Economic Growth Analysis of Japan by Dynamic General Equilibrium Model with R&D Investment *

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Abstract

The purpose of this study is to evaluate the role of R&D capital stock in Japan's economic growth and consider the effects of science and technology policies, including public R&D expenditures, on the economic growth based on a general equilibrium dynamic model incorporating the nature of R&D capital as a public good. After developing data of R&D capital investment and capital stock and arranging SNA data on the basis of 2008SNA, we develop a numerical model by estimating or calibrating the structural parameters of a two-sector dynamic general equilibrium model. A growth account using the production function of final goods production and R&D production shows that the TFP growth rates of both final goods production and R&D production have declined since the 1990s. As a result of a policy simulation incorporating the role of the spillover effects of public R&D into the model, it is found that an increase in public R&D investment significantly increases not only R&D production but also final goods production and household consumption.

Key words: Research and Development Investment, Dynamic General Equilibrium Model, Growth Accounting, Policy Simulation

JEL Classification: E01, E17, E22, O38, O41

I. Introduction

This study evaluates the role of research and development (R&D) investment and its capital stock in Japan's economic growth and examines the effect of public R&D expenditure of science and technology policy on economic growth using a dynamic general equilibrium model that includes the public goods property of R&D capital.

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In recent years, the field of macroeconomic statistics is seeing an increase in studies that focus on the roles of intangible capital stock, such as R&D capital and human capital, in production. Corrado et al. (2009), a representative example of such a study, estimates the investment of intangible assets of developed countries and the capital stock of such investment. The authors conclude that such investment contributed significantly to economic growth.

Furthermore, the System of National Accounts (SNA) of the 2008 estimation standard recommends statistical agencies that the SNA records various forms of intangible capital, such as R&D capital. Even in the field of macroeconomics, studies are increasingly including intangible capital in conventional dynamic general equilibrium models. A representative example of such a study is McGrattan and Prescott (2010, 2012), which improved the explanatory power of its model for US business cycles by introducing intangible capital.

In Japan, the research has focused on the method of recording R&D capital investment and its stock as one type of intangible capital. Economic and Social Research Institute (ESRI) in the Cabinet Office has been a prominent contributor to such research in the lead up to the 2016 fiscal introduction of the 2008 SNA.[1] Arato and Yamada (2012) estimates the intangible capital stock owned by companies from financial statement data, and report Japan's ratio of intangible capital to tangible capital as being close to the ratios estimated for the United States and the United Kingdom. Miyagawa and Hisa (2013) estimate various types of intangible capital investment and its stock, and particularly that of R&D capital. Hayashi and Prescott (2002) were pioneers in the study of Japan's economy using dynamic general equilibrium models. They explained the "lost decade" using the real business cycle (RBC). Then Kobayashi and Inaba (2006) and Otsu (2008) evaluated interpretability using the business cycle accounting (BCA) method. Based on these studies that consider intangible capital as important production factor, this study evaluates the effect of the macroeconomic environment of intangible capital investment in Japan on the growth path and the steady state of the economy.

BCA was first proposed by Chari, Kehoe, and McGrattan (2007). It is an empirical method, which evaluates the discrepancies between output of a calibrated dynamic general equilibrium model and observed data as measured market distortions (called as "wedges"). When the wedges are measured, it is possible to evaluate the effects of these wedges on endogenous economic variables in the model. BCA is also used in this study to explain the divergence between real-world data and data outputted by the dynamic general equilibrium model as multiple wedges.

The contributions of this study are as follows. Initially, Japan's R&D capital stock was estimated from a database compiled by the Statistics Bureau of Japan's Survey of Research and Development. Furthermore, human capital stock was estimated by referring to the

[1] Examples of studies in Japan that introduce R&D capital investment and its stock include Kawasaki (2006), Fukao et al. (2009), Cabinet Office, ESRI (2010), Shigeno (2012), Tonogi, Kitaoka, and Kobayashi (2014), and Tonogi, Kitaoka, and Li (2015).

average years of schooling, as per Barro and Lee (2013), and the wage function was estimated as in Miyazawa (2011). In addition, the structural parameters of the production functions of final goods and of R&D production that share common R&D capital stock were estimated from R&D capital stock and tangible capital stock. A two-sector dynamic general equilibrium model is then constructed. The proposed model adopts the production functions of both the final goods production and R&D production sectors. Then, after estimating (calibrating) the remaining structural parameters, the results are used in the subsequent analysis. First, growth accounting is applied. The factors of production and total factor productivity (TFP) are evaluated to determine their respective contributions to production by applying growth accounting to the final goods production and R&D production sectors. Second, a policy simulation is conducted to extrapolate exogenous variables, such as future population, human capital, TFP, tax rate, and so on. Then, we estimate the effects of these variables on the steady-state equilibrium of science and technology policy (specifically, on public R&D expenditure) and on economic growth paths.

The main results are as follows. First, the share of R&D capital in final goods production is small, at 0.017, while the share of R&D capital in the R&D sector is high, at 0.37. Second, we apply growth accounting to the estimated final goods and R&D production functions. Here, the results show that the TFP rate of increase for final goods has been decreasing since the 1990s, as has the rate of TFP growth in R&D production. Third, the policy simulation conducted using a model that assumes a spillover effect from public R&D indicates that increases in public R&D investment will increase the TFP of the R&D sector and boost R&D production considerably. Furthermore, the simulation shows that the same increases in public R&D will increase production in the final goods production sector and household consumption.

This paper is organized as follows. Section 2 develops a two-sector dynamic general equilibrium model, including R&D investment, based on McGrattan and Prescott (2010). Section 3 explains the estimations of final goods production, R&D production, tangible capital stock, R&D capital stock, the human capital level, and labor input for both sectors. This section also explains the method of estimation for the consumption tax rate, labor income tax rate, capital income tax rate, enterprise profit tax (corporate tax) rate, and property tax rate on tangible capital. Then, the structural parameters of the production function and the utility function are estimated, and the other structural parameters are calibrated. In Section 4, growth accounting is applied to the estimated production, production factor data, and production function. In Section 5, the divergence between the observational data from 1980 to 2011 and the endogenous variables outputted by the model is measured as wedges based on the BCA method. In Section 6, a policy simulation is performed on an extrapolated area using the calibrated numerical model and estimated wedges. The simulation enables an examination of how public R&D investment affects the steady-state and equilibrium path of the macro economy.

II. Dynamic General Equilibrium Model with R&D Investment

In this section, a dynamic general equilibrium model incorporating endogenous R&D capital investment and exogenous human capital growth is developed. The developed model is based on a model of non-neutral technological progress that incorporates intangible capital investment, as proposed by McGrattan and Prescott (2010) (hereinafter, "MP model"). The first major difference from the MP model is that different shares are given for the production function of the final goods production (FGP) sector and the Cobb–Douglas production function of the R&D production sector. This is because it is possible to estimate the respective production functions of the FGP and R&D sectors by estimating Japanese R&D capital stock, under certain assumptions. The second major difference is that by introducing intangible capital with two different properties, the R&D capital contribution to production is reflected in R&D capital revenue, and the human capital contribution to production is reflected in wages. For this reason, this study does not include the MP model's concept of sweat equity, which divides a single intangible capital contribution to production into capital income and labor income. The third difference is that in addition to incorporating consumption tax, corporate tax, labor income tax, and property tax, the proposed model incorporates R&D investment tax credit, which is thought to have a large impact on R&D investment.

II-1. Representative household

Suppose the utility function of the representative household in period t is

$$U(c_t, l_t) = \ln(c_t) + \psi \ln(l_t), \tag{1}$$

where, c_t ($c_t \geq 0$) is the consumption level of the final goods, and l_t ($l_t \in [0,1]$) is leisure time. Then, ψ represents the logarithmic utility weight of leisure time. Households have one unit of time per period, divided into leisure time, l_t, and labor input, h_t.

Next, suppose the inter-temporal utility function at period t is

$$\sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, l_{t+j}\right), \tag{2}$$

where, β ($1 > \beta > 0$) is a time preference factor. Here, a lower value of β means future utility is evaluated as lower than current utility, and vice versa. For the representative household, it is necessary to decide labor input, leisure time, tangible capital investment, R&D capital investment, and household consumption for every fiscal period in order to maximize the inter-temporal utility of (2) with satisfying the following budget constraint equation in all fiscal periods:

$$c_t + x_t^T + q_t x_t^I = r_t^T k_t^T + r_t^I k_t^I + w_t h_t + \zeta_t - \tau_t. \tag{3}$$

The left side of equation (3) represents household expenditure, and the right side represents household disposable income. Then, x_t^T and x_t^I denote tangible capital investment and R&D capital investment, respectively; k_t^T and k_t^I denote tangible capital stock and R&D

capital stock, respectively; r_t^T and r_t^I are the returintangible capital and the return on R&D capital, respectively; w_t is the wage per unit of labor input; and q_t is the relative price of the R&D product. In addition, ζ_t is the transfer income of the lump sum from the government, which is added to household income, and τ_t is the total amount of tax borne by households, which is deducted from household income. The way that τ_t depends on endogenous variables is shown as

$$\begin{aligned}\tau_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k k_t^T - \kappa_t^I q_t x_t^I + \tau_t^p\left(r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I\right) \\ + \tau_t^d\big[r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I \\ - \tau_t^p\left(r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I\right)\big],\end{aligned} \tag{4}$$

where, τ_t^c, τ_t^h, τ_t^k, τ_t^p, and τ_t^d are the consumption tax rate, labor income tax rate, property tax rate, enterprise profit tax rate (e.g., corporate tax), and capital income tax rate (e.g., interest income tax, dividend income tax), respectively. Then, κ_t^I is the tax credit rate for R&D investment. Therefore, $\tau_t^c c_t$ is the consumption tax amount, $\tau_t^h w_t h_t$ is the labor income tax amount, $\tau_t^k k_t^T$ is the property tax amount, $\tau_t^p\left(r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I\right)$ is the enterprise profit tax amount, and $\kappa_t^I q_t x_t^I$ indicates the R&D tax credit amount. The capital income tax amount is obtained by multiplying τ_t^d by the enterprise profit after tax (inside the parentheses of the sixth item on the right side of equation (4)). The tangible capital stock owned by households, k_t^T, and R&D capital stock, k_t^I increase or decrease according to the following capital transition equations:

$$k_{t+1}^T = [(1-\delta^T)k_t^T + x_t^T](1+\eta_t)^{-1}, \tag{5}$$

$$k_{t+1}^I = [(1-\delta^I)k_t^I + x_t^I](1+\eta_t)^{-1}, \tag{6}$$

where, δ^T is the tangible capital depletion rate, and δ^I is the R&D capital depletion rate. Then, η_t is the growth rate of the number of households (population) in the economy, and the population, N_t, grows exogenously, as follows:

$$N_{t+1} = (1+\eta_t)N_t. \tag{7}$$

When the number of households increases at rate η_t in period $t+1$, it is assumed that the capital stock per household decreases at rate η_t.

