

# Aggregate Productivity Growth Decomposition: an Overview

---

Tetsushi Murao\*

Associate Professor, Department of Economic Engineering, Kyushu University

---

## Abstract

In this article, we review some recent developments of decomposition methods of the aggregate productivity growth by using firm/plant level datasets. These methods have been uncovering the sizable impacts of resource reallocation for the aggregate productivity growth. We categorize these methods into (i) *reduced form decomposition methods* (BHC-FHK decomposition, OP decomposition, and PL decomposition) and (ii) *the structural decomposition method* that is based on the structural estimation of an endogenous growth model (LM decomposition), and review each of them. By assessing the potential pros and cons of existing reduced form decomposition methods, we highlight features of the structural decomposition method. As an example of the use of the structural decomposition method, we introduce Murao and Nirei (2015) who quantitatively evaluate counterfactual deregulation of firm entry.

Keywords: aggregate productivity growth decomposition; resource reallocation; deregulation of entry

JEL Classification: O43, O47

---

## I. Introduction

Recent empirical research has uncovered the fact that it is crucial for aggregate productivity growth that how resources such as workers and capital are relocated among firms smoothly. Laws and regulations related to economic activity affect the macro economy sometimes by promoting and other times by interfering with such “resource reallocation”. In this article, we review some recent developments of aggregate productivity growth (APG) decomposition methods that served as a trigger to recognize the importance of such resource reallocation on the aggregate economy. Below, we categorize these methods into (i) *reduced form APG decomposition* (BHC-FHK decomposition, OP decomposition, and PL decomposition) and (ii) *the structural APG decomposition* that is based on the structural estimation of an endogenous growth model (LM decomposition), and review each of them.

The purpose of this article is as follows. First, by assessing the potential pros and cons of existing reduced form APG decomposition methods to highlight features of the structural

---

\* Correspondence: murao@econ.kyushu-u.ac.jp.

decomposition method. As far as the author notices, these points are not raised or discussed by the existing literature. Second, we introduce Murao and Nirei (2015) to review the possibility of using the structural decomposition method to counterfactual policy simulations.<sup>1</sup> While the reduced form APG decomposition methods have been widely used as ex-post policy evaluation, the most significant advantage of the structural decomposition method is the possibility of applying for the counterfactual (or ex-ante) policy evaluation.

The results of Murao and Nirei (2015) are summarized as follows. First, there is a U-shape relationship between the strength of the entry barrier and the APG. Thus, deregulation of entry is not necessarily associated with an increase in APG. Second, (a moderate amount of) deregulation of entry from the status quo increases APG in Japan, while the same policy decreases APG in Denmark.

The organization of this paper is as follows. In Section II, we introduce several reduced form APG decomposition methods: BHC-FHK decomposition, OP decomposition, and PL decomposition. In Section III, we review critiques on some existing reduced form APG decomposition methods. In Section IV, we argue the pros and cons of reduced form decomposition methods in general, that are not discussed in existing literature. In Section V, we introduce the structural APG decomposition methods, proposed by Lentz and Mortensen (2008). In Section VI, we provide a brief overview of Murao and Nirei (2015), who analyze quantitative impacts of counterfactual entry deregulation policy, by extending Lentz and Mortensen's framework. In Section VII, we conclude the paper.

## II. Reduced form APG decomposition methods

Existing APG decomposition methods are broadly divided into two categories, that is, (i) reduced form APG decomposition, and (ii) the structural APG decomposition. In this section, we introduce the former methods (i.e., BHC-FHK decomposition, OP decomposition, and PL decomposition).

### II-1. BHC-FHK decomposition

Firstly, we explain BHC-FHK decomposition, based on a simple two period exposition provided by Melitz and Polanec (2015).<sup>2</sup> With firm level panel data on market share and productivity in hand, aggregate productivity in period  $t$  is given by:

$$\Phi_t = \sum_i s_{it}\varphi_{it},$$

<sup>1</sup> Lentz and Mortensen (2008) do not conduct any counterfactual simulation.

