The Securities-Correlation Risks and the Volatility Effects in the Japanese Stock Market*

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Abstract

This article decomposes the market volatility risks into the variance components of individual securities and the correlation components regarding upside and downside risks, defining and analyzing the corresponding sub factors to those components, and then studies the nature of the volatility factors such as pricing and explores the relationship between the sensitivity to the market average volatility and the volatility effects.

First, I propose the method of decomposing volatility into the above-mentioned four parts as volatility’s approximate descriptions. Then I apply this approach to the Japanese stock market and can get a high approximate accuracy.

When I analyze the Japanese stock market by using these sub factors, it turns out that the fluctuations of the market volatility are likely to be the risk factors accompanied by the assessment of premiums, and that they are consistent with the volatility effects. They are regarded as showing the demand for hedging against the increase in the market volatility, and the fluctuations in the volatility levels of individual securities are assessed as significant about both upside and downside risks, although to a different degree from each other. On the other hand, although the fluctuation risks of the correlation levels is assessed as not so significant, there is a case where it is assessed as significant as the demand for hedging at the time of high correlations due to stock price drops, and this tendency has become more obvious recently. I study the strategies making use of these natures in our analysis on the future returns. High volatility securities that tend to produce low returns due to the volatility effects rise independently in the bull market and drops together with many other securities in the bear market, showing that I cannot expect the effects of hedging on these securities.

Key words: volatility effects, downside risks, upside risks, average individual securities variance, average correlation coefficient, volatility decomposition

JEL classification: G12

I. Introduction

There are empirical evidences of stock markets in many countries that low volatility

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stocks tend to produce higher return than high volatility stocks, known as anomaly against standard evaluation theories to assume reward suitable for a risk, called the volatility effect. As reported in Ishibe, Kakuda, Sakamaki [2009], many studies use the return differences among the volatility ranked portfolios for evidences of the volatility effect. In addition, the idiosyncratic volatility anomaly is reported by a similar method in Ang et al. [2006, 2009]. These studies show a property that the high risk stocks are low returns. It means if low risk stocks are relatively high return, then it is an attractive property for investment.

As the case with the Lehman shock or the Great East Japan Earthquake disaster, the volatility of the many stocks often rises suddenly at the same time in the recent stock market. Therefore, it can be imagined with importance the increasing need for the action to control excessive risk for investors. In response to this demand, asset management companies trying to provide a new product that is lower-risk than before is increasing. For the tendency of recent years, the products using the volatility effect with a property of low volatility and high return increase remarkably. They decide the possession ratio of stocks by the quantitative technique that these products can statistically expect low risk. For instance, “the global minimum variance portfolio,” “risk parity portfolios” and “equal-weighted portfolios” are categorized of these products. There are some traditional active investment approaches to achieve low-volatility and high return strategies. They assume that the stocks of low business risk companies are likely to be low-volatility. The fund managers invest then in the stocks where business risk is low. As a point of view evaluating the lower business risk companies, they belong to types of industries considered to be the domestic and defensive demand or they get a stable position for reasons of having an overwhelming technique in the industry. These ideas depend on the individual stocks’ characteristics such as the property of the return of individual stocks with low volatility and stability of the revenue base of the individual companies.

On the other hand, investment funds are usually the portfolios which incorporate many stocks. In other words, the main risk management target is the volatility of a portfolio, not an individual stock. Well known in modern portfolio theory, the volatility of the portfolio is decided by the variances of individual securities and those correlations. However, in the strategies of “risk parity portfolios” and “equal-weighted portfolios,” correlations are not evaluated and are not used. The difference of investment performances by the difference of the construction policy of the portfolio, to invest in stocks decided by only lower volatilities, or lower volatilities and the lower correlations, is not argued enough.

Furthermore, it is also important that we investigate a time-variation of correlation. When the profit by the volatility effect rises with a low risk as an individual stock, if the correlations between stocks change and holds a lot of high-correlated stocks in the portfolio subsequently then the portfolio becomes high-risk. It is different from the property that investors who want to reduce risk expect. Particularly, a rise of volatility and the correlation of the whole market are observed at a time to be called “a shock” and “a crisis” from market participants, and the risk of the portfolio is more likely to rise. In those cases, the stocks with low-volatility and stable low-correlation are relatively high-return, so the stocks become the hedge of the whole
portfolio. Hedge demand can occur for the stocks as the hedge of the loss to occur when a change of correlative structure extends broadly. Conversely, it is thought that the stocks in which the correlation with other stocks is unstable have low demand, because risk control is more difficult than the stocks in which correlation is stable. Therefore, the variation of the correlative structure of the market may become one of the risk resources in the portfolio, it is significant to make quantitative analysis. In the previous studies, they often provided evidence of the volatility effect using the return of the rank portfolio for the volatility. They usually build those portfolios as equal weighted portfolios or capitalization-weighted portfolios. Those constitution-weights are decided without considering return correlations. It is important that we understand the influence of the correlation on volatility effect deeply from a practical point of view to design attractive investment products. It seems to be provided knowledge for asset management, such as “a shock” and “the crisis,” for the period investors request to reduce risk more. In these years, as risk-reducing needs of investors increase and investment products by the volatility effect are more popular, there is an urgent need for the study of the volatility effect when volatilities and correlations of the whole market increase.