II-2. Optimization behavior of household

Now that the prices, q_t, r_t^T, r_t^I, and w_t are given for the representative household, the consumption, c_t, tangible capital investment, x_t^T, R&D capital investment, x_t^I, labor input, h_t, and leisure time, l_t, are determined for the representative household with satisfying to each fiscal period's budget constraints (equation (3)) in order to maximize the inter-temporal utility function of equation (2). For capital transitions, we follow equations (5) and (6).

The problem is expressed in Lagrangian form as follows:

$$L = \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t \{ r_t^T k_t^T + r_t^I k_t^I + w_t^1 h_t + \zeta_t - \tau_t \\ - [c_t + x_t^T + q_t x_t^I] \},$$

where λ_t is the Lagrangian multiplier of period t. Endogenous variables are subjected to the following constraints:

$$l_t = 1 - h_t,$$
$$x_t^T = (1 + \eta_t) k_{t+1}^T - (1 - \delta^T) k_t^T,$$
$$x_t^I = (1 + \eta_t) k_{t+1}^I - (1 - \delta^I) k_t^I,$$
$$\tau_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k k_t^T - \kappa_t^I q_t x_t^I + \tau_t^p \left(r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I \right) \\ + \tau_t^d \left[r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I - \tau_t^p \left(r_t^T k_t^T + r_t^I k_t^I - \delta^T k_t^T - \tau_t^k k_t^T - q_t x_t^I \right) \right].$$

The first-order condition for maximizing the inter-temporal utility for household consumption is as follows:

$$\lambda_t = \frac{U_c(c_t, l_t)}{1 + \tau_t^c}.$$

Here, U_c represents the first-order partial differential with respect to household consumption, c_t, in the utility function of equation (1). The first-order condition for household consumption in period *t* shows that the Lagrangian multiplier is equal to the marginal utility of consumption divided by the acquisition cost for one consumer good unit in period *t*.

The first-order condition of labor input is that the ratio of the marginal substitution rate to price for consumption is equal to the same ratio for leisure, as follows:

$$\frac{U_c(c_t, l_t)}{1 + \tau_t^c} = \frac{U_l(c_t, l_t)}{w_t \left(1 - \tau_t^h \right)}.$$

Here, U_l represents the first-order partial differential with respect to leisure time, l_t, in the utility function of equation (1). For the first-order condition relating to tangible capital, the ratio between period t and period $t + 1$ for the marginal utility per acquisition cost unit of household consumption is shown to be equivalent to the rate of return of tangible capital after tax:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = (1 + \eta_t)^{-1} \left[1 + \left(r_{t+1}^T - \delta^T - \tau_{t+1}^k \right) \left(1 - \tau_{t+1}^d - \tau_{t+1}^p + \tau_{t+1}^d \tau_{t+1}^p \right) \right].$$

For the first-order condition relating to R&D capital, the ratio of period t to period $t + 1$ for the marginal utility per acquisition cost unit of consumption is equal to the rate of return of R&D capital after tax:

$$\frac{\lambda_t}{\beta\lambda_{t+1}} = \left[\frac{r_{t+1}^I + q_{t+1}(1-\delta^I)(1-\kappa_{t+1}^I) + (r_{t+1}^I + q_{t+1}(1-\delta^I))(-\tau_{t+1}^p - \tau_{t+1}^{\mathrm{d}} + \tau_{t+1}^{\mathrm{d}}\tau_{t+1}^p)}{q_t\left[(1-\kappa_t^I) - \tau_t^{\mathrm{p}} - \tau_t^d + \tau_t^d\tau_t^p\right](1+\eta_t)}\right].$$

In other words, the rate of return from R&D capital after tax is equal to the rate of return from tangible capital after tax:

$$\begin{aligned}1 + &\left(r_{t+1}^T - \delta^T - \tau_{t+1}^k\right)(1 - \tau_{t+1}^d - \tau_{t+1}^p + \tau_{t+1}^d\tau_{t+1}^p) \\ &= \frac{r_{t+1}^I + q_{t+1}(1-\delta^I)(1-\kappa_{t+1}^I) + (r_{t+1}^I + q_{t+1}(1-\delta^I))(-\tau_{t+1}^p - \tau_{t+1}^{\mathrm{d}} + \tau_{t+1}^{\mathrm{d}}\tau_{t+1}^p)}{q_t\left[(1-\kappa_t^I) - \tau_t^{\mathrm{p}} - \tau_t^d + \tau_t^d\tau_t^p\right]}.\end{aligned}$$

In this model, the rate of return from the second term on the left side of the above equation is the rate of return, r_t, from capital, after adjusting for the effect of the tax rate and the capital depletion rate.

In order to maximize the utility of households, it is necessary to eliminate the divergent path of capital held by households. For the two types of capital, the following two termination conditions are imposed:

$$\lim_{i\to\infty}\prod_{j=0}^{i}\frac{1}{1+r_{t+j}}k_{t+i+1}^T = 0, \tag{8}$$

$$\lim_{i\to\infty}\prod_{j=0}^{i}\frac{1}{1+r_{t+j}}k_{\mathrm{i}+1}^I = 0. \tag{9}$$

II-3. Representative enterprise

The representative enterprise produces final goods, Y_t, used for household consumption or for tangible capital investment, and knowledge, X_t^I, which is used for R&D investment.

As in the MP model, it is assumed that R&D capital stock is non-competitive because it is knowledge, and that it can be used simultaneously in the FGP sector and in the R&D sector when used as a factor of production within an enterprise.

The production function of final good production, Y_t, is

$$Y_t = (K_t^{T1})^{\theta_1}(K_t^I)^{\phi_1}(A_t^1 Z_t H_t^1)^{\varphi_1}, \tag{10}$$

where, $\theta_1 + \phi_1 + \varphi_1 = 1$ and $\theta_1, \phi_1, \varphi_1 > 0$. The production function is the Cobb–Douglas type, with the production factors of tangible capital, K_t^{T1}, R&D capital, K_t^I, and labor input, H_t^1. In other words, θ_1, ϕ_1, and φ_1 correspond to the tangible capital share, R&D capital share, and labor share, respectively, in the income of the FGP sector. Then, Z_t is the level of human capital, and A_t^1 is a Harrod-neutral TFP in the final good production function, and changes exogenously.

The production function of R&D production, X_t^I, is given as follows:

$$X_t^I = (K_t^{T2})^{\theta_2}(K_t^I)^{\phi_2}(A_t^2 Z_t H_t^2)^{\varphi_2}. \tag{11}$$

Here, $\theta_2 + \phi_2 + \varphi_2 = 1$ and θ_2, ϕ_2, $\varphi_2 > 0$. As in the final goods sector, the production function of the R&D sector is also the Cobb–Douglas type, with production factors of tangible capital, K_t^{T2}, R&D capital, K_t^I, and labor input, H_t^2. In other words, θ_2, ϕ_2, and φ_2 correspond to the tangible capital share, R&D capital share, and labor share, respectively, in the income of the R&D sector. Then, A_t^2 is a Harrod-neutral TFP in the R&D production function, and changes exogenously.

The total factor productivity values of sectors A_t^1 and A_t^2 grow according to the common-trend growth rate, γ, and the individual growth rates, a_t^1 and a_t^2, as follows:

$$A_{t+1}^1 = (1 + a_t^1)(1 + \gamma)A_t^1, \tag{12}$$

$$A_{t+1}^2 = (1 + a_t^2)(1 + \gamma)A_t^2. \tag{13}$$

Human capital level, Z_t, varies exogenously by growth rate z_t:

$$Z_{t+1} = (1 + z_t)Z_t. \tag{14}$$

The profit of the representative enterprise is as follows:

$$\Pi_t = Y_t + q_t X_t^I - [r_t^T(K_t^{T1} + K_t^{T2}) + r_t^I K_t^I + w_t(H_t^1 + H_t^2)]. \tag{15}$$

II-4. Optimization behavior of representative enterprise

Given the prices q_t, r_t^T, r_t^I, and w_t for the representative enterprise, and supposing the production functions given in equations (10) and (11), the amounts of input of the production factors are determined for K_t^{T1}, K_t^{T2}, K_t^I, H_t^1, and H_t^2 in order to maximize the profit function of equation (15). The first-order conditions of optimization are as follows.

From the first-order condition of K_t^{T1}, the tangible capital return is equal to the marginal productivity of the tangible capital of the FGP sector:

$$r_t^T = \theta_1 \frac{Y_t}{K_t^{T1}}.$$

From the first-order condition of K_t^{T2}, the tangible capital return is equal to the marginal productivity of the tangible capital of the R&D sector:

$$r_t^T = \theta_2 \frac{q_t X_t^I}{K_t^{T2}}.$$

That is, from the above two equations, the following relationship holds for the price q_t of the R&D product when the enterprise maximizes profit:

$$q_t = \frac{\theta_1}{\theta_2}\frac{Y_t}{X_t^I}\frac{K_t^{T2}}{K_t^{T1}}. \tag{16}$$

From the first-order condition of K_t^I, the R&D capital return is the sum of the marginal productivities of R&D capital of the FGP and R&D sectors:

$$r_t^I = \frac{\phi_1 Y_t + \phi_2 q_t X_t^I}{K_t^I}.$$

Next, from the first-order condition of H_t^1, the labor wage is equal to the marginal productivity of the labor input of the FGP sector:

$$w_t = \varphi_1 \frac{Y_t}{H_t^1}.$$

From the first-order condition of H_t^2, labor wages are also equal to the marginal productivity of the labor input of the R&D sector:

$$w_t = \varphi_2 \frac{q_t X_t^I}{H_t^2}.$$

That is, from the above two equations, the following relationship holds for the price q_t of the R&D product when the enterprise maximizes profit:

$$q_t = \frac{\varphi_1}{\varphi_2}\frac{Y_t}{X_t^I}\frac{H_t^2}{H_t^1}. \tag{17}$$

In the case that the enterprise behaves in a way that maximizes profit, because this satisfies equations (16) and (17) simultaneously, the following relationship holds for the ratio of tangible capital input between the FGP and R&D sectors and the ratio of working hour input between the FGP and R&D sectors:

$$\frac{\theta_1}{\theta_2}\frac{K_t^{T2}}{K_t^{T1}} = \frac{\varphi_1}{\varphi_2}\frac{H_t^2}{H_t^1}. \tag{18}$$

II-5. Conditions of Dynamic General Equilibrium

In the economy in which there are N representative households, and one representative enterprise and government, the conditions of dynamic general equilibrium are defined as follows:

- Given the initial values of capital stock, k_0^T and k_0^I, and, the prices, q_t, r_t^T, r_t^I, and w_t, all the first-order conditions for household's inter-temporal optimization are established with satisfying its budget constraint and the capital transitions of formula, (5) and (6) for all fiscal periods.