<sup>2</sup> Melitz and Polanec (2015) argue that BHC-FHK decomposition has a potential bias such that entry/exit effects are over-estimated. To cope with this problem, they propose an alternative method called DOPD decomposition where entry/exit is incorporated into the OP decomposition.

where  $s_{it}$  is market share of firm  $i$  in the period  $t$ ;  $\varphi_{it}$  is productivity (in logarithmic term) of firm  $i$  in the period  $t$ . Using this expression, the rate of APG in the economy is given by  $\Delta\Phi = \Phi_2 - \Phi_1$ . Contribution for  $\Delta\Phi$  from each firm,  $(s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1})$ , is classified into three categories by firm's activity status:

- (i) Existing firms (active in both periods):  $s_{i1} > 0, s_{i2} > 0$
- (ii) Entry firms (only active in period 2):  $s_{i1} = 0, s_{i2} > 0$
- (iii) Exiting firms (only active in period 1):  $s_{i1} > 0, s_{i2} = 0$

With this observation, Baily, Hulten and Cambell (1992) propose a method to decompose APG:

$$\Delta\Phi = \sum_{i \in S} (s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1}) + \sum_{i \in E} s_{i2}\varphi_{i2} - \sum_{i \in X} s_{i1}\varphi_{i1},$$

where  $S$  is a set of indexes for existing firms;  $E$  is a set of indexes for entry firms;  $X$  is a set of indexes for exiting firms. In this equation, the first term is a contribution from existing firms; the second term is a contribution from entrants; and the third term is a contribution from exiting firms. Adding and subtracting  $\sum_{i \in S} (s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1})$  to and from the above equation, the following BHC decomposition formula can be obtained:

$$\Delta\Phi = \sum_{i \in S} s_{i1}(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in S} (s_{i2} - s_{i1})\varphi_{i2} + \sum_{i \in E} s_{i2}\varphi_{i2} - \sum_{i \in X} s_{i1}\varphi_{i1},$$

where the first term is counterfactual APG if the individual firm share would be held constant (*within effect*); the second term is APG through changes in the firm share (*between effects*); the third term is the *entry effect*; and the fourth term is the *exit effect*. Rewriting the above equation, the following BHC-FHK decomposition formula can be obtained:

$$\begin{aligned} \Delta\Phi = & \sum_{i \in S} s_{i1}(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in S} (s_{i2} - s_{i1})\varphi_{i1} + \sum_{i \in S} (s_{i2} - s_{i1})(\varphi_{i2} - \varphi_{i1}) \\ & + \sum_{i \in E} s_{i2}\varphi_{i2} - \sum_{i \in X} s_{i1}\varphi_{i1}, \end{aligned}$$

where the first term is the *within effect*; the second term is the *between effect*; and the third term is the *cross effect*.

## II-2. OP decomposition

Olley and Pakes (1996) propose an alternative decomposition method, called OP decomposition. Let  $\bar{s}_t \equiv (1/n) \sum_i s_{it}$ ,  $\bar{\varphi}_t \equiv (1/n) \sum_i \varphi_{it}$ ,  $\Delta s_{it} \equiv s_{it} - \bar{s}_t$  and  $\Delta \varphi_{it} \equiv \varphi_{it} - \bar{\varphi}_t$ , aggregate productivity is written as follows:

$$\begin{aligned}\Phi_t &\equiv \sum_i s_{it} \varphi_{it} = \sum_i (\bar{s}_t + \Delta s_{it})(\bar{\varphi}_t + \Delta \varphi_{it}) \\ &= \sum_i \bar{s}_t \bar{\varphi}_t + \bar{s}_t \sum_i \Delta \varphi_{it} + \left( \sum_i \Delta s_{it} \right) \bar{\varphi}_t + \sum_i \Delta s_{it} \Delta \varphi_{it}.\end{aligned}$$

In the second line of this equation, the second term and the third term is zero, since these two terms are the summation of deviation from the mean. Consequently, we obtain the following OP decomposition formula:

$$\Phi_t = \bar{\varphi}_t + \sum_{i \in E} (s_{it}^H - \bar{s}_t)(\varphi_{it}^H - \bar{\varphi}_t),$$

where the first term is mean productivity which captures the effect from a shift of the productivity distribution; the second term is called *covariance effect*. While the “cross effect” of the BHC-FHK decomposition captures intertemporal covariance between productivity and market share for the same firm, this “covariance effect” captures the cross-section covariance between productivity and market share in the same period. As will be clear in the next section, this difference is related to the theoretical consistency of these two methods.