Furthermore, we can easily imagine that people have particularly strong risk-reduction needs for a stock market plunge. In other words, it is thought that the hedge demand is asymmetry whether the average return of the market is a plus or minus and may influence risk structure. In Ishibe, Kakuta, Sakamaki [2011], they decompose a volatility structure into upside and downside risks and they find that the volatility effect may be influenced by low returns of stocks with high upside-risks. And they argue that the asymmetry of upside and downside risks are related with return reversal. In Ilmanen [2012], if investors would prefer a positively skewed distribution, they also prefer the thin left-tailed and thick right-tailed return distribution, then they pointed out the possibility of the difference in distribution with positive return and negative return. They explain that the thin left-tail means an insurance that can be explained as rational hedge demand, and the thick right-tail means a lottery that is difficult to explain rationally. Based on the point of view of these previous studies, we investigate the influence of the variation of the correlation by the decomposition of the upside and downside risk.

Furthermore, I examine the relations with the return reversal by comparing short-term properties with long-term stable properties.

With the above motivations, I investigate the following themes in this article.

1. Does the correlation structure have time series variation?
2. Is the variation of market volatility recognized as a risk factor to require premium by investors?
3. Is it the upside volatility or the downside volatility where premium is required?
4. When the variation of market volatility is recognized as a risk factor, which individual volatility variation and correlation variation of a premium is required?
5. For the upside volatility, is a premium required for individual volatility variation and correlation variation?
6. For the downside volatility, is a premium required for individual volatility variation
and correlation variation?

(7) Is there a difference in the stable return property about the market average volatility sensitivity and the temporary property?

The constitution of this article is as follows.

In section II, I review previous studies and describe the volatility decomposition method of this article and calculate average stocks’ volatility and average stocks’ correlation for upside and downside risks and check a statistical property. In section III, I analyze the characteristics of the volatility premium using the sub factors of volatility made from the volatility decomposition introduced in section II. From relations with the returns of the near future as the volatility effect and the premium on factor loading estimated by long-term data, I consider the cause of the volatility effect. In section IV, I investigate the relations with the volatility effect and the components of volatility, sub factors, and using ranked portfolios. Some concluding remarks are provided in section V.

II. The sub factors of the volatility risk of the portfolio

II.1. The average variance and the average correlation generated by the decomposition of aggregated market variance

In this article, after I develop the average variance (AV) and the average correlation (AC) generated by the decomposition of aggregated market variance and semi-variances, I estimate the factor loadings to report relations of stock returns.

Pollet and Wilson [2010] introduce approximation of aggregated market variance equal to the average variance multiplied by the average correlation, that variances of all stocks are assumed equal. They argue that innovations for the average correlation are negatively correlated with aggregate stock return shocks and forecast future market excess returns about large-cap stocks in the US, while the average variance has no forecasting power. They suggest that the average correlation has high premium, but the average variance has zero. In Chen and Petkova [2012], they expand their sample including small stocks and report that the idiosyncratic volatility effect depends on average variance, not average correlation, and the risk premium variation are mainly contributed by average variance. In this study, I define the factors of average variance and average correlation as these literatures. Pollet and Wilson [2010] defined the AV for simple average, but I apply capitalized weighted average in this study. This method is consistent to our interest to the variation of whole market volatility as the purpose of this study. As well as total risk, I define sub factors generated by the decomposition of aggregated factors, the average variance of upside risk (AV⁺), the average correlation of upside risk (AC⁺), the average variance of downside risk (AV⁻) and the average correlation of upside risk (AC⁻) to examine the relations with the premium difference of upside and downside risks such as shown in Ishibe, Kakuda, Sakamaki [2011]. These sub factors are discussed in II-3.
Here, I define the sub factors AC and AV by the decomposition of aggregated portfolio variance $V$ and under the standard assumption,

$$V = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j$$  \hspace{1cm} (1)

where $w_i$ is the weight of stock $i$ ($i = 1, \ldots, N$) at time $t$ in the portfolio where the weights must sum to one. $\rho_{ij}$ is the correlation coefficient between stock $i$ and $j$. Volatility $\sigma_i$ is the standard deviation of the return of stock $i$.

I express the standard deviation of individual stocks as sum of the average in the capitalization weighted average variance $\sigma_{\text{avg}}$ and the difference with its average $\sigma_{\text{g}}$.

$$\sigma_i = \overline{\sigma} + \epsilon_i.$$  \hspace{1cm} (2)

Substitute equation (2) into equation (1),

$$V = \overline{\sigma}^2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} + \overline{\sigma} \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \left( \epsilon_i + \epsilon_j \right) + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \epsilon_i \epsilon_j$$  \hspace{1cm} (3)

We use the right side first term ($V_a$) as an approximation of the left side of equation (3). Assuming that all stocks have the same variances, this approximation would be equal. In this approximation equation, aggregated portfolio variance would be the product of the capitalization average of variance $\overline{\sigma}^2$ and the capitalization average of correlation $\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij}$. If we apply the market capitalization weight to $w_i$, aggregated market variance $V$ can approximate to $V_a = AC \times AV$.

