market. The market portfolio does not deliver any reward. The better portfolios in the market are difficult to be known in advance. In this section, we show evidence to indicate that the past return data included information about the current shape of efficient frontier.

The portfolio strategies that are applied in DeMiguel, Garlappi, and Up-pal [5] are the tangent portfolio. The tangent portfolio is very sensitive to input parameters. Furthermore, it is typically a very risky portfolio as we can immediately see in Figure 3. Because of these properties of the mean-variance portfolio approach, the parameter estimation errors are amplified in the process of portfolio construction, and the out-of-sample returns of the portfolio becomes extremely shaky. It is thus important to reduce both errors in estimating parameters and in constructing a portfolio.

The tangent portfolio $\varphi_T$ is decomposed into two parts;

$$\varphi_T = \frac{1}{1^\top \Sigma^{-1} (\mu - r_f \mathbf{1})} \Sigma^{-1} (\mu - r_f \mathbf{1})$$

$$= \frac{1}{1^\top \Sigma^{-1} (\mu - r_f \mathbf{1})} ((1^\top \Sigma^{-1} \mu) \varphi_\mu - r_f (1^\top \Sigma^{-1} \mathbf{1}) \varphi_g)$$

$$= \varphi_g + \frac{1^\top \Sigma^{-1} \mu}{1^\top \Sigma^{-1} (\mu - r_f \mathbf{1})} (\varphi_\mu - \varphi_g),$$

where

$$\varphi_g = \frac{1}{1^\top \Sigma^{-1} \mathbf{1}} \Sigma^{-1} \mathbf{1} \quad \text{and} \quad \varphi_\mu = \frac{1}{1^\top \Sigma^{-1} \mu} \Sigma^{-1} \mu.$$

That is, $\varphi_g$ is the global minimum variance portfolio and $\varphi_\mu$ is the tangent portfolio when
The risk-free rate is zero. The tangent portfolio $\varphi_T$ is decomposed into the full-investment in GMVP and the zero-investment in $\varphi_g$.

The portfolio weight $\varphi_g$ of GMVP does not use the expected return vector $\mu$, GMVP is free from mean return prediction error. In this sense, GMVP is the portfolio with the smallest estimation error among portfolios on the efficient frontier.

The estimation error in the expected return vector is amplified by the second part of the tangent portfolio. What is wrong in forming a mean-variance portfolio is easy to understand. Figure 3 shows mean and standard deviation of two arbitrarily chosen portfolios from FF25. The tangent portfolio that is constructed using two portfolios has high risk. Because two portfolios have different mean returns, the tangent portfolio tries to take full advantage of the difference in mean returns. These mean returns, however, are observed with estimation errors. Because of large long and short positions, the estimation error is amplified.

In order to control errors in portfolio construction, we consider the portfolio problem that maximizes expected return under the constraints on the deviation from GMVP.

\[
\begin{align*}
\max_{\varphi} & \quad (\varphi - \varphi_g)^T \mu \\
\text{s.t.} & \quad (\varphi - \varphi_g)^T \Sigma (\varphi - \varphi_g) = v^2 \\
& \quad \varphi^T 1 = 1,
\end{align*}
\]

where $v^2$ is a tracking error volatility (TEV) from GMVP. The constraints on the TEV controls not only the total volatility but also the errors in portfolio construction. If the out-of-sample performance of the solution portfolio is improved from GMVP, then we can conclude that historical return data helps us to identify the efficient frontier at least locally near GMVP.

In order to see how the idea works, we first estimate the covariance matrix of FF25 portfolio returns at the beginning of each month $t$ using 60 month return data from month $t - 60$ to month $t - 1$. Using the same return data, we make mean return predictions by the sample mean, CAPM, and FF three factor models. Then the tangent portfolio is obtained for these three predictions of mean returns. We set TEV to be 25 so that annualized standard deviation of tracking error is 5%. $1/N$ portfolio and GMVP are also constructed at the beginning of each month.

Table 5 reports the out-of-sample returns of these portfolios for the full sample period from September 1982 to September 2011. Table 6, Table 7, and Table 8 report the results for the first, the second, and the last ten years. Overall, the Sharpe ratio of the market portfolio is beaten by $1/N$ portfolio, and $1/N$ is lost against GMVP. In the most recent ten years in Table 8, $1/N$ portfolio is better than GMVP. Since the FF25 portfolio is a large portfolio and is well diversified, the volatility of GMVP is not dramatically reduced from each component. On the other hand, return of GMVP is better than the market and $1/N$ portfolio in many cases, although GMVP does not use the mean return prediction.

