資料③－３

Optimal Fiscal Policy Rule to achieve Fiscal Sustainability and Economic Growth

Naoyuki Yoshino
Dean, Asian Development Bank Institute (ADBI)
Kasumigaseki Building 8F
3-2-5 Kasumigaseki Chiyoda-Ku Tokyo 100-6008 Japan
nyoshino@adbi.org
and

Tetsuro Mizoguchi
Associate Professor of Economics,
Reitaku University 2-1-2 Kashiwa-shi, Chiba, 277-8686, Japan
tmizo@reitaku-u.ac.jp

3. Revised Domar condition and the Bohn' condition combining with the bond market

The Domar condition and the Bohn's condition are often used to determine whether budget deficits are sustainable or not.

The Domar condition is obtained from government budget constraints as follows.

$G_t + r_t B_{t-1} = \Delta B_t + T_t$ Government budget constraint (1)

Equation (1) states that government spending (G_t) ＋interest payments $(= r_t B_{t-1})$

＝new issue of government bonds (ΔB_t) ＋tax revenue (T_t)

Divide Equation (1) by GDP (Y_t) and rewrite Equation (1)

$b_t - b_{t-1} = \frac{(r_t - \eta_t)}{1+\eta_t} b_{t-1} + g_t - t_t$ The Domar Condition[1] (2)

where $b_t = B_t / Y_t, \eta_t = \Delta Y_t / Y_t, g_t = G_t / Y_t$, and $t_t = T_t / Y_t$

If $r_t > \eta_t$ then b_t will become larger and larger, namely the budget deficits explode.

If $r_t < \eta_t$ then b_t will become small and smaller namely the budget deficits converge.

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[1] We also mention that we can derive the Domar condition from our government objective function (Equation (?)). From this equation, $\frac{\partial L}{\partial b_{t-1}} = 2w_5 \frac{\partial \Delta b_t}{\partial b_{t-1}} = \frac{r_t - \eta_t}{1+\eta_t}$ setting up other policy weights as zero, *i.e*, $w_i = 0, i = 1, \ldots, 4$.

Equation (1) denotes the government budget constraint which describes the supply of government bonds. Dividing Equation (1) by Y_t and a few transformation, the Domar condition in Equation (2) is obtained. If the interest rate is higher than the growth rate of the economy, budget will explode. On the other hand, if the interest rate is lower than the growth rate of the economy the budget deficits will converge into stable manner. However, the Domar condition focuses only on the supply of government bond and it does not take into account of the demand for government bonds.

4. Mathematical Model

The paper describes the equation of both the Government Bond Supply and the Government Bond Demand based on the model set by Yoshino and Mizoguchi (2010, 2013). Our model is summarized as follows:

$G_t + r_t B_{t-1} = \Delta B_t + T_t$ Government Budget Constraint=Supply of government bonds (3)

A simple macro model which includes the demand side of the government bond can be constructed as follows.

Equation (3) is the disposable income where wage income, transfer payment from the government and interest receipt from the government bonds minus tax payments. The disposable income is distributed consumption and savings. Savings are allocated to purchase of government bonds, increase in domestic deposits and investment into foreign countries. For simplicity, the foreign assets holdings is regarded as exogenous.

The disposable income is defined as income (Y_t) plus government transfer to households (θG_t) plus interest receipt of government bond ($r_t B_{t-1}$) by households minus tax payment (T_t) as follows. The disposable income is divided into consumption (C_t) and savings (S_t)

$YD_t = Y_t + \theta G_t + r_t B_{t-1} - T_t = C_t + S_t$

where $S_t = \Delta B_t + \Delta W_t^D - \Delta W_t^F$ (4)

Savings(S_t)=Government bonds(ΔB_t)+Domestic Deposits(ΔW_t^D)–Foreign assets(ΔW_t^F)

$\Delta W_t^F = \Delta \overline{W}^F$ Foreign assets are assumed to be constant (5)

$C_t = c_0 + c_1 YD_t$ Consumption Equation (6)

$W_t^D = d_0 + d_1 YD_t + d_2 \Delta r_t$ where $\Delta r_t = r_t - r_{t-1}$ Deposit Equation (7)

Consumption depends on the disposable income for simplicity. The deposit market is expressed as the supply of deposits and the demand for deposits in equation (7). The demand for deposits is explained by the disposable income and the interest rate of government bonds. If the interest rate on government bonds increases, households want to buy much more government bonds and reduce the amount of deposits. All the deposits are used for bank loans to satisfy investment

From equations (4)-(7), the demand for government bonds can be obtained as follows.