Then I explain how to calculate these factors in the empirical test. I use all listed stocks in the First Section of the Tokyo Stock Exchange (TSE1) for the proxy of market. First, I calculate standard deviation $\sigma_{i,t}$, $\sigma_{j,t}$ and correlation $\rho_{i,j,t}$ at $t$ given by daily excess returns of stock $i$, $j$ in month $t$. Weight $w_{i,t}$ and $w_{j,t}$ are the ratio of individual stock’s capitalization to aggregated capitalization in TSE1 at the end of month $t$. Therefore, the aggregated variance $V_s$, the average variance $AV$s, and the average correlation $AC$s are monthly data. These data are total stocks listed and total returns from January 1992 to September 2012 from the Nikkei-NEEDS database. I show the time series of monthly $V$, AV and AC in Figure 1. The vertical axis is logarithmic scale.

In Figure 1, $V$, AV and AC are all fluctuated over time. The level of the $V$ and AV is between around 0.1 and much more then 10, and the fluctuation range is more than 100 times. Because the AC is a correlation coefficient, it must be a value between 1 out of -1. All ACs are positive in the sample period, but the value fluctuates from around 0.1 to 0.9. For a tendency of monthly changes, we find both AV and AC fluctuate in the same direction as $V$.

The correlation coefficients table is listed in the left upper part of Figure 1. There is a very high correlation of 0.898 between AV and V. Moreover, their values are almost the same.
level throughout Figure 1, obviously. On the other hand, the correlation coefficient of AV and AC is relatively small, 0.285. For example, their variations are different in the well-known high market risk periods. The most indent remarkable sample is in several years of the IT bubble around 2000, AC remained low-level, but V rose by increases of AV. It’s thought there was a surfacing of the bad-loan problem of the financial institutions and the influence of the Internet bubble, because specific stocks are bought or sold by advanced selection, AV increased and the AC decreased. After the burst of the IT bubble, the macroeconomic environment was improved and the economy had begun to expand, and many stock prices were stable, then AV decreased a little, but AC increased, and V remained at the same level.

After 2005, the so-called Goldilocks Economy in the United States, V suddenly decreased with a drop of the AV. When the Lehman shock of 2008 and Great East Japan earthquake disaster of 2011 happened, V increased suddenly. AC did not rise in the former so much, but jumped in the latter. We find that AC fluctuates in this way and confirm that correlative structure is changed over time.

II.2. The relations of the variation of AC and the investment strategies

In the previous section, we find that the correlation structure fluctuates over time and V fluctuates by the increase and decrease of the AC being different from the variation of the AV. It means that even if the increment of the market risk is equal, the increment of the AC may not be equal. When the market risk increase by a large increment in AC, because the correlation of the market return and individual stock returns increases, the market risk sensitivity about most stocks may rise. In other words, the efficiency of diversified investments will fall
sharply at such a time. In consideration of this point, we try to think about investment strategies. I suppose there are risk-averse investors and they like lower-risk investments. Such investors will expect to make diversified investments to control portfolio risk. The rise of the correlation is a risk for these investors to enjoy the effect of diversified investments because it is undesirable. In particular, they will strongly wish the hedge of such risk when the stock market decline. I try to explain this preference in risk sensitivity.

I think that the investors measure excess returns for their profit\(^1\). In addition, because we are interested in portfolio management, these investors use the risk hedge by diversified investments. We divide stock market situations into rising and declining price-movements and divide them into increasing and decreasing correlation-levels of individual stock returns, about rising and declining sensitivity (market \(\beta\)) of the market stock prices and about increasing and decreasing sensitivity (AC \(\beta\)) of the correlation, we try to examine desirable sensitivity strategies from the viewpoint of diversified investment effects. **Table 1** shows desirable sensitivity strategies in four situations. Note that high AC \(\beta\) means a return is higher when AC is increasing, but a return is lower when AC is decreasing. There is a big problem for these investors as to how to hedge their portfolio when the stock market declines. This situation is the row on the right-side of **Table 1**, and a low market \(\beta\) strategy is preferable. Particularly, in the situation of AC declining, you should perform a market \(\beta\) low strategy by a well-diversified portfolio, because diversified investments are effective. On the other hand, the effect of diversified investments decreases in the situation of AC rising, and market \(\beta\)s of the individual stocks rise. I guess that if I can invest high AC \(\beta\), it would help a hedge in this situation. Therefore, we find that an attractive strategy that we could expect stable profits from in the market is “the market \(\beta\) is low and AC \(\beta\) is high,” Vice versa, a not so attractive strategy is “the market \(\beta\) is high and AC \(\beta\) is low.” Then, the hedge of a market correlation increasing may interpret the hedge for the rise of market \(\beta\) at the same time.

**Table 1. The Sensitivity Strategies to become a High Return**

<table>
<thead>
<tr>
<th>AC</th>
<th>A stock market return</th>
</tr>
</thead>
<tbody>
<tr>
<td>rise</td>
<td>effect decrease</td>
</tr>
<tr>
<td></td>
<td>effect increase</td>
</tr>
<tr>
<td>decline</td>
<td>effect decrease</td>
</tr>
<tr>
<td></td>
<td>effect increase</td>
</tr>
</tbody>
</table>

\(^1\) In Ishibe, Kakuta, Sakamaki [2011], they assume the existence of investors for the absolute returns and excess returns not market relative returns seen in the benchmark of institutional investors and try to explain the low volatility effect.
However, for example, if there are very few stocks with high AC β for demand, those stocks become expensive and may be very low expected returns except for high AC periods. This is the same way of thinking as Ishibe, Kakuta, Sakamaki [2009] and a lot of documents reported that the stocks with high market β could expect leveraged high risk premium by low cost, but those stocks resulted in low returns empirically.