### II-3. PL decomposition

The BHC-FHK decomposition and OP decomposition are designed to capture aggregate TFP growth through resource reallocation from lower TFP firms to higher TFP firms. Meanwhile, Petrin and Levinsohn (2012) propose an alternative method which decomposes APG into improvement in technical efficiency and resource reallocation, called PL decomposition. Moreover, Petrin and Levinsohn (2012) discuss that there are some shortcomings in decomposition methods such as the BHC-FHK which solely rely on the information in technical efficiency. In what follows, we introduce the PL decomposition method.

Here we ignore the entry/exit of firms for simplicity.<sup>3</sup> Let production function for firm  $i$  be given by:<sup>4</sup>

$$y_{it} = f(k_{it}, \ell_{it}, \varphi_{it}),$$

where  $y$  is (real) output;  $k$  is capital;  $\ell$  is labor input; and  $\varphi$  is productivity. Then, APG is defined as the amount of change in Solow residual between different periods:

<sup>3</sup> See the Appendix of Petrin and Levinsohn (2012) for the case with firm entry and exit.

<sup>4</sup> For simplicity, we also ignore intermediate inputs which are also take into consideration in Petrin and Levinsohn (2012).

$$APG(t) = \sum_{i=1}^n p_i(t) dy_i(t) - \sum_{i=1}^n r_i(t) dk_i(t) - \sum_{i=1}^n w_i(t) d\ell_i(t),$$

where  $p$  is good price;  $w$  is wage; and  $r$  is capital cost. Based on this relationship, Petrin and Levinsohn (2012) propose the following formula to decompose APG into the technological efficiency term and the reallocation term:

$$APG \equiv TE + RE,$$

and

$$TE \equiv \sum_i p_i d\varphi_i,$$

$$RE \equiv \sum_i \left( p_i \frac{\partial y_i}{\partial k_i} - r_i \right) dk_i + \sum_i \left( p_i \frac{\partial y_i}{\partial \ell_i} - w_i \right) d\ell_i,$$

where TE is the *technical efficiency* term and RE is the *reallocation* term, respectively. The economic interpretation of these two terms are as follows. The technical efficiency term captures an increase in aggregate productivity by an increase in output, when amount of the production input would be holding fixed. The intuition behind the reallocation term is as follows. Firstly, the reallocation term can be seen as an aggregate of a multiple of (i) “gap” between value of marginal products and nominal prices ( $p \cdot MPL - w$ ,  $p \cdot MPK - r$ ) and (ii) changes in each production input. Following Petrin and Levinsohn (2012), let us take labor input as an example. Consider now the case where unit labor is reallocated from firm  $j$  to firm  $i$  (wage is assumed to be the same across firms):  $d\ell_i = 1$ ,  $d\ell_j = -1$ . Then, if aggregate labor input is held fixed, aggregate output increases by:

$$p_i \frac{\partial y_i}{\partial \ell_i} - p_j \frac{\partial y_j}{\partial \ell_j}.$$

Thus, by reallocating inputs from firms with low value of marginal products to high value of marginal products, total output can be increased without any increase in total input in the whole economy and/or the improvement in the technical efficiency. In general, the total output is increased if resources are reallocated from firms with a smaller “gap” between the value of marginal products and nominal input price to firms with a larger “gap”, this increase in output is what the reallocation term of PL decomposition captures exactly. In other words, the PL decomposition’s reallocation term is positive if (i) there is a positive “gap” and if (ii) resources are reallocated from low- to high- gap firms.<sup>5</sup>

<sup>5</sup> As is shown in Corollary 3 of Petrin and Levinsohn (2012), there are two extreme cases where the PL reallocation term turns out to be zero: (i) there is no moving of input because of sufficiently large frictions ( $dL = 0$ ,  $dK = 0$ ) or (ii) “gap” is zero because of no input market friction.