$B_t = (1 - c_1 - d_1)(Y_t + \theta G_t - T_t) + (-d_2 + (1 - c_1 - d_1)B_{t-1})r_t - d_2 r_{t-1} + B_{t-1} + W_{t-1}^D +$
$(\Delta \overline{W}^F - c_0 - d_0)$ (8)

By using equation (3) and (4), we obtain temporally equilibrium as follows:

$$r_t = \frac{1}{d_2 - (c_1 + d_1)B_{t-1}}[(1 - c_1 - d_1)Y_t - W_{t-1}^D - \Delta \overline{W}^F - (c_1 + d_1)T_t - (1 - \theta + (c_1 + d_1)\theta)G_t + (c_0 + d_0) + d_2 r_{t-1}] \qquad (9)$$

$$B_t = \left[\frac{(1 - c_1 - d_1)B_{t-1}}{d_2 - (c_1 + d_1)B_{t-1}}\right] Y_t + B_{t-1} + \left[\frac{d_2 - (1-\theta)(1 - c_1 - d_1)B_{t-1}}{d_2 - (c_1 + d_1)B_{t-1}}\right] G_t - \left[\frac{d_2}{d_2 - (c_1 + d_1)B_{t-1}}\right] T_t + \left[\frac{B_{t-1}}{d_2 - (c_1 + d_1)B_{t-1}}\right](c_0 + d_0 - W_{t-1}^D - \Delta \overline{W}^F) + \left[\frac{d_2 B_{t-1}}{d_2 - (c_1 + d_1)B_{t-1}}\right] r_{t-1} \quad (10)$$

and in the long run, we can obtain steady state level of the government bond and interest rate be setting $\Delta B_t = 0$.

$\overline{B_t} = \frac{d_2}{(c_1 + d_1)}$ is the steady-state equilibrium level of government debt. (11)

$\overline{r_t} = \frac{(1 - c_1 - d_1)Y_t^f - \overline{W^D} - \overline{W}^F + (c_1 + d_1)\overline{T} - (1 - \theta + (c_1 + d_1)\theta)\overline{G} - (c_0 + d_0)}{1 + d_2}$ is the steady state equilibrium level of interest rate. (12)

Now we check the necessary and sufficient condition of our local stability of steady state equilibrium. From Equation (7) and (8), the system of the equations are described by the following:

$$\begin{pmatrix} b_t \\ \gamma_t \end{pmatrix} = \begin{pmatrix} \overline{r_t} + 1 & 0 \\ -d_2 + (1 - c_1 - d_1)\overline{r_t} + 1 & -d_2 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \qquad (13)$$

where $b_t = B_t - \overline{B_t}$ and $\gamma_t = r_t - \overline{r_t}$. Equation (13) will be rewritten as:

$$\begin{pmatrix} \Delta b_t \\ \Delta \gamma_t \end{pmatrix} = \begin{pmatrix} \overline{r_t} & -1 \\ -d_2 + (1 - c_1 - d_1)\overline{r_t} + 1 & -d_2 - 1 \end{pmatrix} \begin{pmatrix} b_t - b_{t-1} \\ r_t - r_{t-1} \end{pmatrix} \qquad (14)$$

where $\Delta b_t = b_t - b_{t-1}$ and $\Delta \gamma_t = r_t - r_{t-1}$

The trace and determinant of the system of the equation (14) can be shown as:

Trace = $\overline{r_t} - d_2 - 1$ and Determinant = $\overline{r_t}(-d_2 - 1) + (-d_2 + (1 - c_1 - d_1)\overline{r_t} + 1) = -d_2(1 + \overline{r_t}) > 0.$ (15)

By Routh=Hurwitz condition, our system shows that Trace < 0, and Determinant > 0 (stable).

5. Fiscal Policy rule and Tax rule to achieve sustainability of the budget and economic growth:

Fiscal policy rule can be obtained as follows:

The objective function of the government is set as follows.

$$L(B_t, Y_t, G_t, T_t) = \frac{1}{2}w_1(B_t - B_t^*)^2 + \frac{1}{2}w_2\left(Y_t - Y_t^f\right)^2 + \frac{1}{2}w_3(G_t - G_{t-1})^2 + \frac{1}{2}w_4(T_t - T_{t-1})^2 + \frac{1}{2}w_5(\Delta B_t - \Delta B_t^*)^2 \quad (16)$$

The government aims to stabilize government debt (B_t) as close as its desired level, GDP (Y_t) as close as its full employment level of GDP (Y_t^f), smooth change of government spending (G_t), smooth change of taxation (T_t), and smooth change of flow of bonds (ΔB_t) . Here w_i $(i = 1, \cdots, 5)$ are the *policy weights* where government can set up.

where

$B_t^* = \frac{d_2}{(c_1+d_1)}$ is the optimal target level of government debt and $r_t^* = \frac{(1-c_1-d_1)Y_t^f - \overline{W^D} - \overline{W}^F + (c_1+d_1)\overline{T} - (1-\theta+(c_1+d_1)\theta)\overline{G} + (c_0+d_0)}{d_2}$ is the optimal-target level interest rate.

Minimize Equation (16) base on the following macroeconomic equations.

$G_t + r_t B_{t-1} = \Delta B_t + T_t$ Government Budget Constraint (5-2)

$Y_t - T_t + r_t B_{t-1} + \theta G_t = C_t + S_t$ where $S_t = \Delta B_t + \Delta W_t^D - \Delta W_t^F$

Disposable income= Consumption(C_t) + Savings(S_t) = Government bonds(ΔB_t) + Domestic Deposits(ΔW_t^D) – Foreign Deposits (ΔW_t^F) where $\Delta W_t^F = \overline{W}^F$

$YD_t = Y_t + \theta G_t + r_t B_{t-1} - T_t = C_t + S_t$

Definition of disposable income consists of income (Y_t), government transfer (θG_t), interest receipt of government bond ($r_t B_{t-1}$) minus tax payment (T_t).