- Given the prices, q_t, r_t^T, r_t^I, and w_t, while supposing the production functions given in equations (10) and (11), the representative enterprise will satisfy all the first-order conditions to maximize the profit function of equation (14) for all fiscal periods.
- Market supply and demand balance conditions hold in all periods. That is, the market clear condition of the final good market, $Y_t = N_t(x_t^T + c_t)$, the market clear condition of the R&D product market, $\mathrm{X}_\mathrm{t}^I = N_t x_t^I$, the market clear condition of the labor market $H_t^1 + H_t^2 = N_t h_t$, the market clear condition of the tangible capital market, $K_t^{T1} + K_t^{T2} = N_t k_t^T$, and the market clear condition of the R&D capital market, $K_t^I = N_t k_t^I$, each hold for all fiscal periods.
- The government's budget constraint equation reaches balance ($\tau_\mathrm{t} = \zeta_t$) in all fiscal periods.
- The terminal conditions (equations (8) and (9)) are established.

II-6. Removal of trends from endogenous variables

To find the steady state of the dynamic general equilibrium, exogenous trend components are removed from the endogenous variables. Specifically, all variables per household are divided by $(1+\gamma)^\mathrm{t} A_0^1 Z_t$. The variable after trend removal is represented with a hat, as follows. That is, the household consumption is $\hat{c}_t = \frac{c_t}{(1+\gamma)^\mathrm{t} A_0^1 Z_t}$, the tangible capital of the FGP sector is $\hat{k}_t^{T1} = \frac{K_t^{T1}/N_t}{(1+\gamma)^\mathrm{t} A_0^1 Z_t}$, the tangible capital of the R&D sector is $\hat{k}_t^{T2} = \frac{K_t^{T2}/N_t}{(1+\gamma)^\mathrm{t} A_0^1 Z_t}$, the R&D capital is $\hat{k}_t^\mathrm{I} = \frac{K_t^I/N_t}{(1+\gamma)^\mathrm{t} A_0^1 Z_t}$, final good production is $\hat{y}_t = \frac{Y_t/N_t}{(1+\gamma)^\mathrm{t} A_0^1 Z_t} = \left(\hat{k}_t^{T1}\right)^{\theta_1} \left(\hat{k}_t^I\right)^{\phi_1} (\hat{a}_t^1 h_t^1)^{\varphi_1}$, and R&D production is $\hat{x}_t^I = \frac{X_t^I/N_t}{(1+\gamma)^t A_0^1 Z_t} = \left(\hat{k}_t^{T2}\right)^{\theta_2} \left(\hat{k}_t^I\right)^{\phi_2} (\hat{a}_t^2 h_t^2)^{\varphi_2}$.

At this time, the TFP of FGP production is $\hat{a}_t^1 = \frac{A_t^1}{(1+\gamma)^t A_0^1 Z_t}$, and the TFP of R&D production is $\hat{a}_t^2 = \frac{A_t^2}{(1+\gamma)^t A_0^1 Z_t}$. Furthermore, we define $h_t^1 = H_t^1/N_t$ and $h_t^2 = H_t^2/N_t$. The first-order conditions of the dynamic general equilibrium model after trend removal are as follows.

The first-order condition for consumption is

$$\hat{\lambda}_t = \frac{1}{\hat{c}_t(1+\tau_t^c)}. \tag{19}$$

The first-order condition for consumption and leisure is

$$\hat{\mathrm{c}}_\mathrm{t} = \frac{\left(1-\tau_t^h\right)}{1+\tau_t^c}\frac{(1-h_t)\hat{w}_t}{\psi}. \tag{20}$$

The first-order condition for the tangible capital of period t and the tangible capital of period $t+1$ is

$$\hat{\lambda}_t = \hat{\beta}_t \hat{\lambda}_{t+1}\left[1 + \left(r^T_{t+1} - \delta^T - \tau^k_{t+1}\right)\left(1 - \tau^d_{t+1} - \tau^p_{t+1} + \tau^d_{t+1}\tau^p_{t+1}\right)\right], \tag{21}$$

where $\hat{\beta}_t = \frac{\beta}{(1+z_t)(1+n_t)(1+\gamma)}$.

The first-order condition for the R&D capital of period t and the R&D capital of period $t+1$ is

$$\hat{\lambda}_t = \hat{\beta}_t \hat{\lambda}_{t+1} \times \left[\frac{r^I_{t+1} + q_{t+1}(1-\delta^I)(1-\kappa^I_{t+1}) + (r^I_{t+1} + q_{t+1}(1-\delta^I))(-\tau^p_{t+1} - \tau^{\mathrm{d}}_{t+1} + \tau^{\mathrm{d}}_{t+1}\tau^p_{t+1})}{q_t\left[(1-\kappa^I_t) - \tau^p_t - \tau^d_t + \tau^d_t\tau^p_t\right]}\right]. \tag{22}$$

The first-order condition for R&D capital and tangible capital is

$$1 + \left(r^T_{t+1} - \delta^T - \tau^k_{t+1}\right)\left(1 - \tau^d_{t+1} - \tau^p_{t+1} + \tau^d_{t+1}\tau^p_{t+1}\right) = \frac{r^I_{t+1} + q_{t+1}(1-\delta^I)(1-\kappa^I_{t+1}) + (r^I_{t+1} + q_{t+1}(1-\delta^I))(-\tau^p_{t+1} - \tau^{\mathrm{d}}_{t+1} + \tau^{\mathrm{d}}_{t+1}\tau^p_{t+1})}{q_t\left[(1-\kappa^I_t) - \tau^{\mathrm{p}}_t - \tau^d_t + \tau^d_t\tau^p_t\right]}. \tag{23}$$

The first-order condition for the labor input of the FGP sector and the labor input of the R&D sector is

$$\hat{w}_t = \varphi_1 \frac{\hat{y}_t}{h^1_t} = \varphi_2 \frac{q_t \hat{x}^I_t}{h^2_t}. \tag{24}$$

The first-order condition for the tangible capital input of the FGP sector and the tangible capital input of the R&D sector is

$$r^T_t = \theta_1 \frac{\hat{y}_t}{\hat{k}^{T1}_t} = \theta_2 \frac{q_t \hat{x}^I_t}{\hat{k}^{T2}_t}. \tag{25}$$

The first-order condition for the R&D capital is

$$r^I_t = \frac{\phi_1 \hat{y}_t + \phi_2 q_t \hat{x}^I_t}{\hat{k}^I_t}. \tag{26}$$

The capital transition equation, equilibrium condition for the final goods market, and the equilibrium fiscal condition for the government are as follows.

$$\hat{x}^T_t = (1+\eta_t)(1+\gamma)(1+z_t)\hat{k}^T_{t+1} - (1-\delta^T)\hat{k}^T_t, \tag{27}$$

$$\hat{x}^I_t = (1+\eta_t)(1+\gamma)(1+z_t)\hat{k}^I_{t+1} - (1-\delta^I)\hat{k}^I_t, \tag{28}$$

$$\hat{c}_t + \hat{x}^{T1}_t + \hat{x}^{T2}_t = \hat{y}_t, \tag{29}$$

$$\hat{\tau}_t = \hat{\zeta}_t. \tag{30}$$

II-7. Steady state

In the steady state, $\hat{x}_t = \hat{x}_{t+1} = \hat{x}_{ss}$ holds for all variables that have had the trend removed. The values in the steady state of production of the FGP sector are as follows:

$$\hat{y}_{ss} = \left(\hat{k}_{ss}^{T1}\right)^{\theta_1}\left(\hat{k}_{ss}^{I}\right)^{\phi_1}(\hat{a}_{ss}^{1}h_{ss}^{1})^{\varphi_1}.$$

The values in the steady state of production in the R&D sector are as follows:

$$\hat{x}_{ss}^{I} = \left(\hat{k}_{ss}^{T2}\right)^{\theta_2}\left(\hat{k}_{ss}^{I}\right)^{\phi_2}(\hat{a}_{ss}^{2}h_{ss}^{2})^{\varphi_2}.$$

The consumption value in the steady state, $\hat{c}_{ss}$, and the leisure value in the steady state, $l_{ss} = 1 - h_{ss}$, must satisfy the following condition:

$$\hat{c}_{ss} = \frac{(1-\tau_{ss}^{h})}{1+\tau_{ss}^{c}}\frac{(1-h_{ss})}{\psi}\widehat{w}_{ss}.$$

The tangible capital return, R&D capital return, and labor wage must satisfy the following five conditions:

$$1 = \hat{\beta}_{ss}\left[1 + (1-\tau_{ss}^{d})\left(1-\tau_{ss}^{p}\right)(\mathrm{r}_{ss}^{\mathrm{T}} - \delta^{T} - \tau_{ss}^{k})\right]$$

$$1 = \hat{\beta}_{ss}\left[\frac{r_{ss}^{I} + q_{ss}(1-\delta^{I})(1-\kappa_{ss}^{I}) + (r_{ss}^{I} + q_{ss}(1-\delta^{I}))(-\tau_{ss}^{p} - \tau_{ss}^{\mathrm{d}} + \tau_{ss}^{\mathrm{d}}\tau_{ss}^{p})}{q_{ss}\left[(1-\kappa_{ss}^{I}) - \tau_{ss}^{p} - \tau_{ss}^{d} + \tau_{ss}^{d}\tau_{ss}^{p}\right]}\right],$$

$$\widehat{w}_{ss} = \varphi_1\frac{\hat{y}_{ss}}{h_{ss}^{1}} = \varphi_2\frac{q_{ss}\hat{x}_{ss}^{I}}{h_{ss}^{2}},$$

$$r_t^{T} = \theta_1\frac{\hat{y}_{ss}}{\hat{k}_{ss}^{T1}} = \theta_2\frac{q_{ss}\hat{x}_{ss}^{I}}{\hat{k}_{ss}^{T2}},$$

$$r_{ss}^{I} = \frac{\phi_1\hat{y}_{ss} + \phi_2 q_{ss}\hat{x}_{ss}^{I}}{\hat{k}_{ss}^{I}}.$$

The capital transition equations, supply and demand balance conditions of the final goods market, and government fiscal balance condition are as follows. These conditions must also be satisfied in a steady state:

$$\hat{x}_{ss}^{T} = [(1+\eta_{ss})(1+\gamma_{ss})(1+z_{ss}) - (1-\delta^{T})]\hat{k}_{ss}^{T},$$
$$\hat{x}_{ss}^{I} = [(1+\eta_{ss})(1+\gamma_{ss})(1+z_{ss}) - (1-\delta^{I})]\hat{k}_{ss}^{I},$$
$$\hat{c}_{ss} + \hat{x}_{ss}^{T1} + \hat{x}_{ss}^{T2} = \hat{y}_{ss},$$
$$\hat{\tau}_{ss} = \hat{\zeta}_{ss}.$$

III. Calibration and Structural Estimation

III-1. Construction of data corresponding to the model

This subsection constructs economic data of the aggregate amounts corresponding to the dynamic general equilibrium model developed in the previous section. The sources of data are mainly the “System of National Accounts” of the Cabinet Office, the “Survey of Research and Development” (SRD), “Population Estimates,” the “Labor Force Survey” conducted by the Ministry of Internal Affairs and Communications, and the “Monthly Labor Statistics” compiled by the Ministry of Health, Labor, and Welfare.