### III. Theoretical consistency of BHC-FHK and OP decomposition

While productivity growth decomposition methods reviewed above have been widely used, there are criticisms on some methods with regard to theoretical consistency. In this section, we review some of them.

#### III-1. BHC-FHK decomposition under stochastic stationary environment

Lentz and Mortensen (2008) argue that BHC-FHK decomposition's between- and cross-term both turn out to be zero in the stochastic stationary environment. Though Lentz and Mortensen (2008) use quite a general environment, below we confirm this in a simple setting.

There are a large number of firms in the economy and each firm is classified into either high productivity (H-type) or low productivity (L-type). There is no firm entry or exit for simplicity. Thus, APG is completely attributed to incumbent's efficiency improvement and resource reallocation. Number of H-type and L-type firms are given by  $n_H$  and  $n_L$ , respectively. Then, contribution to aggregate productivity by H-type firms and L-type firms are given by  $\Phi_{it}^H = \sum_{i \in I_H} s_{it}^H \varphi_{it}^H$  and  $\Phi_{it}^L = \sum_{i \in I_L} s_{it}^L \varphi_{it}^L$ , respectively, where  $I_H$  is a set of index of type-H firms and  $I_L$  is a set of index of type-L firms. Thus, aggregate productivity can be rewritten as below:

$$\Phi_t \equiv \sum_i^n s_{it} \varphi_{it} = \sum_{i \in I_H}^{n_H} s_{it}^H \varphi_{it}^H + \sum_{i \in I_L}^{n_L} s_{it}^L \varphi_{it}^L \equiv \Phi_t^H + \Phi_t^L.$$

Therefore,

$$\Delta \Phi^H = \sum_{i \in I_H} s_{i1}^H (\varphi_{i2}^H - \varphi_{i1}^H) + \sum_{i \in I_H} (s_{i2}^H - s_{i1}^H) \varphi_{i1}^H + \sum_{i \in I_H} (s_{i2}^H - s_{i1}^H) (\varphi_{i2}^H - \varphi_{i1}^H),$$

where the first term is within effect; the second term is between effect; and the third term is cross effect.

Lentz and Mortensen (2008) argue that, if (i) firms are classified into a finite number of efficiency types (that is,  $\varphi_{it}^H = \varphi_t^H, \forall t$ ) and (ii) share of an individual firm's market share varies stochastically, then the firm share of each type should be constant in the stationary equilibrium (that is,  $(1/n_H) \sum_{i \in I_H} s_{it}^H \equiv \bar{s}_t^H = \bar{s}^H, \forall t$ ). Thus plugging  $\varphi_{it}^H = \varphi_t^H$  and  $(1/n_H) \sum_{i \in I_H} s_{it}^H \equiv \bar{s}^H$  into the above equation, we have:

$$\begin{aligned} \Delta \Phi^H &= (\varphi_2^H - \varphi_1^H) \sum_{i \in I_H} s_{i1}^H + \varphi_1^H \sum_{i \in I_H} (s_{i2}^H - s_{i1}^H) + (\varphi_2^H - \varphi_1^H) \sum_{i \in I_H} (s_{i2}^H - s_{i1}^H) \\ &= (\varphi_2^H - \varphi_1^H) \sum_{i \in I_H} s_{i1}^H + \varphi_1^H (\bar{s}_t^H - \bar{s}_t^H) + (\varphi_2^H - \varphi_1^H) (\bar{s}_t^H - \bar{s}_t^H) \\ &= (\varphi_2^H - \varphi_1^H) \sum_{i \in I_H} s_{i1}^H. \end{aligned}$$

Thus, between term and cross term both turn out to be zero in this case. Gross reallocation term, the sum of the between and the cross term, also becomes zero. From the above result, Lentz and Mortensen (2008) argue that BHC-FHK decomposition is not consistent with a stochastic stationary environment. Note that, since actual data contains some measurement errors and transitory shocks, these terms would not necessarily be zero in reality. Lentz and Mortensen (2008) conduct BHC-FHK decomposition using Danish data to report that, while the between and the cross term is not exactly zero (and both not significant), the measured gross reallocation term (between + cross) is very close to zero.