In the following empirical analysis, I examine the difference of return in these situations. For this purpose, I would do sensitivity analysis for stock price rising and declining situations, and check the preferability of high AC β stocks of the market decline situation.

II.3. The sub factors for upside and downside risk

To measure an individual stocks sensitivity for AC about the market-rising situations and the market-declining situations, about the definitions of the AV and the AC, I would like to define sub factors about upside and downside risks. For concise notations, without changing the letter expression for the risk categories, I decided to add subscripts (+) and (-) to express upside and downside risks.

Because upward risk and downward risk definitions that are used widely are similar to semi-variance, if we extend the expression of the deviation of the return to use in the definition about upside and downside risks, we can define the variance and the coefficient of correlation of the portfolio corresponding to upside and downside risks like volatility.

When I show deviation $d_i$, for return $R_i$ of individual stock $i$ concretely,

$$
\begin{align*}
\tilde{R}_i & \text{ is the period average return of } R_i. K \text{ is the threshold value of return between upside returns and downside returns. Ishibe, Kakuta, Sakamaki [2011] argue the threshold value and they insist that zero is a recommended value because of the high interpretability of risk-return relation, the consistency with prospect theory and easily intuitive understanding. This article also follows it.}

V_+, AC_+, AV_+, V_-, AC_- and AV_- are determined by using the upside partial return deviation of $d_i^+$ or the downside partial return deviation of $d_i^-$ in substitution for return deviation $d_i$ of the individual stock $i$ which is used to determine $V$, $AC$ and $AV$. Because these statistics are provided by decomposing volatility as a risk, I can consider them to be the components of the market risk. I call them “sub factors” in this article. After section III, I investigate stock returns by using volatility and these sub factors as risk explanation factors. Basic statistics of factors are exhibited in panel A of Table 2. Because the original series factor $s$ are variance statistics, they are extremely high when news is reported, such as when a market crisis occurs. They have distribution with very large skewness and kurtosis. Because these distributions transform to around normal distributions, I
Table 2. Approximate Precision of the Market Volatility by Sub Factors

Panel A  Basic statistic of monthly data of $V$, $V_a$, and sub factors

<table>
<thead>
<tr>
<th></th>
<th>Original Series</th>
<th>Log-transformed series</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$V$</td>
<td>0.784</td>
<td>1.209</td>
</tr>
<tr>
<td>$AV$</td>
<td>0.559</td>
<td>0.473</td>
</tr>
<tr>
<td>$AC$</td>
<td>0.381</td>
<td>0.135</td>
</tr>
<tr>
<td>$V^+$</td>
<td>0.912</td>
<td>1.505</td>
</tr>
<tr>
<td>$AV^+$</td>
<td>0.647</td>
<td>0.534</td>
</tr>
<tr>
<td>$AV^-$</td>
<td>0.365</td>
<td>0.142</td>
</tr>
<tr>
<td>$V^-$</td>
<td>0.767</td>
<td>1.218</td>
</tr>
<tr>
<td>$AV^-$</td>
<td>0.458</td>
<td>0.401</td>
</tr>
<tr>
<td>$AC^-$</td>
<td>0.432</td>
<td>0.182</td>
</tr>
<tr>
<td>$Va$</td>
<td>0.757</td>
<td>1.128</td>
</tr>
<tr>
<td>$Va^+$</td>
<td>0.848</td>
<td>1.314</td>
</tr>
<tr>
<td>$Va^-$</td>
<td>0.733</td>
<td>1.135</td>
</tr>
</tbody>
</table>

Panel B  Approximate precision of the market volatility $V$ by $V_a$ and sub factors

(Original Series)

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable (Va and sub factors)</th>
<th>Coefficient of determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$Va$ $V^+$ $V^-$ $AV$ $AC$ $AV^+$ $AC^+$ $AV^-$ $AC^-$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.060</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>0.481 $0.434$</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>1.963 $-0.433$</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>0.066 $0.494$ $2.299$ $-0.831$</td>
<td>0.824</td>
</tr>
<tr>
<td>$V^+$</td>
<td>1.927 $-0.301$</td>
<td>0.752</td>
</tr>
<tr>
<td>$V^-$</td>
<td>2.333 $-0.305$</td>
<td>0.830</td>
</tr>
</tbody>
</table>

Panel C  (Log-Transformed Series)

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable (Va and sub factors)</th>
<th>Coefficient of determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$Va$ $V^+$ $V^-$ $AV$ $AC$ $AV^+$ $AC^+$ $AV^-$ $AC^-$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.037 $1.043$</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>$-0.024$ $0.515$ $0.453$</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>1.242 $1.062$ $0.995$</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>1.246 $0.518$ $0.573$ $0.503$ $0.434$ $0.989$</td>
<td></td>
</tr>
<tr>
<td>$V^+$</td>
<td>1.300 $1.065$ $1.037$</td>
<td>0.991</td>
</tr>
<tr>
<td>$V^-$</td>
<td>1.266 $1.067$ $0.984$ $0.985$</td>
<td></td>
</tr>
</tbody>
</table>
logarithmically transformed them to the original series\(^2\). After log transformation, as we can see in the statistics, the distributions of the factors became near the representative distributions, TOPIX index of skewness (-0.082) and kurtosis (0.578) in same period and the normal distribution of skewness (0) and kurtosis (0). In the factor analysis of the stocks return, I use the logarithm transformed factors. We can guess a linear relation among \(V\), \(V_+\) and \(V_-\) because there is a linear relation \(d_i = d_{i+} + d_{i-} - \bar{R}_t\) among upside deviations, downside deviations and deviations\(^3\).