First, for the series of R&D total production, X_t^I, a method of the Cabinet Office of the Economic and Social Research Institute (2010) is used. This data was obtained from the “research expenses for internal use” of enterprises, universities, and research institutes in the SRD[2]. In this case, R&D total expenditure = R&D total investment, without considering R&D imports from and exports to other countries.

$$\text{R\&D total production} = \text{Internal R\&D research cost from SRD.}$$

Next, we define the total production of the final goods, Y_t, as something from which the total R&D production of the government and non-profit institutions serving households (NPISH) from the 1993 SNA gross national income (GNI) is deducted, and from which the depletion of R&D capital of the government and NPISH is deducted:

$$\begin{aligned}&\text{Final goods total production} = \\ &\text{GNI}^{93} - (\text{Government R\&D total production} - \text{Government R\&D capital depletion}) \\ &\qquad - (\text{NPISH R\&D total production} - \text{NPISH R\&D capital depletion}).\end{aligned}$$

The tangible capital total investment in the FGP sector, X_t^{T1}, is calculated as follows:

$$\begin{aligned}&\text{Tangible capital investment in FGP sector} = \\ &\text{Gross domestic capital formation} + \text{Net exports} - \text{Tangible capital total investment in R\&D sector.}\end{aligned}$$

The tangible capital total investment in the R&D sector, X_t^{T2}, is calculated as follows:

$$\begin{aligned}&\text{Tangible capital investment in R\&D sector} = \\ &= \text{Tangible fixed asset purchase cost from SRD.}\end{aligned}$$

Household consumption, $C_t = N_t c_t$, is the sum of private final consumption expenditure and government final consumption expenditure:

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2 Personnel expenses of universities, junior colleges, and laboratories attached to universities in the SRD are adjusted using a full-time conversion factor. See Appendix A for details of the calculation.

Household consumption =

Private final consumption expenditure + Government final consumption expenditure.

The above totals are realized by using the corresponding deflators.

The tangible capital stock of the FGP sector, K_t^{T1}, the tangible capital stock of the R&D sector, K_t^{T2}, and the R&D capital stock, K_t^{I}, are calculated according to the following capital transition equations:

$$K_t^{T1} = X_t^{T1} + (1 - \delta^T)K_t^{T1},$$
$$K_t^{T2} = X_t^{T2} + (1 - \delta^T)K_t^{T2},$$
$$K_t^{I} = X_t^{I} + (1 - \delta^I)K_t^{I}.$$

Next, we calculate the labor input per holding time per labor force, h_t, as follows:

$$h_t = \frac{\text{Annual labor time}}{\text{Annual holding time}},$$

where,

$$\text{Annual labor time} = \text{Average monthly work hours} \times 12 \times \text{Number of workers},$$

$$\begin{aligned}\text{Annual holding time}\\ &= \text{Holding time of one day (16 hours)}\\ &\times \text{Number of prescribed working days per year (250 days)}\\ &\times \text{Population aged 15 years or more.}\end{aligned}$$

With regard to the share of labor in the FGP sector and in the R&D sector, we use equation (18) as the working hours, corresponding to the tangible capital stock ratios in the FGP and R&D sectors:

$$h_t^1 = \left(1 + \frac{\varphi_2}{\varphi_1}\frac{\theta_1}{\theta_2}\frac{k_t^{T2}}{k_t^{T1}}\right)^{-1} h_t,$$
$$h_t^2 = h_t - h_t^1.$$

The level of human capital, Z_t, is calculated using the average number of years of schooling and the Mincer earnings function, as follows:

$$Z_t = \left(\frac{\alpha_M}{1 - \psi_M}(s_t)^{1-\psi_M}\right). \tag{31}$$

Here, s_t is the average number of years of schooling in year *t*. The average number of years of schooling in Japan is estimated based on Barro and Lee (2013). For the Mincer earnings function parameters, the parameters $\alpha_{\mathrm{M}} = 0.32$ and $\psi_M = 0.28$ are used, based on the research of Miyazawa (2011).

III-2. Tax rate calculation

Various tax rates that affect household decision-making behavior are incorporated into the model. Specifically, consumption tax, labor income tax, property tax, enterprise profit tax (corporate tax etc.), and asset income tax (dividend tax, interest income tax) are used.

The consumption tax rate, τ_t^c, uses the actual consumption tax rates: 0% from 1980 to 1988, 3% from 1989 to 1996, and 5% from 1997 to 2011.

The labor income tax rate, τ_t^h, is the sum of salary income tax, retirement income tax, compensation income tax, non-resident income tax, and local residence tax (individuals)[3], divided by the SNA's nominal compensation of employee. The average value of this tax rate from 1980 to 2011 is calculated and applied to the whole period. For the property tax rate, τ_t^k, the standard tax rate of 1.4% is used for the whole period.

For the enterprise profit tax rate, τ_t^p, the corporate tax of national tax, enterprise tax of local tax, local corporation special tax, and inhabitant tax (corporate portion) are taken as enterprise profit tax. This is divided by the pre-tax enterprise profit, obtained by subtracting the fixed capital depletion, internal research funds, and property tax rate × tangible capital stock amount from the operating surplus. The average value of this tax rate from 1980 to 2011 is calculated and applied to the whole period.

For the capital income tax rate, τ_t^d, we calculate the sum of the amount of national tax income and dividend income, divided by the pre-tax corporate income, less the corporate income tax amount. The average value of this tax rate from 1980 to 2011 is calculated and applied to the whole period.

The R&D investment tax credit rate, κ_t^I, is obtained by dividing the deductions of experimental research expenses by the R&D investment amount. This tax rate is assumed to fluctuate every year and, thus, an average value is not applied to the whole period.

III-3. Calibration and estimation of structural parameters

In the dynamic general equilibrium model, the basic parameters of the economy that are not affected by government policies or changes in exogenous variables are called structural parameters. Production function parameters, utility function parameters, the capital depletion rate, and so on, correspond to such structural parameters.

[3] Data was obtained from the tax status of withholding income tax from the National Tax Agency in the case of national taxes, and from the "White Paper on Local Public Finance" in the case of local taxes.

Capital depletion rate

For the capital depletion rate of tangible capital, δ^T, the average value of the rate of fixed capital depletion to capital stock from 1980 to 2011 is used. For the capital depletion rate of R&D capital, δ^I, the value of 15% is applied, as in prior studies by the BEA (2006) and the Cabinet Office of the Economic and Social Research Institute (2010), for the R&D capital stock estimation.

Utility functions

The utility function for the weight of the utility from leisure, ψ, is given the value that made the 1980–2010 average value of the labor input wedge rate, ω_t^L (see Section 5), become 0. The time preference factor of the representative household, β, is assumed to be 0.98.

Production functions

The structural parameters for the production functions of the FGP and the R&D sectors are estimated by the GMM technique, using the model's first-order conditions (i.e., the conditions for tangible capital return of both FGP and R&D sectors, and simultaneous substitution) after the labor share parameters of both sectors were estimated from the data.

The labor share parameter for the FPG sector, φ_1, is given the 1980–2011 average of the ratio of total employer compensation and mixed income (SNA data) to the GNI.

The labor share parameter for the R&D sector, φ_2, is given the 1980–2011 average of the ratio of the internal research cost to R&D personnel expenses.

Next, for the tangible capital share and the R&D capital share for the production functions of the FGP and R&D sectors (i.e., $\theta_1, \phi_1, \theta_2$ and, ϕ_2), the moment condition is given by a GMM estimation made with the following settings and using equations (23) and (25), which are the first-order conditions developed in Section 2:

$$\min_{\theta_1,\theta_2} \Gamma(\mathrm{M};\theta_1,\theta_2) = \chi(\mathrm{M};\theta_1,\theta_2)\Sigma^{-1}\chi(\mathrm{M};\theta_1,\theta_2)'. \tag{32}$$

Here, $M = \{m_{1980},\cdots,m_{2010}\}, m_t = \left\{\hat{y}_t, \hat{x}_t^I, \hat{k}_t^{T1}, \hat{k}_t^{T2}, \hat{k}_t^I, \kappa_t^I, \kappa_{t+1}^I, \tau_t^p, \tau_{t+1}^p, \tau_t^d, \tau_{t+1}^d, \tau_{t+1}^k\right\}$.

In addition,

$$\chi(M;\theta_1,\theta_2) = \begin{bmatrix} \frac{1}{T}\sum_t f_1(m_t;\theta_1,\theta_2) \\ \frac{1}{T}\sum_t f_2(m_t;\theta_1,\theta_2) \end{bmatrix},$$

and $f_1(m_t;\theta_1,\theta_2)$ and $f_2(m_t;\theta_1,\theta_2)$ are

$$f_1(m_t;\theta_1,\theta_2) = 1 + \left(r^T_{t+1} - \delta^T - \tau^k_{t+1}\right)\left(1 - \tau^d_{t+1} - \tau^p_{t+1} + \tau^d_{t+1}\tau^p_{t+1}\right) - \frac{\left(r^I_{t+1} + q_{t+1}(1-\delta^I)(1-\kappa^I_{t+1})\right)\left(1 - \tau^p_{t+1} - \tau^d_{t+1} + \tau^d_{t+1}\tau^p_{t+1}\right)}{q_t\left[(1-\kappa^I_t) - \tau^p_t - \tau^d_t + \tau^d_t\tau^p_t\right]},$$

$$f_2(\mathrm{X};\theta_1,\theta_2) = \theta_1\frac{\hat{y}_t}{\hat{k}^{T1}_t} - \theta_2\frac{q_t\hat{x}^I_t}{\hat{k}^{T2}_t},$$

where

$$r^I_t = \frac{\phi_1\hat{y}_t + \phi_2 q_t\hat{x}^I_t}{\hat{k}^I_t},$$

$$\varphi_1 = 1 - \phi_1 - \theta_1,$$

$$\varphi_2 = 1 - \phi_2 - \theta_2.$$

For q_t, the value of $q_t = 1$ is assumed for all fiscal periods in the parameter estimation of the production function. After obtaining the parameters of the production function, equation (16) is used to calculate and use the relative price of R&D products, q_t. The parameters are estimated by repeating the method of reducing the search grid every time an estimation round by a nonlinear grid search advances, until $\Gamma(\mathrm{M};\theta_1,\theta_2)$ becomes sufficiently small. For the weight matrix (Σ), the first estimation round uses

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The second and subsequent estimation rounds use the diagonal matrix of the eigenvalues of the variance–covariance matrix of the estimation error of the moment condition of the previous round, $Cov(f_1(m_t;\theta_1,\theta_2), f_2(m_t;\theta_1,\theta_2))$:

$$\Sigma = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

From the production function obtained in this manner, the TFP in the final goods sector (i.e., A^1_t) is found and γ is calculated as its average growth rate for the period 1980–2011. The calibrations of the structural parameters and the estimation results are shown in Table 1.