### III-2. OP decomposition for stochastic stationary environment

Meanwhile, does OP decomposition have the same problem with BHC-FHK decomposition? Because Lentz and Mortensen (2008) do not discuss this point, below we assess this by using the same environment as before. Let  $\bar{s}_t \equiv (1/n) \sum_i s_{it}$ ,  $\bar{\varphi}_t \equiv (1/n) \sum_i \varphi_{it}$ ,  $\Delta s_{it}^H \equiv s_{it}^H - \bar{s}_t$ , and  $\Delta \varphi_{it}^H \equiv \varphi_{it}^H - \bar{\varphi}_t$ , contribution of H-type firms to aggregate productivity is given by:

$$\begin{aligned} \Phi_t^H &= \sum_{i \in I_H}^{n_H} s_{it}^H \varphi_{it}^H = \sum_{i \in I_H}^{n_H} (\bar{s}_t + \Delta s_{it}^H)(\bar{\varphi}_t + \Delta \varphi_{it}^H) \\ &= \sum_{i \in I_H}^{n_H} \bar{s}_t \bar{\varphi}_t + \bar{s}_t \sum_{i \in I_H}^{n_H} \Delta \varphi_{it}^H + \left( \sum_{i \in I_H}^{n_H} \Delta s_{it}^H \right) \bar{\varphi}_t + \sum_{i \in I_H}^{n_H} \Delta s_{it}^H \Delta \varphi_{it}^H. \end{aligned}$$

A similar calculation can be done for L-type firms to obtain  $\Phi_{it}^L$ . Summing the second term for the type-H and the type-L, and the third term for the type-H and the type-L respectively, then these two terms turn out to be zero (these two terms reduce to the sum of deviation from the mean). Consequently, the above equation is given by:

$$\Phi_t^H = n_H \bar{s}_t \bar{\varphi}_t + \sum_{i \in I_H}^{n_H} (s_{it}^H - \bar{s}_t)(\varphi_{it}^H - \bar{\varphi}_t),$$

As discussed in the previous subsection, in the stochastic stationary equilibrium, (i) firms with the same type have the same productivity ( $\varphi_{it}^H = \varphi_t^H, \forall t$ ) and (ii) firm type share is constant ( $(1/n_H) \sum_{i \in I_H} s_{it}^H \equiv \bar{s}_t^H = \bar{s}^H, \forall t$ ). Thus, the above equation can be rewritten as:<sup>6</sup>

$$\Phi_t^H = n_H \bar{s}_t \bar{\varphi}_t + n_H (\bar{s}^H - \bar{s})(\varphi_t^H - \bar{\varphi}_t).$$

The first term in this equation is a contribution of an H-type firm to a mean productivity; the second term is a contribution of an H-type firm to “covariance term”. Using these results,

<sup>6</sup> Note that  $\bar{s} = \bar{s}_t = (1/n) \sum_i s_{it} = 1/n, \forall t$ .

the time variation of  $\Phi_t^H$  is given by:

$$\Delta\Phi^H \equiv \Phi_2^H - \Phi_1^H = n_H(\bar{s}_2\bar{\varphi}_2 - \bar{s}_1\bar{\varphi}_1) + n_H(\bar{s}^H - \bar{s})[(\varphi_2^H - \varphi_1^H) - (\bar{\varphi}_2 - \bar{\varphi}_1)].$$

As noted above, the first term in this equation is a contribution of H-type firms to mean productivity and the second term is a contribution of H-type firms to the ‘‘covariance’’ term. Thus, the covariance term in the OP decomposition cannot be zero even when we postulate the stochastic stationary environment. In other words, in contrast to the BHC-FHK decomposition, we can conclude that the OP decomposition is consistent with a stochastic stationary environment.