Noting the relation and the approximation \(\ln V \approx \ln V_a = \ln AV + \ln AC\), I guessed the linear approximation of aggregated market variance by sub factors

\[
\ln V \approx b_0 + b_1 \ln AV_+ + b_2 \ln AV_- + b_3 \ln AC_+ + b_4 \ln AC_-.
\]

To examine the influence of the logarithm transformation for \(V\) approximation, we consider linear regression without intercept term. In that, dependent variables are \(V\) or log-transformed \(V\) and independent variables are each of the original series sub factors or each of the log-transformed sub factors. Panel B of Table 2 shows the regression coefficients and the coefficient of determinations for the original series and panel C shows the log-transformed series. In Panel B, we find that each regression shows the high R-square then original series sub factors have high explanation power. However, the regression by \(AV\) and \(AC\) shows the opposite sign of coefficients. A coefficient of the \(AV\) is a positive and a large value around 2 whereas a coefficient of the \(AC\) is negative. There is an unstable suspicion for these coefficients, because these sub factors show positive correlation and the value of \(V\) may become extremely big in Figure 1. The results of upside and downside sub factors and the results of regressions for downside risk \(V_-\) and upside risk \(V_+\) show similar tendencies.

To distinguish log-transformed variables, I use a small letter in Panel C. We find that they are almost completely explained by meaning of R-square. These regressions may be stable because coefficients of \(av\) and \(ac\) are positive together. It is thought that the coefficient’s instability in the original system is caused by an extremely big volatility, and high approximation precision was provided by the appropriate handling of these data.

From the result of Panel C, I assess that the market variance factor \(V\) can decompose by 4 sub factors, \(AC_+\), \(AV_+\), \(AC_-\), and \(AV_-\), and perform the following analysis.

### III. Risk premium analysis by the linear model

I add the sub factors which explain the volatility effect to the linear multi-factor model which explains stock returns and investigate factor premium of those factors. First, the factor sensitivities of individual stocks are estimated by factor returns after the \((0, 1)\) standardization. Then, factor premiums are estimated as implied returns for the factor loadings. The average

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\(^2\) A correlation \(AC\) may be negative by definition and could not log-transform. But all ACs are positive throughout the sample period.

\(^3\) About the guessed linear approximation, the equality is attained if \(K = \bar{R}_t\).
of these implied returns may not be zero. I try to interpret the time changes of the factor premiums. In addition, I discuss the factor whose premium is significant may be an omission factor of the existing multi-factor model.

III.1. The analysis method

The risk premiums are investigated along the cross-sectional regression analysis of Fama-MacBeth (1973). I investigated the risk premium. This procedure has three steps.

In the first step, factor returns, $X_k$, of the k-th factor are estimated. The base multi-factor model is the 4-factor model (FF4) of the momentum included in the Fama-French model introduced in Carhart (1997). Therefore, market (mkt), market capitalization (smb), book-to-market ratio (hml), and 12 months return momentum (wml) are used for the risk factors. In the factor return of the market factor ($R_m - R_f$), I use the TOPIX total returns as the market return ($R_m$) and the return of the mean interest rate for the MUTAN call over-night rate as the risk-free factor ($R_f$). And I add the changes of sub factors ($\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$, $\omega_5$, $\omega_6$, $\omega_7$) to these 4 factors. Note that these variables that I added as additional factor returns are different from factors of FF4. The dimension of the independent variables matches the dimension of the dependent variables in the regression analysis because all 4 factors of FF4 are provided as stock portfolio returns in the market. On the other hand, absolute levels of additional sub factors about volatility are not related directly to the stock market because these sub factors are not portfolio returns but exogeneous variables of the market. Therefore, I use standard scores of the monthly differences of the log-transformed sub factors of volatility for a factor return. These standard scores are derived by subtracting the cross-sectional population mean from an individual raw score and then dividing the difference by the cross-sectional population standard deviation (the $(0, 1)$ standardization), then their distribution is zero mean and unit standard deviation. The absolute level influences for intercepts in the regressions are neutralized by being standardized.

In the second step, the risk factor $j$ loadings ($\beta_{ij}$) of stock $i$ are estimated by regressions of the monthly excess return series in factor $j$’s return series. I estimate the loadings for an all excess return during a sample period.

In the third step, I estimate factor risk premium ($\gamma_j$) by panel regressions throughout the periods and stocks. There are the excess returns of stock $i$ for an independent variable and risk factor sensitivities ($\beta_{ij}$) about factor $j$ of stock $i$.