In Table 5, the out-of-sample performances of the portfolio that tries to move up along the efficient frontier with TEV constraints are good if we use sample mean or FF three factor
### Table 6: Portfolio Return Statistics (1982.9 - 1992.8)

<table>
<thead>
<tr>
<th></th>
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<th>S.D.</th>
<th>Sharpe Ratio</th>
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<tbody>
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<td>$R_M$</td>
<td>12.34</td>
<td>20.91</td>
<td>0.30</td>
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<tr>
<td>$R_f$</td>
<td>6.02</td>
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<td>0.00</td>
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<td>SMB</td>
<td>3.30</td>
<td>15.18</td>
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<td>HML</td>
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<td>11.23</td>
<td>0.24</td>
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<tr>
<td>$1/N$</td>
<td>17.72</td>
<td>21.64</td>
<td>0.54</td>
</tr>
<tr>
<td>GMVP</td>
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<td>0.59</td>
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<td>0.74</td>
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<td>GMVP+ CAPM mean tangent</td>
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<td>GMVP+ FF mean tangent</td>
<td>20.16</td>
<td>20.21</td>
<td>0.70</td>
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### Table 7: Portfolio Return Statistics (1992.9 - 2002.8)

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<td>0.00</td>
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<td>0.34</td>
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<tr>
<td>$1/N$</td>
<td>-0.27</td>
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<td>-0.12</td>
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<td>GMVP</td>
<td>13.29</td>
<td>19.93</td>
<td>0.54</td>
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<tr>
<td>GMVP+ sample mean tangent</td>
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<td>0.56</td>
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### Table 8: Portfolio Return Statistics (2002.9 - 2011.9)

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<th>Sharpe Ratio</th>
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</thead>
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<tr>
<td>$R_f$</td>
<td>1.31</td>
<td>0.09</td>
<td>0.00</td>
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<td>8.83</td>
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<td>0.79</td>
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<tr>
<td>$1/N$</td>
<td>6.51</td>
<td>18.61</td>
<td>0.28</td>
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<tr>
<td>GMVP</td>
<td>6.00</td>
<td>20.98</td>
<td>0.22</td>
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</tbody>
</table>
models to predict mean returns. In both cases, the Sharpe ratios are improved from GMVP. However, prediction by CAPM does not improve the performance. For subsample periods in Table 6, Table 7, and Table 8, the results are similar, although the CAPM prediction is slightly better than the FF3 prediction in Table 7. As opposed to DeMiguel, Garlappi, and Uppal [5], the result suggests that the tangent portfolio contains useful information to identify the shape of the efficient frontier.

It is also interesting to see that sample mean prediction does better than FF3 predictions. One possible reason is the regression that is used in the FF3 prediction increases the prediction error. The coefficients on three factors are estimated with errors and the sample means of three factors may not predict the mean return of factors very well. Since the FF25 portfolio is a well diversified portfolio, the sample mean of them can be better estimated than predictions that use regression.

In summary, we conclude that the past return data are valuable to know the shape of the efficient frontier. If we carefully construct the portfolio so that parameter estimation error is not amplified in the process of portfolio construction, then the tangent portfolio can be used to identify the efficient frontier.

5. Portfolio management against benchmark

In previous sections, we see the market portfolio is not a good portfolio in Japanese equity market data because it is not profitable and CAPM does not seem to hold. The market portfolio is however often used as a benchmark portfolio in the financial industry, when asset management is delegated to portfolio managers. In this final section, we consider properties of portfolio management against benchmark portfolio. In particular, we investigate potential problems when the benchmark portfolio is not really efficient.

The importance of portfolio delegation is already known. For example, see Sharpe [15] and Elton and Gruber [6]. Since we cannot completely monitor portfolio managers, there exists principal and agent problems in portfolio delegation. In order to deal with the problem, the market portfolio is used as a benchmark portfolio. Portfolio managers are constrained by the tracking error and are expected to earn higher return than the benchmark portfolio.