### III-3. Criticism by Petrin and Levinsohn (2012)

Petrin and Levinsohn (2012) discuss a shortcoming of the BHC-FHK decomposition from a different perspective. Consider an economy with two firms. The sole production input is labor and (inelastic) aggregate labor supply is given by  $\bar{\ell}$ . The production function satisfies standard conditions:  $f_\ell > 0$ ;  $f_{\ell\ell} < 0$ ;  $f_\omega > 0$ . Assume that the productivity of firm 1 is higher than that of firm 2 ( $\varphi_1 > \varphi_2$ ). In this case, the amount of labor inputs that maximize economy wide output is given by  $(\ell_1^*, \ell_2^*)$  such that:

$$\frac{\partial y(\ell_1^*, \varphi_1)}{\partial \ell} = \frac{\partial y(\ell_2^*, \varphi_2)}{\partial \ell},$$

$$\ell_1^* + \ell_2^* = \bar{\ell}.$$

Under the above efficiency condition, if the unit labor is reallocated from the lower productivity firm (firm 2) to the higher one (firm 1), labor input is now given by  $\ell_1 > \ell_1^*$  and  $\ell_2 < \ell_2^*$  and that:

$$\frac{\partial y(\ell_1, \varphi_1)}{\partial \ell} < \frac{\partial y(\ell_1^*, \varphi_1)}{\partial \ell} = \frac{\partial y(\ell_2^*, \varphi_2)}{\partial \ell} < \frac{\partial y(\ell_2, \varphi_2)}{\partial \ell}$$

Therefore, we have:

$$\frac{\partial y(\ell_1, \varphi_1)}{\partial \ell} - \frac{\partial y(\ell_2, \varphi_2)}{\partial \ell} < 0$$

In other words, such reallocation reduces an economy wide output. However, as BHC-FHK decomposition solely relies on information of technical efficiency, the aggregate productivity index  $\Phi$  can be *increased* by such a reallocation.

## IV. Pros and cons of decomposition methods based on production function estimation

The advantage of reduced form decomposition methods stem from the fact that these



methods are quite simple. In particular, there are the following two advantages in these methods.

First, these methods are easy to conduct: thus, comparable results have been steadily accumulated by applying the methods to datasets from many countries and many different periods. These studies have been contributing to our understanding of firm and industry productivity dynamics. For a recent example, Bartelsman, Haltiwanger and Scarpetta (2013) apply OP decomposition to internationally comparable datasets of several countries to reveal that the OP covariance term plays a significant role for aggregate productivity. One another example is a series of research about the long stagnation of the macro economy in Japan. Fukao and Kwon (2006), Nishimura, Nakajima and Kiyota (2005) and Kwon, Narita and Narita (2015) study causes of aggregate productivity slowdown in Japan after the bubble burst from the perspective of productivity dynamics.

Moreover, a vast amount of exercises of productivity decomposition have built a momentum that various economic phenomena are reexamined from the perspective of the relationship between resource reallocation and aggregate productivity. For example, Melitz (2003) proposed a new mechanism about “gains from trade” that trade liberalization promotes resource reallocation from lower to higher productivity firms to raise economic welfare.<sup>7</sup> Hsieh and Klenow (2009) use plant level datasets to reveal that about half of the difference of TFP between the U.S. and the India can be attributed to the (in)efficiency of resource reallocation.

Second, reduced form decomposition methods have a wide range of possibilities for extension. See, for example, Collard-Wexler and De Loecker (2015) or Nishiwaki and Kwon (2013) for such an application.

While reduced form decomposition methods have many virtues, there are at least three limitations. First, there is a limitation regarding the maintained assumption. At first glance, these methods have less concern about misspecification, since it seems to not rely on specific theoretical models. However, it is worth noting that, these methods have all postulated implicit assumptions about productivity dynamics: it is assumed that labor input does not affect technical efficiency at all. However, as a vast amount of empirical research about R&D has revealed, (at least a part of) growth of technical efficiency is caused by innovation (in a broader sense). Thus, firms optimally determine the level of labor input for R&D (in a broader sense) by taking innovation’s future benefit into consideration. In this respect, as in PL decomposition, defining reallocation effects in terms of a gap between (static) value of marginal products and input price is undesirable. Moreover, extensively used methods of production function estimation, such as OP (Olley and Pakes, 1996) or LP (Levinsohn and Petrin, 2003) assume that firm productivity follows an exogenous stochastic process. Since it is necessary to pose the same assumptions for the estimation of production function and the APG decomposition, reduced form methods widely share this problem.