$C_t = c_0 + c_1 YD_t$ Consumption Equation

$W_t = d_0 + d_1 YD_t + d_2 \Delta r_t$ Deposit Equation

The temporal equilibrium value of the bond market is as follows:

$$r_t = \frac{1}{d_2-(c_1+d_1)B_{t-1}}[(1-c_1-d_1)Y_t - W_{t-1}^D - \Delta\bar{W}^F - (c_1+d_1)T_t - (1-\theta+(c_1+d_1)\theta)G_t + (c_0+d_0) + d_2 r_{t-1}]$$

$$B_t = \left[\frac{(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right]Y_t + B_{t-1} + \left[\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right]G_t - \left[\frac{d_2}{d_2-(c_1+d_1)B_{t-1}}\right]T_t + \left[\frac{B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right](c_0 + d_0 - W_{t-1}^D - \Delta\bar{W}^F) + \left[\frac{d_2 B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right]r_{t-1} \quad \text{(5-6)}$$

$$\frac{\partial L}{\partial G_t} = w_1(B_t - B_t^*)\left(\frac{\partial B_t}{\partial G_t}\right) + w_2\frac{\partial Y_t}{\partial G_t}\left(Y_t - Y_t^f\right) + w_3(G_t - G_{t-1}) + w_5(\Delta B_t - \Delta B_t^*)\left(\frac{\partial \Delta B_t}{\partial G_t}\right) = 0 \text{ where}$$

$$\frac{\partial B_t}{\partial G_t} = \left[\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right], \frac{\partial Y_t}{\partial G_t} = \theta, \frac{\partial \Delta B_t}{\partial G_t} = \left[\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right]$$

$$G_t = G_{t-1} + \frac{w_1}{w_3}(B_t - B_t^*)\left(\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right) - \frac{w_2}{w_3}\theta\left(Y_t - Y_t^f\right) + \frac{w_5}{w_3}(\Delta B_t - \Delta B_t^*)\left(\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right)$$

(17)

$$G_t - G_{t-1} = \alpha_1(B_t - B_t^*) + \alpha_2(\Delta B_t - \Delta B_t^*) + \alpha_3\left(Y_t - Y_t^f\right) \quad \text{(18)}$$

where $\alpha_1 = \frac{w_1}{w_3}\left(\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right), \alpha_2 = \frac{w_5}{w_3}\left(\frac{d_2-(1-\theta)(1-c_1-d_1)B_{t-1}}{d_2-(c_1+d_1)B_{t-1}}\right), \alpha_3 = -\frac{w_2}{w_3}\theta$

$$\frac{\partial L}{\partial T_t} = w_1(B_t - B_t^*)\left(\frac{\partial B_t}{\partial T_t}\right) + w_2\frac{\partial Y_t}{\partial T_t}\left(Y_t - Y_t^f\right) + w_4(T_t - T_{t-1}) + w_5(\Delta B_t - \Delta B_t^*)\left(\frac{\partial \Delta B_t}{\partial T_t}\right) = 0$$

where $\frac{\partial B_t}{\partial T_t} = -\frac{d_2}{d_2-(c_1+d_1)B_{t-1}}, \frac{\partial Y_t}{\partial T_t} = -1, \frac{\partial \Delta B_t}{\partial T_t} = -\frac{d_2}{d_2-(c_1+d_1)B_{t-1}}$

$$T_t - T_{t-1} = \beta_1(B_t - B_t^*) + \beta_2(\Delta B_t - \Delta B_t^*) + \beta_3\left(Y_t - Y_t^f\right) \quad \text{(19)}$$

$$\beta_1 = \frac{w_1}{w_4}\left(\frac{d_2}{d_2-(c_1+d_1)B_{t-1}}\right), \beta_2 = \frac{w_5}{w_4}\left(\frac{d_2}{d_2-(c_1+d_1)B_{t-1}}\right), \beta_3 = \frac{w_2}{w_4}$$

From these two FOCs, we can find the relationship between $G_t, T_t, (B_t - B_t^*)$, and $(\Delta B_t - \Delta B_t^*)$.

$$PB_t - PB_{t-1} = (\alpha_1 - \beta_1)(B_t - B_t^*) + (\alpha_2 - \beta_2)(\Delta B_t - \Delta B_t^*) + (\alpha_3 - \beta_3)\left(Y_t - Y_t^f\right) \quad \text{(20)}$$

6. Conclusion

Bohn's condition does not satisfy economic recovery. It only gives a condition that the budget balance is retained. Therefore the optimal government spending rule and the tax revenue rule have to follow equations (19) and (20) respectively. The optimal fiscal policy rule has to watch how the government debt diverges from the desired level and the GDP gap.

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