III.2. The relations of FF4 factors and the sub factors

Relations with additional volatility sub factors and the FF4 factors are not clear, because sub factors are exogeneous variables. Therefore I assess statistical relations. The correlation coefficients throughout the sample period are exhibited in Table 3. I highlight coefficients which are more than 0.75 as the threshold of very high correlation. Most of the correlation
coefficients among the FF4 factors and the volatility sub factors are low and almost zero. However, the correlations among av, ac, av, av+, and ac, which are the decomposition of v, are around 0.5 (slightly high). The correlation between v- and v+ which are also the decomposition of v is at such a level. Furthermore, the correlations between v and sub factors are around 0.8 (very high). Considering the above mentioned points, the sub factors in this article seem to be worth mentioning as additional factors of the FF4 model. But when I adopt plural factors, we treat them as a set of factors, {v}, {v-, v+}, {av, ac}, {av-, ac-, av+, ac+}, in consideration of the decomposition. The aim of this sub factor grouping is controlling the mixture of sub factors with the same meaning for the stability of the risk premium estimation.4

III.3. The estimation of factor loadings (β)

As the result of the estimation in step 2, the means and the standard deviations of the risk factor loading (βi) are presented in Table 4. The value in the braces under the coefficient means the standard deviation of the coefficient. The factor loading value of each stock estimated through the sample period.

To investigate the stability of the coefficient, the factor loadings are estimated by 25 models in which adopting factors are different. Table A in the Appendix lists the model names and their factors. Because 13 models from factor model m0 to m12 are simple regressions of one factor, for the saving of space, factor loadings are displayed in one line in Table 4. Panel A provides equal weighted statistics and Panel B provides capital-weighted statistics for the calculations.

The factor loading of the FF4 factor is almost equal in having additional factors or not. And the signs of factor loadings are consistent between the simple regression and other regression analysis. Therefore, factor loading estimations are stable. On the other hand, the large standard deviations are shown at m18, m23, and m24, then factor loading estimations are not stable, because those models include the factors having high correlation.

4 The sample means of FF4 factors are -1.50%(mkt), -0.16%(smb), 2.89%(hml) and -1.56%(mom) par annum.

Table 3. The Correlation Coefficients among the FF4 Factors and the Volatility Sub Factors

<table>
<thead>
<tr>
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<th>smb</th>
<th>hml</th>
<th>mom</th>
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<tr>
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<td>1.00</td>
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<table>
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<tr>
<th></th>
<th>Δ v</th>
<th>Δ v-</th>
<th>Δ v+</th>
<th>Δ av</th>
<th>Δ av-</th>
<th>Δ av+</th>
<th>Δ ac</th>
<th>Δ ac-</th>
<th>Δ ac+</th>
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<tr>
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<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>hml</td>
<td>-0.07</td>
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<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>mom</td>
<td>-0.07</td>
<td>0.02</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The sample means of FF4 factors are -1.50%(mkt), -0.16%(smb), 2.89%(hml) and -1.56%(mom) par annum.
III.4. Risk premium analysis

Table 5 provides the result of the estimated factor risk premium ($\gamma$) throughout the sample period in step 3. The factor loadings in the regression are common in all periods by individual stocks. Therefore, it is expected that the bias of the premium estimation by the
short-term reverse effect is controlled. Such bias is seen using the factor loading estimated by the returns just before. The number of asterisks * shows the statistical level of significance being *** 1%, ** 5%, and * 10%.

The notable result is that the premiums of sub factor v, av and av are significantly negative and stable and these factors may be candidates of the omission factor of the multifactor model. As for this result, the increase in downside risk might be the factor which investors wish to hedge including a hedge demand in the market and it might be reflected by a stock price evaluation.

We can observe that the many premiums are negative. This means that the stocks in which prices seem to rise to increase the target risk have low return on average. In other words, there is a hedge demand. For example, because the hml factor has a historically positive factor return (2.89% par annum), the positive hml factor loading strategy brings positive profit. This means that there is not a demand to the strategy which brings a high return when the hml factor return is high, then the premium is significantly low. Therefore, there is reverse hedge demand for the hml factor. Similarly, the volatility factors which I added here have negative premium. This means that there is a demand for the strategy which brings a low return when volatility factor return is high. The v factor has negative premium. There is a hedge demand to the stock which brings a high return when volatility increases, such stock has a low average return. The ac is not significant, but the av is negative significant. It’s consistent with the argument of Chen and Petkova [2012]. By the comparison of upside and downside risks, the upside risk premium v is not significant and the downside risk premium v is significant negative. The stocks which get the high return when downside volatility increases have hedge demand. They are consistent with the argument of Ishibe,
Kakuta, Sakamaki [2011].

In four decomposed sub factors, the most significant was \( \Delta v \), followed in order by \( \Delta v^- \), \( \Delta v^+ \), and \( \Delta c^- \). Hence, I find that about the risk premium, the variance of individual stocks is more important than correlation, then the downside risk is more important than the upside risk.

In addition, the level of significance is 10%, but the demand seen in \( \Delta c^- \) and \( \Delta c^+ \) is not so,
as I assumed in Table 1. Table 5 exhibits the result of the equal weighted regression, but Table B in the Appendix exhibits the result of the capital weighted regression. The general characteristics are the same, but ac_ is negative and significant, and ac_ was not significant. It is interesting that the ability to follow when market rise is evaluated on large-capital stock. In large-capital stock, the strategy to hedge a rise of the correlation (ac ) at the time of market fall is not evaluated, it suggests the possibility that the investor can constitute a hedged portfolio without it becoming expensive.