---

<sup>7</sup> Theoretical and empirical research on recent trade models with heterogeneous firms, see Melitz and Redding (2014).

Second, the mechanism behind measured decomposed components is kept in a black box. This may be particularly problematic when these methods are applied for policy analyses. When researchers can access data both before and after the policy change, it is possible to quantify how the policy affects the value of each decomposed term. For example Pavcnik (2002) conduct OP decomposition before and after the Chilean trade liberalization to study the effect of trade liberalization on resource reallocation. Petrin and Sivadasan (2013) conduct PL decomposition using the same Chilean plant dataset to study the effect of change in the strength of employment protection on resource reallocation. However, it is still unclear that what kind of behavioral response by individual firms induce changes in “measured values”. Identifying the mechanism behind changes in each decomposed term caused by policy change is critical to predict the effects of similar policy changes in different contexts.

Third and most important, these methods can apply to policy evaluation only when researchers could access datasets that contain both before and after the policy change. In other words, these methods cannot be applied to the counterfactual (or ex-ante) policy evaluation that sometimes policy makers are most interested in.

The above problems are all results from the fact that these methods do not have a mechanism for which the value of each of the measured terms are determined.

## **V. The structural APG decomposition method – LM decomposition**

Lentz and Mortensen (2008, hereafter LM) utilize an endogenous growth model developed by Klette and Kortum (2004, hereafter KK) to derive a new formula for APG decomposition. The most distinctive feature of the formula is that, the contribution of each decomposed component to APG is completely described by the structural model. By estimating structural parameters from firm panel data, each decomposed component can be evaluated quantitatively. Though Lentz and Mortensen (2008) do not conduct any counterfactual simulation, Murao and Nirei (2015) incorporate free entry condition and entry cost into their framework and evaluate quantitative impacts of counterfactual entry deregulation policy. In this section, we briefly overview the LM decomposition formula and its application of counterfactual policy simulation by Murao and Nirei (2015).

### *V-1. Overview of LM model*

Since the LM decomposition formula is derived directly from a structural model, we first introduce an endogenous growth model of Klette and Kortum (2004) and its extension of Lentz and Mortensen (2008).

These models are based on a “quality ladder” framework which is proposed and developed by Grossman and Helpman (1991) and Aghion and Howitt (1992). There is only one consumption good in the economy. This consumption good is produced from many intermediate input goods.

The quality ladder models focus entirely on R&D aimed at improvement in these intermediate goods. Each intermediate input is produced by only one firm: the firm with the highest quality of each good produces the good. Below, the intermediate good firm that is undergoing production is called as *incumbent firm*. In addition, there are many *potential entrants* who engage in (process) R&D: if a potential entrant could successfully improve the quality of an intermediate good, then it enters the market and drives out the previous producer. Innovation which induces such “creative destruction” is nothing less than the source of economic growth in quality ladder models. In other words, quality ladder models are a rigorous mathematical representation of Schumpeter’s original idea of growth through creative destruction in the capitalist economy.

Since the 1990s, emerging empirical research of firm microdata observes that there are non-negligible amounts of dispersion in productivity or firm size, even after controlling the precise industry classification. Klette and Kortum (2004) introduce firm size heterogeneity in the original quality ladder model to provide an appropriate framework to replicate such circumstances. Departing from the original quality ladder model, firms in the KK economy have a possibility to produce a multiple number of goods. This extension provides R&D incentives for incumbent firms, because not only a potential entrant but an incumbent firm has opportunity to take over the new intermediate goods market. Since the number of intermediate goods differ from firm to firm, a large amount of dispersion in the firm size (measured in number of employees) emerges in the KK model.