Roll [14] criticized this way of portfolio management, because the portfolios that are held by portfolio managers are irrelevant to the indicated benchmark. Furthermore, the selected portfolio tends to increase the total volatility but rarely reduces it. In this section, we simplify Roll’s model to understand what is really wrong.

We suppose that the principal shows benchmark portfolio $\varphi_B$, which is a full-investment portfolio and $\varphi_B^T 1 = 1$, to the portfolio managers and asks them to manage their portfolio with constraints on the tracking error from the benchmark. The performance of the portfolio manager is measured by excess return against the benchmark. Roll [14] supposed that portfolio managers minimize the tracking error under constraints of the target return, using their own predicted mean return vector $\mu$. Given the covariance matrix $\Sigma$, the problem is given by
where $\bar{\mu}$ is a target excess return of the portfolio manager. Roll [14] carefully studies the solution to (3). It turns out that the solution is chosen independently of the given benchmark and may increase the total return volatility over the benchmark. The latter however depends on the model parameter combination. We thus here simplify the problem (3) to see essential problems underlying it.

One of the difficulties in identifying the current shape of the efficient frontier is to predict mean returns of assets. The global minimum variance portfolio (GMVP) is the only one portfolio that locates on the frontier and is free from estimation error of mean returns. By using notations in Cochrane [4], GMVP and the efficient frontier is given as follows. Let $\mathbf{R}^e$ be a set of returns by zero-investment portfolio strategies. Then the return $R_i$ of any asset or portfolio $i$ is given using an excess return $R_i^e \in \mathbf{R}^e$ and a scalar $w_i$,

$$R_i = R_f + (w_i - R_f)R_i^e + n_i.$$  \hspace{1cm} (4)

Here $R_i^e$ is defined as a projection of the constant vector 1 to the space of excess return $\mathbf{R}^e$. Thus $R_i^e$ is the excess return vector that is closest to arbitrage trading. The vector $n^i$ is in the space $\mathbf{R}^e$ with $E(n^i) = 0$ and $E(R_i^e n^i) = 0$. The noise vector $n_i$ increases the variance but does not increase the expected return. Then as Cochrane [4] Theorem 5.3b shows, the necessary and sufficient condition of return $R_i^{mv}$ to be on the efficient frontier is that there is a positive scalar $w$ such that

$$R_i^{mv} = R_f + (w - R_f)R_i^e.$$  

Since the tangent portfolio return $R_T$ and GMVP return $R_g$ are on the efficient frontier, there are two scalars $w_T$ and $w_g$ such that

$$R_T = R_f + (w_T - R_f)R_i^e$$  \hspace{1cm} (5)

$$R_g = R_f + (w_g - R_f)R_i^e.$$  \hspace{1cm} (6)

Thus $R_i^e$ is given by $R_T - R_g$.

Suppose that the benchmark portfolio $B$ is not on the efficient frontier. Then there are two scalars $w_B$ and $n_B$, which are perpendicular to the space $R_i^e$, such that

$$R_B = R_f + (w_B - R_f)R_i^e + n_B.$$  \hspace{1cm} (7)
Given (7), portfolio managers identify the trade-off of the risk and return by two elements, that is, the deviation from efficient frontier \( n_B \) and excess return \( R^{*} \) closest to arbitrage.

We now simplify the problem (3). We suppose that portfolio managers try to find the combination of two vectors \( n_B \) and \( R^{*} \) so that excess return is minimized under the constraint on the tracking error from the benchmark. Equations (5), (6), and (7) show that the reduction of \( n_B \) and extension of \( R^{*} \) are done by using two zero-investment portfolio \( \Delta \varphi_g \) and \( \Delta \varphi_T \). Assuming that \( x \) and \( y \) are amount invested in two portfolio, the total portfolio \( \varphi_P \) is given by

\[
\varphi_P = \varphi_B + x \Delta \varphi_g + y \Delta \varphi_T.
\]

Thus the problem (3) is changed to a simplified two-asset problem:

\[
\max_{(x,y)} \begin{pmatrix} x \\ y \end{pmatrix} \tilde{\mu} \\
\text{s.t.} \quad \begin{pmatrix} x \\ y \end{pmatrix} \tilde{\Sigma} \begin{pmatrix} x \\ y \end{pmatrix} = v^2, 
\]

where \( \tilde{\mu} \) and \( \tilde{\Sigma} \) are given by

\[
\tilde{\mu} = \begin{pmatrix} \Delta \varphi_g^\top \mu \\ \Delta \varphi_T^\top \mu \end{pmatrix}, \quad \tilde{\Sigma} = \begin{pmatrix} \Delta \varphi_g^\top \\ \Delta \varphi_T^\top \end{pmatrix} \Sigma \begin{pmatrix} \Delta \varphi_g & \Delta \varphi_T \end{pmatrix}.
\]

These are the expected return and covariance matrix of \( \Delta \varphi_g \) and \( \Delta \varphi_T \).