It is clear that the quantity of discounts by hedge demand are estimated differences between the premium (\( \gamma \)) and the factor return (X). I estimated the monthly risk premium variations by monthly excess returns as the dependent variable. Figure 2-1 shows the accumulated value of monthly risk premiums (\( \gamma X \)), \( \Delta v \), \( \Delta v_+ \), and \( \Delta v_- \); Figure 2-2 shows \( \Delta av_- \), \( \Delta av_+ \), \( \Delta ac_- \), and \( \Delta ac_+ \).

\( \Delta v \) in Figure 2-1 has declined throughout. The stock whose return is high at the time of rising market volatility is relatively low in return. Therefore, the demand decreases in the long term. \( \Delta v_+ \) has not declined to long-term, then low demand are the stocks that returns are provided at the time of increase of downside risk. But after the IT bubble Lehman shock, and Great East Japan Earthquake disaster, the demand increased. We can read a similar tendency in Figure 2-2. We find that the high return stocks when the market rises and correlation increases is in demand. After March 2011, the stock prices rose more than hedge demand. As shown in Figure 1, the Great East Japan Earthquake disaster resulted in a fall in the market with increasing AC, then it was thought that the high AC \( \beta \) stocks were continuing in demand as shown in the factor strategy at the top right corner of Table 1.

III.5. The relations of factor loadings and future returns

In this sub section, I investigate the influence of the ex-post evaluation of sub factor loadings and try to examine the relations with the volatility effect.

In the previous sub section, we examine the characteristics of the long-term premium of stocks estimated by the risk factor loadings throughout the sample period. This premium can be considered the expectation return of the buy and hold strategy, if the investor could know average properties of individual stocks from past to future. Because the investor could know only past characteristics with fact, I check the out of sample strategy.

I investigate return relations between individual stock returns of next month and risk factor loadings estimated by past information. In addition, the volatility effect is also the relation between past return volatility and with the near future returns. Note that, unlike the previous sub section, the analysis includes the influence of the price fluctuation such as the short-term return reversal and momentum.

Specifically, I estimate risk factor loadings by the return data froma past 60-month moving window, and then I perform the cross-sectional regression analysis using these factor loadings as independent variables and the excess return of the next month as dependent variables. Only stocks whose return data was more than the past 30 months were available
Table 6. The Mean of the Factor Return in the Next Month using the Factor Loadings Estimated by Past Return Data

<table>
<thead>
<tr>
<th>mkt</th>
<th>smb</th>
<th>hml</th>
<th>mom</th>
<th>Δav−</th>
<th>Δav+</th>
<th>Δac−</th>
<th>Δac+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.058%</td>
<td>0.008%</td>
<td>0.202%</td>
<td>−0.153%</td>
<td>−0.035%</td>
<td>−0.071%</td>
<td>0.019%</td>
<td>−0.006%</td>
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<tr>
<td>(0.204)</td>
<td>(0.066)</td>
<td>(1.777)</td>
<td>(−0.884)</td>
<td>(−0.977)</td>
<td>(−2.493)</td>
<td>(0.008)</td>
<td>(−0.208)</td>
</tr>
</tbody>
</table>

Figure 3. The Cumulative Factor Return (Δav−, Δav+, Δac−, Δac+).

estimated factor loadings. Therefore, the next month return that I used for a regression analysis was from January 1995 to September 2012. Table 6 lists the mean of the factor return in the next month of the m22 model (the independent variables are mkt, smb, hml, mom, Δav−, Δav+, Δac−, and Δac+). The t value is enclosed in parenthesis.

As is well known, the factor return of the hml factor is stable positive. The factor return of Δav− factor is more stable negative. The stocks which achieve large positive returns when the variance of the upside risk increase in a market are significantly low returns. About the in sample relation in Table 5, both av− and av+ are significant negative. But about the out of sample relation in Table 6, only av− is significant negative. It is natural by the return reversal effect even if it became the large av− factor loading for the large negative return before. However, the future return of the stock with the large av− factor loading may be stable negative. Therefore, a short-selling strategy using the stock with the large av− factor loading may be highly profitable. This is consistent with a result of Ishibe, Kakuta, Sakamaki [2011] who pointed out the low return of stocks that are high upside risk similar to the return reversal effect as return properties of the individual stock. In this article, this effect generated from systematic factors by market volatility variation is different.

To observe a characteristic by time, Figure 3 exhibits cumulative factor returns of each month for four sub factors (Δav−, Δav+, Δac−, Δac+). As Figure 1, a different variation is seen when volatility of the market increases, for example, the Lehman shock of September 2008 and the Great East Japan Earthquake disaster of March 2011. A high factor return is observed
in the former in $\Delta \text{av}$, $\Delta \text{ac}$, and $\Delta \text{ac}_+$, but the latter is not seen in the big change. But $\Delta \text{av}$, and $\Delta \text{ac}$ are positive trend after March, 2011 and are continued after the Lehman shock in $\Delta \text{ac}_+$. It is thought that these trends meant the investors’ preference for high return stocks when the downside risk of market is increasing or the market is in decline with high aggregated correlation. This result is consistent with the result of Figure 2-2 using the factor loading estimated by throughout the sample period. The risk was that the correlation of the market increases was not evaluated very much. But because the investors experienced that increase of correlation at the market collapse brings a serious increase of risk, the evaluation of the stock to have a hedge characteristic at such a time was re-evaluated. If it is on the re-evaluation road now, because the demand for high ac factor loading (high ac $\beta$) stocks is stable, it will be evaluated and expensive and it will be more likely to be low profit like the stocks where factor loading for av is high (high av $\beta$).