Table 1. Structural parameter values

β	Time Discount Factor	0.980000000000000
γ	Trend Growth Rate of TFP in FGP Sector	0.009624326842325
ψ	Weight of Utility from Leisure	1.730726793404400
θ^1	Tangible Capital Share in FGP Sector	0.396956117920397
ϕ^1	R&D Capital Share in FGP Sector	0.017111500728346
φ^1	Labor Share in FGP Sector	0.585932381351257
θ^2	Tangible Capital Share in R&D Sector	0.153413459575458
ϕ^2	R&D Capital Share in R&D Sector	0.369837767290490
φ^2	Labor Share in R&D Sector	0.476748773134053
δ^T	Depreciation Rate of Tangible Capital	0.073503400483774
δ^I	Depreciation Rate of R&D Capital	0.150000000000000

IV. Growth Accounting

This section applies growth accounting to the Japanese economy. Specifically, two cases are examined: the case of a standard production function and the case where the R&D capital stock adopted here is shared by the two sectors. First, the result of growth accounting using the production function is as follows:

$$Y_t = (K_t^T)^{1-\varphi_1}(A_t H_t)^{\varphi_1}.$$

Here, φ_1 is the labor share of the FGP sector estimated in Section 3. Table 2 shows the growth accounting results. Hayashi and Prescott (2002) pointed out the declining growth rate of labor input and the slowing of TFP growth as causes of the slump in Japan's economic growth rate in the 1990s, which is confirmed by the result. Furthermore, in the 2000s, labor input and TFP growth continued on a similar trend, also influenced by the deceleration of capital stock growth, and the growth rate of output declined even further than it did in the 1990s.

Table 2. Growth accounting by capital stock, labor input, and TFP

Contribution (% point)

	GDP			
		Labor Input	Capital Input	TFP
1960-1970	10.13	0.32	4.74	5.06
1970-1980	4.35	0.13	3.89	0.32
1980-1990	4.24	0.55	1.95	1.74
1990-2000	1.26	-0.40	1.29	0.37
2000-2011	0.34	-0.34	0.36	0.32

Table 3 shows the growth accounting result in the case where the human capital level estimated in Section 3 is added as a production factor. The production function has the following form:

$$Y_t = (K_t^T)^{1-\varphi_1}(A_t Z_t H_t)^{\varphi_1}.$$

In Table 3, the contribution of TFP growth, measured as a residual, is decreasing owing to the addition of human capital growth. In the 1970s and 1990s, the growth rate of human capital was high, and the growth rate of TFP was negative.

The last growth accounting to be applied is based on the two-sector growth model introduced in Section 2 and the final goods production and R&D production data constructed in Section 3. The production functions of the FGP and R&D sectors are the same as those shown in equations (10) and (11):

$$Y_t = (K_t^{T1})^{\theta_1}(K_t^I)^{\phi_1}(A_t^1 Z_t H_t^1)^{\varphi_1},$$
$$X_t^I = (K_t^{T2})^{\theta_2}(K_t^I)^{\phi_2}(A_t^2 Z_t H_t^2)^{\varphi_2}.$$

Here, Y_t is final goods production, and X_t^I is R&D production. The respective values of the structural parameters of each production function are those estimated in Section 3. Table 4 shows the results of this new growth accounting. The notable characteristic of the final goods sector is the growth contribution of R&D capital stock, which declined during the 1990s and 2000s. For other production factors, there are no significant differences from the version of growth accounting shown in Table 3. With regard to the R&D sector, the growth rate of R&D production declined during the 1990s and 2000s. In addition, over this time, the TFP growth rate of the R&D sector is negative[4].

Table 3. Growth accounting by capital stock, labor input, human capital levels, and TFP

Contribution (% point)

	GDP	Labor Input × Human Capital			Capital Input	TFP
			Labor Input	Human Capital		
1960-1970	10.13	0.42	0.32	0.10	4.74	4.96
1970-1980	4.35	0.65	0.13	0.51	3.89	-0.19
1980-1990	4.24	0.87	0.55	0.31	1.95	1.42
1990-2000	1.26	-0.01	-0.40	0.39	1.29	-0.02
2000-2011	0.34	-0.11	-0.34	0.23	0.36	0.09

[4] With regard to labor input data, assuming that the human capital levels of the FGP sector and the R&D sector are the same, total labor input is split using equal conditions for wages of the FGP sector and the R&D sector, and equal conditions for physical capital return. In fact, because the human capital growth rates of the FGP sector and the R&D sector are considered different, it is highly likely that TFP as a residual exhibits a different trend. This point is left for future research.

Table 4. Growth accounting by model with R&D investment

Contribution (% point)

	Final Goods Production Sector									
		Labor Input			Capital Stock					TFP1
			Work Hour	Human Capital		Tangible Capital	R&D Capitall			
								Corporate	Gov. and NPISH	
1960-1970	10.13	0.41	0.31	0.10	5.02	4.83	0.19	0.14	0.05	4.70
1970-1980	4.37	0.66	0.14	0.51	3.89	3.78	0.10	0.08	0.02	-0.17
1980-1990	4.24	0.84	0.52	0.31	1.97	1.87	0.10	0.09	0.02	1.43
1990-2000	1.25	-0.02	-0.40	0.39	1.31	1.23	0.07	0.06	0.02	-0.04
2000-2011	0.35	-0.11	-0.34	0.23	0.38	0.34	0.04	0.03	0.00	0.08

	R&D Investment Sector									
		Labor Input			Capital Stock					TFP2
			Work Hour	Human Capital		Tangible Capital	R&D Capitall			
								Corporate	Gov. and NPISH	
1960-1970	11.57	0.96	0.88	0.08	5.74	1.69	4.04	3.01	1.05	4.87
1970-1980	2.94	0.16	-0.26	0.42	3.42	1.17	2.25	1.83	0.48	-0.63
1980-1990	7.63	2.10	1.84	0.26	3.46	1.18	2.28	1.93	0.39	2.07
1990-2000	1.99	0.15	-0.16	0.32	2.14	0.53	1.61	1.25	0.36	-0.30
2000-2011	0.90	0.04	-0.14	0.19	0.94	0.17	0.77	0.66	0.09	-0.09

V. Divergences between Model and Data

This section measures the distortions (wedges) in the Japanese economic market from 1980 to 2011, using the dynamic general equilibrium model with the given numerical values for the structural parameters. Chari, Kehoe, and McGrattan (2007) proposed estimating multiple wedges as divergences between observed macroeconomic data and data generated by the standard RBC model. By incorporating the estimated wedge into the model as an unobserved market distortion, other than tax, it is possible to perfectly match the data of the endogenous variable generated by the model with the observed economic data. This study uses tangible investment wedge, R&D investment wedge, and labor input wedge as non-tax market distortions, and productivity wedge of the final goods sector and productivity wedge of the R&D sector production sector as factors in the production function that cannot be explained by the production factor inputs or the growth trend.

V-1. Introduction of wedges to model

When the labor wedge, tangible capital investment wedge, and R&D investment wedge are added to the household budget constraint of equation (3), it can be written as follows:

$$c_t + (1 + \omega_t^T)x_t^T + (1 + \omega_t^I)q_t x_t^I = r_t^T k_t^T + r_t^I k_t^I + (1 - \omega_t^L)w_t h_t^T + \zeta_t - \tau_t. \quad (33)$$

Here, ω_t^T and ω_t^I denote the tangible capital investment wedge rate and R&D capital investment wedge rate, respectively. When these wedge rates are positive, they act in the same way as the tax rate on investment. Then, ω_t^L is the labor wedge rate. If $\omega_t^L > 0$, the wedge rate acts in the same way as the labor income tax rate, and if $\omega_t^L < 0$, the wedge rate acts in the same way as the labor subsidy rate.

Assuming that these wedges are present, the first-order conditions of the inter-temporal utility maximization of households, (20), (21), and (22), can be rewritten as equations (34), (35), and (36), respectively.

By incorporating the labor input wedge, $\Omega_t^L \equiv 1 - \omega_t^L$, equation (20) can be rewritten as follows:

$$\Omega_t^L = (1 - \omega_t^L) = c_t \frac{1 + \tau_t^c}{1 - \tau_t^h} \frac{\psi (1 - h_t)^{-1}}{w_t} = \left(\frac{1 + \tau_t^c}{1 - \tau_t^h} \right) \left(\frac{\hat{c}_t}{\hat{y}_t} \right) \frac{\psi}{\varphi_1} \left(\frac{h_t^1}{l_t} \right). \tag{34}$$

By incorporating the tangible capital investment wedge, $\Omega_t^T \equiv 1 + \omega_t^T$, the condition of intertemporal substitution for tangible capital, equation (21) can be rewritten as follows:

$$\Omega_t^T = \hat{\beta}_t \frac{\hat{c}_t}{\hat{c}_{t+1}} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left\{ r_{t+1}^T + (1 - \delta^T) \Omega_{t+1}^T - \tau_{t+1}^k + (-\tau_{t+1}^d - \tau_{t+1}^p + \tau_{t+1}^d \tau_{t+1}^p) (r_{t+1}^T - \delta^T - \tau_{t+1}^k) \right\}$$

$$\Leftrightarrow$$

$$\Omega_{t+1}^T = \frac{\frac{1}{\hat{\beta}_t} \frac{\hat{c}_{t+1}}{\hat{c}_t} \left(\frac{1 + \tau_{t+1}^c}{1 + \tau_t^c} \right) \Omega_t^T - r_{t+1}^T + \tau_{t+1}^k - (-\tau_{t+1}^d - \tau_{t+1}^p + \tau_{t+1}^d \tau_{t+1}^p)(r_{t+1}^T - \delta^T - \tau_{t+1}^k)}{1 - \delta^T}. \tag{35}$$

By incorporating the R&D investment wedge, $\Omega_t^I \equiv 1 + \omega_t^I$, the condition of intertemporal substitution for R&D capital, equation (22) can be rewritten as follows:

$$\begin{aligned} &q_t \left[(\Omega_t^I - \kappa_t^I) - \tau_t^d - \tau_t^p + \tau_t^d \tau_t^p \right] \\ &\quad = \hat{\beta}_t \frac{\hat{c}_t}{\hat{c}_{t+1}} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left[r_{t+1}^I + q_{t+1} (1 - \delta^I)(\Omega_{t+1}^I - \kappa_{t+1}^I) \right. \\ &\quad \left. + (r_{t+1}^I + q_{t+1}(1 - \delta^I))(-\tau_{t+1}^d - \tau_{t+1}^p + \tau_{t+1}^d \tau_{t+1}^p) \right] \end{aligned}$$

$$\Leftrightarrow$$

$$\begin{aligned} &(\Omega_{t+1}^I - \kappa_t^I)(1 - \delta^I) q_t = \\ &q_t \left[(\Omega_t^I - \kappa_t^I) - \tau_t^d - \tau_t^p + \tau_t^d \tau_t^p \right] \frac{1}{\hat{\beta}_t} \frac{\hat{c}_{t+1}}{\hat{c}_t} \left(\frac{1 + \tau_{t+1}^c}{1 + \tau_t^c} \right) - (r_{t+1}^I + q_{t+1}(1 - \delta^I))(-\tau_{t+1}^d - \tau_{t+1}^p \\ &+ \tau_{t+1}^d \tau_{t+1}^p) - r_t^I. \end{aligned} \tag{36}$$