\textbf{Proposition 1.} The solution to problem (8) is irrelevant to the benchmark choice in the sense that it is unchanged even if a different benchmark is used. The solution also does not reduce the vector \( n_B \) and the deviation from efficient frontier is unchanged.

\textbf{Proof 1.} See Appendix.

Although problem (8) is a different problem as Roll [14] and Jorion [11], the essential difficulties in portfolio management against benchmarks are easier to understand through this simplified problem. Roll [14] and Jorion [11] suggest to introduce constraints on beta against benchmark and on the total risk level. These constraints may work well in some cases, but as Jorion [11] points out, the combination of the constraints should be carefully chosen. Jaganathan and Ma [9] pointed out that to add constraint conditions is essentially the same as to change the portfolio manager’s prediction. Under additional constraints, the performance of the portfolio managers becomes difficult to measure since we cannot see how their performances are determined, by their own ability or additional constraints.

Many Japanese investors have used the market portfolio as their benchmark portfolio. The benchmark turns out to be inefficient and furthermore unprofitable. Active managers are
expected to add values on the benchmark, but many of them are not very successful to earn positive excess return. Furthermore, all of them surely increase the total risk of the whole portfolio. Last but not least, they charge expensive management fees.

6. Conclusion

Mainly based on the CAPM, the market portfolio is often chosen by investors as a benchmark portfolio. Japanese equity market data, however, shows that the market portfolio is not efficient and moreover, not profitable. The empirical support for CAPM in the Japanese market is weak. There has been, however, some evidence that there are risk sources that reward. Although to identify the efficient frontier in advance is not easy at all, (there is evidence) to suggest past return data contains useful information. Portfolios should be carefully constructed so that parameter estimation risk is not amplified in the process of portfolio formation. Overall, Japanese investors experienced a hard time because they used the market portfolio as their benchmark without careful investigation and adopted many active managers against the benchmark. Not only because very few active managers consistently beat the benchmark but also because they surely increase the total risk, the total portfolio of Japanese investors typically have shown disappointing performance.

7. Appendix: Proof of Proposition 1

Let $\lambda$ be the Lagrange multiplier of problem (8). From the first order condition, the solution satisfies

$$(x \quad y)^\top = \frac{1}{2\lambda} \Sigma^{-1} \bar{\mu}. \tag{9}$$

Substitute into the constraint condition yields

$$\lambda = \frac{1}{2} \sqrt{\frac{\mu^\top \Sigma^{-1} \bar{\mu}}{TEV}}. \tag{9}$$

Thus the optimal solution is given by

$$(x \quad y)^\top = \sqrt{\frac{TEV}{\mu^\top \Sigma^{-1} \bar{\mu}}} \Sigma^{-1} \bar{\mu}. \tag{9}$$

From (5), (6), and (7), we have

$$\Delta \varphi_T^\top R = Y \Delta \varphi_T^\top R - n_B,$$

where $Y \equiv (w_g - w_B)/(w_T - w_g)$. Thus $\bar{\mu}$ and $\Sigma$ are given by
Then we have
\[
\hat{\mu} = \begin{pmatrix} Y \\ 1 \end{pmatrix} \text{E}[R_T - R_g], \quad \hat{\Sigma} = \begin{pmatrix} Y^2 & Y \\ Y & 1 \end{pmatrix} \text{Var}[R_T - R_g] + \begin{pmatrix} \text{Var}[n_B] & 0 \\ 0 & 0 \end{pmatrix}.
\]

Therefore, portfolio managers do not hold \( \Delta \varphi_g \), which is a move from the benchmark to GMVP. Since \( \Delta \varphi_T \) is a move from GMVP to the tangent portfolio, the deviation from efficient frontier \( n_B \) is not reduced. If the target excess return is positive, then the portfolio manager increases the vector \( R^* \) but does not reduce \( n_B \).

References