IV. The relations of the volatility effect and the sub factors

Many studies use the future return differences among the volatility ranked portfolios for evidences of the volatility effect. Here I check the influence of sub factors on the volatility effect using the volatility ranked portfolio. Unlike the previous section, we can observe a non-linear characteristic with the ranked portfolio.

Figure 4 exhibits the characteristics of the 10 ranked portfolios by the volatility level of the return for the past 60 months. The bar chart shows the average of the sub factor loadings in a ranked portfolio. The line chart shows the average return in the last month and next month. We can observe the volatility effect with the ranked portfolios which are more volatile than 4th rank, the mean of next month’s returns is low so that volatility is high (the right axis).
The sub factor loading level tends to increase and decrease with the rank. This fact suggests the possibility that we can explain the volatility effect in the aggregated variance factor. The volatility level is consistent with the upside risk factor loading level. Therefore, the volatility effect has possibilities caused by the sharp fall of the stocks whose prices rose and the correlation of the whole market. In other words, the large positive factor loading of the aggregated upside variance (av+) whose factor return is significant negative may be the cause of the volatility effect. The influence of the aggregated upside variance (av+) is larger than the aggregated upside correlation (ac+). About the downside risk, the aggregated downside correlation factor loading (ac-) is as the same level as the aggregated downside variance factor loading (av-) after the 7th rank. In recent years, the demand for higher factor loading of av- and ac- was observed in previous section, but I was not able to read the relations with the volatility effect here. The high volatility stocks had higher factor loading for all sub factors than the low volatility stocks. Therefore, the low return of high volatility stocks may occur by a sharp fall of the volatility and the correlation of the whole market. It is thought that the causes of the volatility effect of high volatility stocks are the return reversal of individual stocks, the volatility reversal and the correlation reversal of the whole market. In section III-4, I pointed out the hedge demand for downside risk to be shown as negative av- factor loading as a long-term cause of the volatility effect. In addition, I pointed out these short-term effects in this section.

The result was similar with β of the market model or idiosyncratic volatility of the FF4 model in substitution for volatility of the return for the past 60 months.

V. Conclusion

This article decomposes the market volatility risks into the variance components of individual securities and the correlation components regarding upside and downside risks, defining and analyzing the corresponding sub factors to those components, and then studies the nature of the volatility factors as pricing one and explores the relationship between the sensitivity to the market average volatility and the volatility effects.

I propose the method of decomposing volatility into the above-mentioned four parts as volatility’s approximate descriptions. Then I apply this approach to the Japanese stock market and can get a high approximate accuracy and the low-distortion distribution properties.

I analyze the Japanese stock market by using these sub factors. I find that the average aggregated correlation is almost positive, but it increases significantly in “a shock” or “a crisis” and is suggested fluctuated correlation structure. It turns out that the fluctuations of market volatility are likely to be the risk factors accompanied by the assessment of premiums, and there is possibility of the additional factor of the FF4 model. Furthermore, the premium of the stocks that a positive return was provided for a rise of market volatility was negative on average during a sample period. It is consistent with the volatility effects. The premium is mainly caused by downside risk factor loading, but high volatility itself is caused by high upside risk factor loading. The variation of individual stock variance is more evaluated in the
whole market as volatility risk than the variation of correlation. About individual stock variance and correlation, the sub factors of the downside risk are evaluated more than ones of the upside risk. The influence of the downside risk for long-term individual stock properties is significant, but the negative influence of the upside risk for short-term stock price properties is more significant. I think that the former is the hedge demand for downside risk, and the latter is a reversal effect of individual stock price, the market return and the market correlation from the reaction that overestimated.

On the other hand, although the fluctuation risks of the correlation levels is assessed as not so significant, there is a case where it is assessed as significant as the demand for hedging at the time of high correlations due to stock price drops, and this tendency has become more obvious recently. The low expected return of the stocks which get high return when the upside variance of individual stocks in the market increases will continue in the future, It may be achieved that the low return of the high upside risk stock affects volatility effect as an evaluation in systematic factor. The stocks which could hedge an increasing of the market correlation at the time of the market fall have not been evaluated much. However, an evaluation continues to increase after the Great East Japan Earthquake disaster, the demand may be increased. If demand is stable, the stocks would be expensive and have low expected return.

In this article, I report the influence to the valuation caused by the variance-covariance structural change, especially the variation of individual volatility levels and individual volatility levels for upside and downside risk. These are only an example of the decompose method. It is a topic for future research, to for ther clarify the pattern of causes and changes and, to perform analysis on the basis of these mechanisms that make evaluations in the stock market clear.

References


### Table A. Models and Factors

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### Figure B. Estimated Risk Premium (Capital Weighted)

The table above shows the estimated risk premium for different models and factors. The models are labeled m0 to m24, and each model has a set of parameters estimated for various factors such as Δv. The table reflects the results of a simple regression analysis.