In the production functions of equations (10) and (11), A_t^1 and A_t^2 correspond to the TFP of the FGP sector and the TFP of the R&D sector, respectively. By dividing the production factors and production volumes of each fiscal period by $(1 + \gamma)^t A_0^1 Z_t N_t$, the TFPs of the FGP sector and the R&D sector in the model after the trend removal are defined, respectively, as follows:

$$\hat{a}_t^1 = \frac{1}{h_t^1}\left(\frac{\hat{y}_t}{\left(\hat{k}_t^{T1}\right)^{\theta_1}\left(\hat{k}_t^I\right)^{\phi_1}}\right)^{\frac{1}{1-\theta_1-\phi_1}}, \tag{37}$$

$$\hat{a}_t^2 = \frac{1}{h_t^2}\left(\frac{\hat{y}_t}{\left(\hat{k}_t^{T2}\right)^{\theta_2}\left(\hat{k}_t^I\right)^{\phi_2}}\right)^{\frac{1}{1-\theta_2-\phi_2}}. \tag{38}$$

V-2. Estimated market distortion

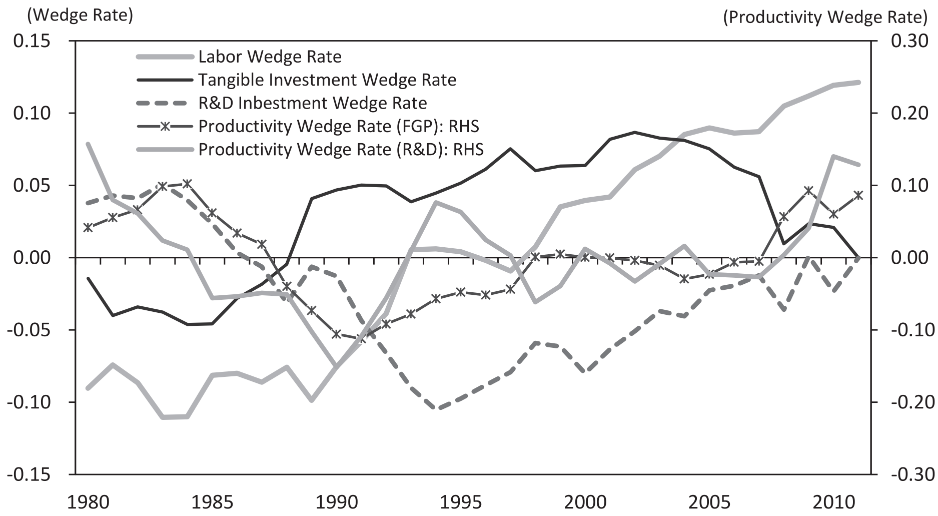
Wedges can be estimated for equations (33) to (38), which give numerical values to the structural parameters, by using the sequence of tax rates defined in Section 3 and the observed data[5].

Figure 1 shows the transition of each estimated wedge rate. The labor wedge rate, ω_t^L, is consistently rising. The tangible capital investment wedge rate, ω_t^T, is negative until the middle of the 1980s, but has been positive since then. The R&D investment wedge rate, ω_t^I, became significantly negative in the 1990s, but has become less so since then. The productivity wedge rate of the FGP sector, $1 - \hat{a}_t^1$, declined through the 1990s and became negative (i.e., productivity exceeded the trend), but the values have been gradually increasing since then. The productivity wedge rate in the R&D sector, $1 - \hat{a}_t^2$, also declined in the 1990s, but then increased sharply, and has been positive since the middle of the 2000s. In Figure 1, when a wedge rate is negative, there is a promoting effect on labor supply, investment, and production, and when a wedge rate is positive, there is a suppressing effect on labor supply, investment, and production.

For 1980 to 2011, to interpolate the estimated wedges as exogenous variables, it matches the observed data and the generated endogenous variables in the model.

[5] For the physical capital investment wedge, Ω_t^T, and the R&D investment wedge, Ω_t^I, there is no criterion for determining the level, because the wedge for period t + 1 depends on the wedge in period t. However, here, the initial values of both wedges are given such that the physical capital investment wedge and the R&D investment wedge become 1.

Figure 1. Estimated wedges



VI. Policy Simulation

This section implements simulations for the extrapolated areas using the numerical model discussed in the previous section. The steady-state equilibrium of the economy and the convergence paths are examined under different exogenous variables and policy environments. One method of generating endogenous variables of paths converging to a steady state in a nonlinear model is the "shooting algorithm," which finds paths that converge to a steady state by successively trying the initial values of the control variables. Since there are multiple control variables in this model, we used the grid search to find the values of the initial control variables converging to a steady state[6]. In this method, which finds the convergence path of this nonlinear model, it is possible to conduct a simulation even when there is a change in the exogenous variable such that the steady-state equilibrium becomes different. The extrapolated section is the period from 2012 to 2070, and a prediction simulation is performed assuming that the economy reaches a steady state at 2070, or earlier.

In order to perform a predictive simulation, it is necessary to prepare a baseline scenario of exogenous variables. For the TFP in the final goods sector, it is assumed to be the observed, A_t^1, from 1980 to 2011. It will grow at trend rate, γ, after 2020. Then, it is assumed that the TFP in the R&D sector to be the observed, A_t^2, and it will grow at the same rate as the TFP of the final goods sector after 2020. Human capital is assumed to grow at the 1980–2011 average growth rate, even after 2020. We assume that the consumption tax rate will increase to 8% in 2014, to 10% in 2017, and thereafter, will be 10%. For tax rates other than the consumption

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[6] See Appendix B for details of the shooting algorithm when there are multiple control variables.

tax, the average tax rate from 1980 to 2011 is determined, and then applied from 2020 onwards. For the tangible capital investment wedge and the R&D capital investment wedge, all periods after 2012 are assumed to take a value of unity. In order to avoid sudden fluctuations of exogenous variables, the values of 2012 to 2019 are filled in using the liner interpolation method, based on the values of 2011 and 2020. Figure 2 shows the transition of exogenous variables in the baseline scenario.

Furthermore, note that there is a question on how to incorporate the increase on social security-related burdens into the scenario as the aging of the population progresses. Although labor income tax is incorporated in the model, the burden associated with the social security system (e.g., payments of defined-benefit pension premiums or of long-term care insurance premiums) due to the aging population is not incorporated. Miyazaki (2009) notes that the increase in the social security burden, which acts like a labor income tax, may create a declining trend of the labor wedge $\Omega_t^L = 1 - \omega_t^L$. Values of the labor wedge after 2012 were extrapolated on the assumption that these values are a function of the aging rate (population over 65 years old ÷ population over 15 years old). When regressing the labor wedge from 1980 to 2011 on the quadratic polynomial of the aging rate, the relationship on the left side of Figure 3 is observed. Using this relationship, if we draw the trend of the labor wedge from the aging rate (medium estimate for "future population estimate") of the extrapolated area, it

Figure 2. Exogenous variables in baseline scenario

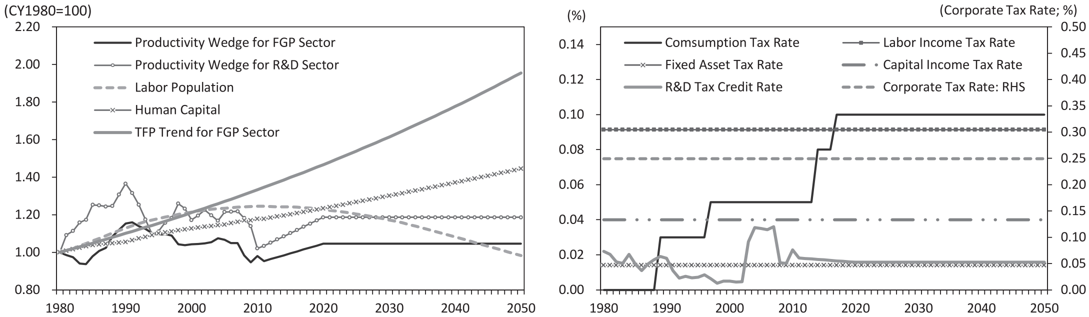
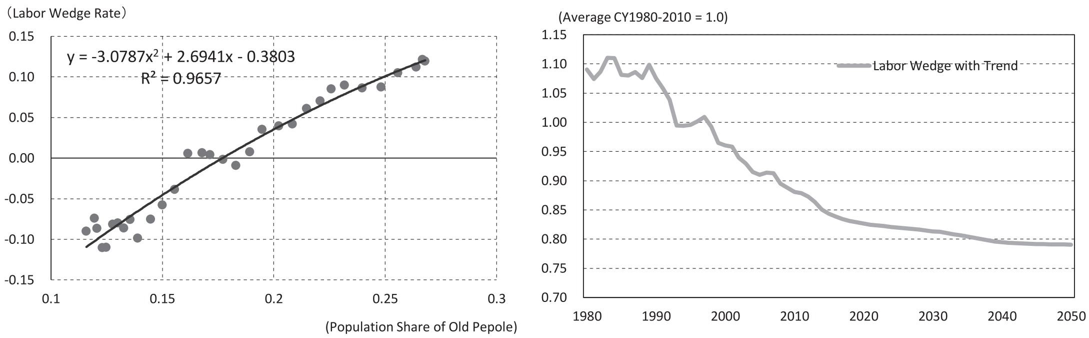


Figure 3. Aging population and labor wedges



will change as shown on the right side of Figure 3. Given the growing social security burden associated with the aging population, the assumption of a labor wedge with such a downward trend is considered appropriate.

Next, the role of public R&D capital stock is incorporated into the model. Despite the rarity of cases of technology being transferred from universities to enterprises by technological transfer or research laboratories, it is thought that public R&D has little direct contribution to the sales and profits of private companies. On the other hand, by providing basic research, it is thought that public R&D promotes R&D and patent acquisition among private enterprises, as well as R&D and organizational growth among public research institutes, with increasing the productivity of R&D production. Ikeuchi et al. (2014) points out that this public R&D has a spillover effect on private enterprise production. This point is recognized as a "science linkage" that leads to the development of patents citing basic research papers and new products based on such patents, and through this, a single research field is being formed. The policy attempted here assumes that the public R&D capital stock is related to the TFP level of R&D production.

It is assumed that R&D production is divided into private enterprise R&D, K_t^{IB}, and public R&D, K_t^{IP}, (government and NPISH R&D). Now, we have the following relationship between public R&D, K_t^{IP}, and the TFP of the R&D sector.

The production function of the final goods sector is as follows. Note that there is no change from what has been done so far:

$$Y_t = (K_t^{T1})^{\theta_1}(K_t^{IB} + K_t^{IP})^{\phi_1}(A_t^1 Z_t H_t^1\ )^{\varphi_1}. \tag{39}$$

On the other hand, the production function of the R&D sector, K_t^{IP} contributes to production via two routes:

$$X_t^I = (K_t^{T2})^{\theta_2}(K_t^{IB} + K_t^{IP})^{\phi_2}(A_t^2(K_t^{IP})Z_t H_t^2\ )^{\varphi_2}. \tag{40}$$

The direct effect of public R&D capital stock on R&D production is taken into account for representative enterprise behavior and for the optimized behavior of representative households holding the capital stock. However, because the effect through the TFP of the R&D sector, $A_t^2(K_t^{IP})$, is a spillover effect, this indirect effect is treated as an exogenous for representative enterprise and representative households.

The effect of public R&D stock on the TFP of the R&D sector is estimated from the public R&D capital stock, K_t^{IP}, of 1965 to 2011, and the TFP of the R&D sector, A_t^2, for the same period, using the following regression equation:

$$\log(A_t^2/\bar{A}^2) = \rho \cdot \log(K_t^{IP}/\bar{K}^{IP}) + e_t. \tag{41}$$

Here, $\bar{A}^2$ and $\bar{K}^{IP}$ are the average throughput of A_t^2 and K_t^{IP}, respectively, and the estimated coefficient is $\rho = 0.3609$. For the extrapolation area of productivity in the R&D sector, public R&D production is grown for the three scenarios, as shown in Table 5.

Table 5. Policy simulation scenarios

	Growth rate of public R&D investment	
Scenario 1	$\frac{X_{t+1}^{IP}}{X_t^{IP}} = 0.995$	Average growth rate for 2001 to 2011
Scenario 2	$\frac{X_{t+1}^{IP}}{X_t^{IP}} = 1.012$	Average growth rate for 1991 to 2011
Scenario 3	$\frac{X_{t+1}^{IP}}{X_t^{IP}} = 1.020$	—

Figure 4 shows the calculated convergence paths to a steady state assuming the spillover of R&D production to the TFP by public R&D stock for each scenario, showing the growth rate of public R&D investment after 2012. Figure 5 shows the policy simulation in 2070. When calculating a convergence path, the model assumes that the economy reaches a steady state in 2070, or earlier. Comparing the endogenous variables in 2070, in scenario 3, the final goods production of each year is about 1% higher than they are in scenario 1 (upper left of Figure 5). In addition, R&D production increased by 42% (upper right of Figure 5), and the relative price of R&D products declined by 29% (lower right of Figure 5). Consumption and the wage rate are similar to the final good production, at 1% higher (second left of Figure 5, and second right of Figure 5), leisure time does not change (third left of Figure 5). This simulation shows that increasing public investment in R&D will increase economic welfare.

Figure 4. Result of simulation of public R&D investment policy (convergence paths)

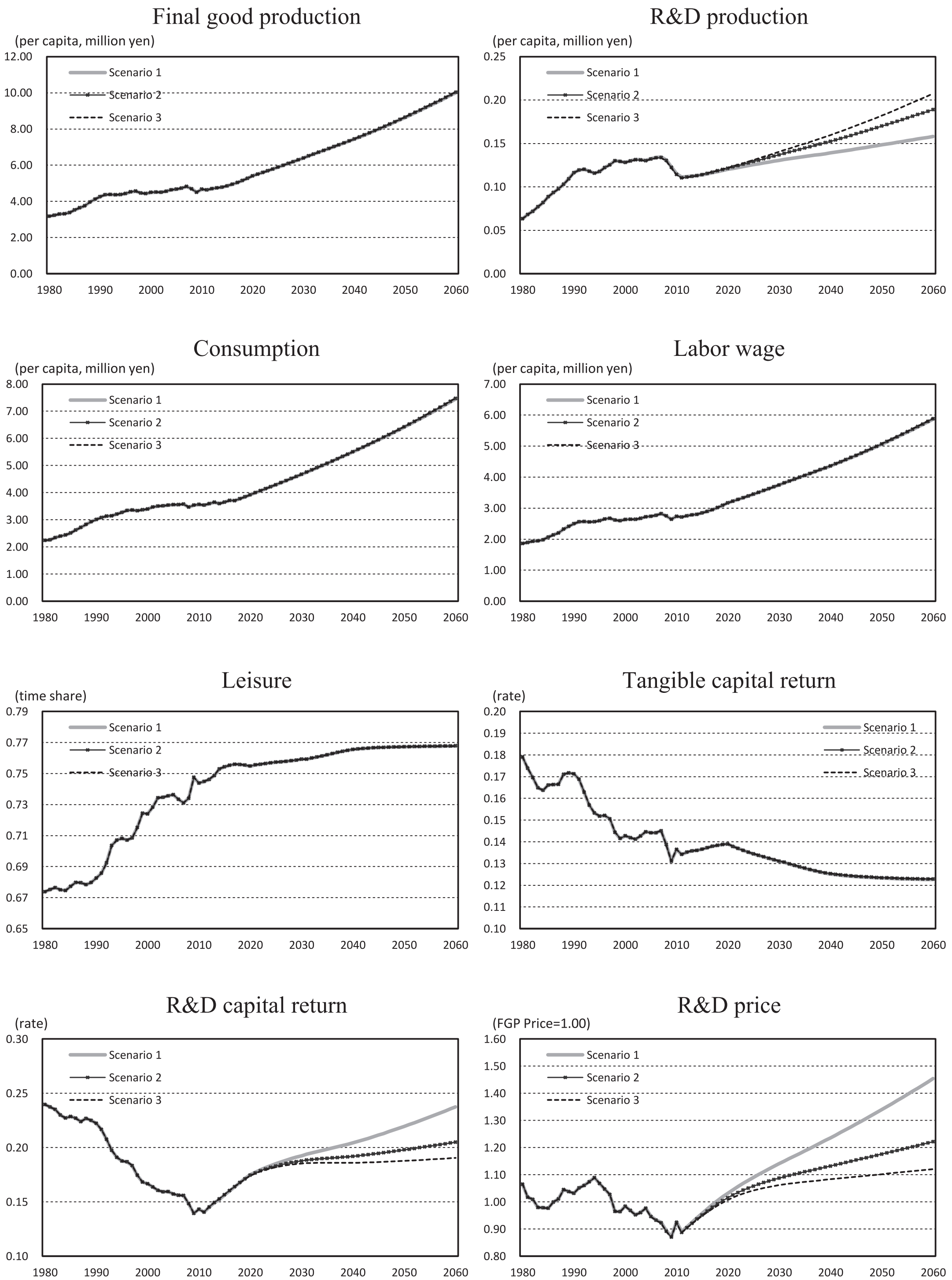
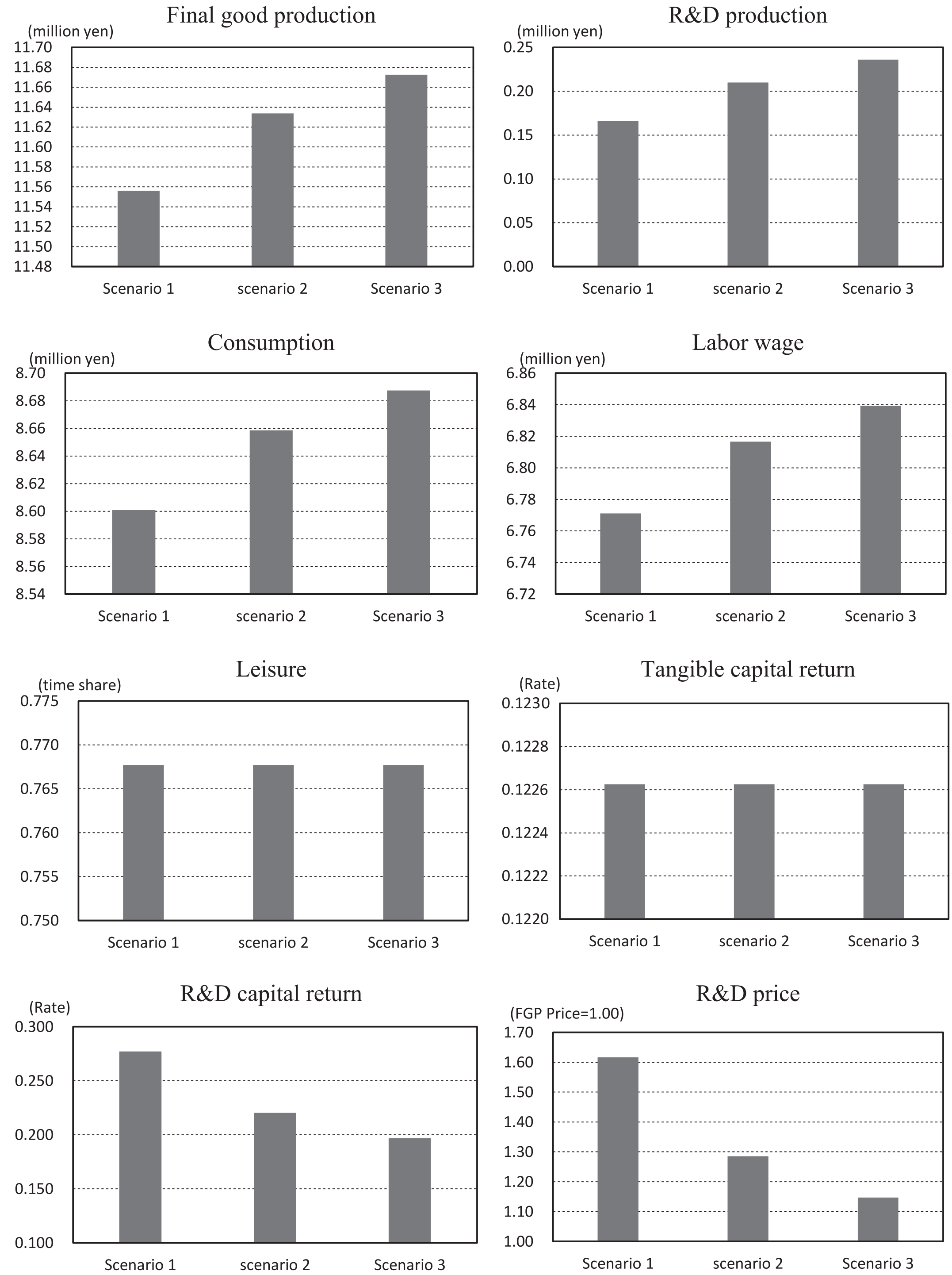


Figure 5. Result of simulation of public R&D investment policy (value in 2070)



VII. Conclusion

In this paper, the stock of Japanese R&D capital was estimated using data mainly from the “Survey of Research and Development.” Human capital stock was estimated by referring to the average years of schooling, as in Barro and Lee (2013), and the Mincer earnings function of Miyazawa (2011).

In addition, using the tangible capital stock and labor input of the final goods production sector and the R&D production sector, the structural parameters of the production functions of final good production and R&D production, which commonly share R&D capital stock, were estimated.

A two-sector dynamic general equilibrium model with the production functions of the final goods production sector and R&D production sector was developed, and the remaining structural parameters were estimated or calibrated and used in the subsequent analysis. First, growth accounting was applied. By applying growth accounting to the final goods production sector and to the R&D production sector, we evaluated the contribution of the respective production factors. Second, a policy simulation was conducted to extrapolate the exogenous variables, such as future population, human capital, TFP, tax rate, and so on, and to evaluate their effect on the steady-state equilibrium of the science and technology policy (i.e., public R&D expenditure) and on the economic growth path.

The main results are as follows. First, the R&D capital share in final goods production is small, at 0.017, while the R&D capital share in the R&D sector is high, at 0.37. Second, the growth accounting using the estimated final goods production and the R&D production function shows that the rate of increase of the final goods TFP declined since the 1990s, as did the growth rate of the R&D production TFP over the same period. Third, the policy simulation assuming a spillover effect of public R&D showed that the increase in public R&D investment causes the R&D sector TFP to increase, boosting R&D production considerably. At the same time, the simulation showed that an increase in public R&D also increases production in the final goods production sector and in household consumption. There are many challenges remaining. In the proposed model, the items analyzed were the non-competitiveness of R&D capital and the spillover of public R&D to R&D productivity. However, in many cases, organizations that carry out public R&D are higher education institutions, including universities and graduate schools, or research organizations where young researchers are being fostered. Therefore, the accumulation of public R&D capital may lead to improvements in human capital, which cannot be measured by the average years of schooling. In order to incorporate the effect of public R&D on educational level (common trend of human capital or TFP growth), it is necessary to include the education sector in the model, in addition to the FGP sector and R&D sector, and to consider a numerical model in which the household performs tangible capital investment, human capital investment, and R&D investment.

Furthermore, although this study included R&D capital and human capital as measurable intangible capital, the 2008 SNA recommends the recording of “Artistic Originals” as other intellectual production products, and that such estimates are actually being performed in the national accounts calculation of the United States and the United Kingdom. The UK's Department of Culture, Media, and Sports (2016) notes that "Artistic Originals" and industries producing mainly software are leading the economic growth of the United Kingdom. Measuring the intangible capital and clarifying the role and action of such capital in that economy are left for future research.

Appendix A: Estimation of R&D Capital Investment and R&D Capital Stock

The estimations of R&D capital investment and R&D capital stock are based on the method of the Economic and Social Research Institute, Cabinet Office (2010).

The R&D production amounts of the research subjects are the sum of personnel expenses, raw material expenses, leasing fees, other expenses, and fixed capital depreciation that were part of the internal research expenses of each research subject of the "Survey of Research and Development" conducted by the Ministry of Internal Affairs and Communications.

At this time, the equivalent full-time personnel cost for a university, junior college, other tertiary school, or university-attached research institute were created using a conversion factor to convert personnel expenses into full-time positions engaged in research activities (from the Ministry of Education, Culture, Sports, Science "Survey Report on Full-Time Position Converted Data at Universities etc. (fiscal 2002)"). The full-time conversion factor is 0.475 for a university, 0.342 for a junior college or other tertiary school, and 0.634 for a university-attached research body. The R&D production amounts for each research subject (enterprise, non-profit organization/private-sector research body, university) were reorganized into the R&D production amounts for each institutional sector, based on the bridge table in Table 6. In this study, there is no adjustment for R&D exports/imports because international balance of payments statistics only provide data on balance of payments of R&D services for 2005 onwards. Therefore, the R&D production amount is set equal to the R&D investment amount. Calculation of the R&D investment amounts in real terms are achieved by dividing

Table 6. Correspondence table of SRD classification and SNA classification

SRD Classification	Institutional Sector in SNA		
	Firm	General Govt.	NPISH
Firm	○		
Public Research Institute		○	
Non-Profit Research Institute			○
Public University		○	
Public Junior College		○	
Reserch Institute in Public University		○	
Private University			○
Private Junior College			○
Reserch Institute in Private University			○

personnel expenses by the Monthly Labor Survey's regular total wage index (industrial total, business establishments with five or more employees), the raw material costs by the corporate price index (intermediate goods), the leasing costs as other expenses by the total consumer price index (total excluding imputed rent), and the fixed capital depreciation by the GDP deflator (total fixed capital formation)[7]. Finally, the capitalization of the R&D investment amount was carried out using the permanent inventory method, based on the following capital transition equation:

$$K_{t+1}^{I} = (1 - \delta^{I})K_t^I + X_t^I.$$

Here, the R&D capital depletion rate (δ^I) is 15%, as in BEA (2006). In addition, the gestation period for R&D investment is assumed to be 0 years.

The initial value for the R&D capital stock amount is provided by the following equation:

$$\mathrm{K}_{1959}^{I} = \frac{(\sum_{t=1960}^{1964} X_t^I)/5}{[\sum_{t=1960}^{1964}(X_{t+1}^I/X_t^I - 1)]/5 + \delta^I}.$$

Appendix B: Calculation of Convergence Path by Grid Search Shooting

For the steady-state equilibrium of the nonlinear dynamic general equilibrium model, assigned the function type and the value of structural parameters $\{\beta, \gamma, \psi, \delta^T, \delta^I, \theta_1, \theta_2, \phi_1, \phi_2\}$, the values of the endogenous variables satisfying the steady-state condition in Section 2.7 are obtained using the numerical nonlinear solution.

The path that converges to the steady-state equilibrium is obtained by searching for a path that converges to a steady state, while satisfying the optimization conditions of a representative household and a representative enterprise, as well as the general equilibrium condition of the economy. The proposed two-sector dynamic general equilibrium model considers three state variables, $\{\hat{k}_t^{T1}, \hat{k}_t^{T2}, \hat{k}_t^{I}\}$, and two control variables, $\{h_t^1, h_t^2\}$, that give the time series of the exogenous variables and the initial values of the state variables, $\{k_1^{T1}, k_1^{T2}, k_1^I\}$. As a result, the model finds the initial values $\{h_1^1, h_1^2\}$ of the control variables on the path converging to a steady state. The grid search shooting method is used to find the initial values $\{h_1^1, h_1^2\}$ of the control variables on the path converging to the steady state. Specifically, following this process, we determine the initial values $\{h_1^1, h_1^2\}$ of the control variables that reach the steady state.

Step 1: A series of exogenous variables, with the initial observation period as one period, is prepared, where the terminal period of the observation period is period T, the initial period of the extrapolation period is period T + 1, and the terminal period of the extrapolation period is period S.

$$\Xi = \{A_t^1, A_t^2, \gamma_t\, z_t, \eta_t, \tau_t^c, \tau_t^h, \tau_t^p, \tau_t^k, \tau_t^d, \kappa_t^I, \Omega_t^T, \Omega_t^I, \Omega_t^L\}_{t=1}^{S}$$

[7] Both price indices used 2005 as 1.0.

- Exogenous variables of observation period:

$$\Xi^{\mathrm{r}} = \left\{A_t^1, A_t^2, \gamma_{\mathrm{t}}, \mathrm{z}_t, \eta_t, \tau_t^c, \tau_t^h, \tau_t^p, \tau_t^k, \tau_t^d, \kappa_t^I, \Omega_t^T, \Omega_t^I, \Omega_t^L\right\}_{t=1}^{T}$$

- Exogenous variables of extrapolation period:

$$\Xi^{\mathrm{f}} = \left\{A_t^1, A_t^2, \gamma_{\mathrm{t}}, \mathrm{z}_t, \eta_t, \tau_t^c, \tau_t^h, \tau_t^p, \tau_t^k, \tau_t^d, \kappa_t^I, \Omega_t^T, \Omega_t^I, \Omega_t^L\right\}_{t=T+1}^{S-1}$$

- Exogenous variables of steady state:

$$\Xi^{\mathrm{s}} = \left\{A_t^1, A_t^2, \gamma_{\mathrm{t}}, \mathrm{z}_t, \eta_t, \tau_t^c, \tau_t^h, \tau_t^p, \tau_t^k, \tau_t^d, \kappa_t^I, \Omega_t^T, \Omega_t^I, \Omega_t^L\right\}_{t=S}$$

Step 2: The values of the steady-state variables $\{k_{ss}^{T1}, k_{ss}^{T2}, k_{ss}^{I}\}$ that satisfy all steady-state conditional equations in Section 2.7 are obtained under the values of the steady-state exogenous variables, Ξ^{s}.

Step 3: Given the initial values of the state variables, $\{\hat{k}_t^{T1}, \hat{k}_t^{T2}, \hat{k}_t^{I}\}$, and the time series of the exogenous variables, Ξ, the values for $\{h_1^1, h_1^2;\ h_1^1, h_1^2 \in [0,1], h_1^1 + h_1^2 \leq 1\}$, $(h_0^1 + h_0^2 \leq 1)$ are selected from the grid for the control variables of the initial period. Then, the values of the endogenous variables, $\{h_2^1, h_2^2, \hat{k}_2^{T1}, \hat{k}_2^{T2}, \hat{k}_2^{I}\}$ are solved so that equations (21) to (33) hold. Here, the nonlinear simultaneous equations will be solved for a numerical solution.

Step 4: Given the state-variable previous-period values $\{\hat{k}_t^{T1}, \hat{k}_t^{T2}, \hat{k}_t^{I}\}$, the endogenous variable current-period values $\{h_t^1, h_t^2\}$, and the exogenous variable time series Ξ, the endogenous variable values $\{h_{t+1}^1, h_{t+1}^2, \hat{k}_{t+1}^{T1}, \hat{k}_{t+1}^{T2}, \hat{k}_{t+1}^{I}\}$ are solved so that equations (21) to (33) hold. This is repeated from t = 2 to t = S – 1 in order to obtain $\{\hat{k}_S^{T1}, \hat{k}_S^{T2}, \hat{k}_S^{I}\}$. If midway through this process, the numerical solution of the endogenous variables cannot be obtained, the calculation is stopped when the value diverges to a sufficiently large value or to a sufficiently small value close to zero.

Step 5: The initial values, $\{h_1^1, h_1^2\}$, of the control variables that generate $\{\hat{k}_S^{T1}, \hat{k}_S^{T2}, \hat{k}_S^{I}\}$ satisfy the following converging condition on the grid:

$$\left[\left(\frac{\hat{k}_{ss}^{T1} - \hat{k}_S^{T1}}{\hat{k}_S^{T1}}\right)^2 + \left(\frac{\hat{k}_{ss}^{T2} - \hat{k}_S^{T2}}{\hat{k}_S^{T2}}\right)^2 + \left(\frac{\hat{k}_{ss}^{I} - \hat{k}_S^{I}}{\hat{k}_S^{I}}\right)^2\right]^{\frac{1}{2}} \approx 0.$$

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