Lentz and Mortensen (2008) extend the KK model by allowing heterogeneity in the ex-ante innovation efficiency which is determined at entry to the market.<sup>8</sup> In consequence of this type of heterogeneity, product share of high-type firms constantly increases within the same firm-age cohort in the LM model. Thus, the LM model has a new source of productivity growth, that is, an expansion of the share of high-type firms which is called the *selection effect*. In sum, the LM model has three channels of APG: that is, (i) entry and exit of firms (entry exit effect), (ii) reallocation of market share among heterogeneous types (selection effect), and (iii) quality improvement itself (within effect). Lentz and Mortensen (2008) estimate these three effects from Danish microdata.

## *V- 2. Characteristic of stationary equilibrium: market share reallocation among incumbent firms*

Since innovation efficiency is heterogeneous in LM economy, R&D incentive is increasing in the innovation efficiency. Therefore, product share of high-type firms constantly penetrates low-type firms’ share. Meanwhile, there exists a stationary equilibrium in this economy where type share in the product space remains constant over time. Though these two facts may seem to contradict each other, they do not because firm-age cohorts on the

---

<sup>8</sup> In the original quality ladder model, “innovation efficiency” is assumed to be homogenous for all firms.

other hand constantly contract over time after entry. That is, in stationary equilibrium, share of products by high-innovation type firms constantly expand while each cohort size is constantly shrinking.

### V-3. LM decomposition formula

Lentz and Mortensen (2008) show that APG in stationary equilibrium in this economy is decomposed as following:

$$g = \underbrace{\eta \sum_{\tau} E[\ln \tilde{q}_{\tau}] \phi_{\tau}}_{\text{entry/exit effect}} + \underbrace{\sum_{\tau} \gamma_{\tau} E[\ln \tilde{q}_{\tau}] [K_{\tau} - \phi_{\tau}]}_{\text{selection effect}} + \underbrace{\sum_{\tau} \gamma_{\tau} E[\ln \tilde{q}_{\tau}] \phi_{\tau}}_{\text{within effect}}$$

where  $\eta$  is the entry rate of new firms;  $\gamma_{\tau}$  is the innovation arrival rate for type- $\tau$  firms;  $E[\ln \tilde{q}_{\tau}]$  is the expected rate of productivity improvement for intermediate goods produced by type- $\tau$  firms;  $K_{\tau}$  is a share of goods produced by type- $\tau$  firms; and  $\phi_{\tau}$  is a share of goods produced by type- $\tau$  firms. Note that exactly the same formula is derived in the Murao and Nirei (2015) economy where the entry cost and free entry condition are introduced into the Lentz and Mortensen (2008) framework: although the decomposition formula does not contain the “strength of the entry barrier” ( $c_e$ , in Murao and Nirei, 2015), the entry barrier affects each decomposed component through changing values of endogenous variables such as innovation arrival rate  $\gamma_{\tau}$  or entry rate  $\eta$ .

Each decomposed component in the LM-decomposition formula can be interpreted as follows. First, the *entry exit effect* is a contribution of the entry and exit of firms, which is given by a multiple of instantaneous entry rate ( $\eta$ ) and the average productivity growth rate ( $\sum_{\tau} E[\ln \tilde{q}_{\tau}] \phi_{\tau}$ ). Second, the *selection effect* is a contribution of market share reallocation from low to high innovation efficiency firms. Third, the *within effect* is a contribution of technical efficiency improvement (when type distribution would be held constant over time).

## VI. Application of LM decomposition formula to counterfactual policy simulation

A distinct feature of the LM decomposition formula is that it is directly derived from a theoretical model. As a consequence, this method can be used as a counterfactual policy simulation by quantifying impacts of such policy changes to each decomposed component, without facing the “Lucas critique”. In contrast, BHC-FHK, OP and PL decompositions are not based on particular theoretical models that describe the firm behavior: these decomposition methods are not suitable for counterfactual policy evaluation. Below, we review Murao and Nirei (2015) who quantify impacts of counterfactual entry deregulation policy by utilizing the LM framework.

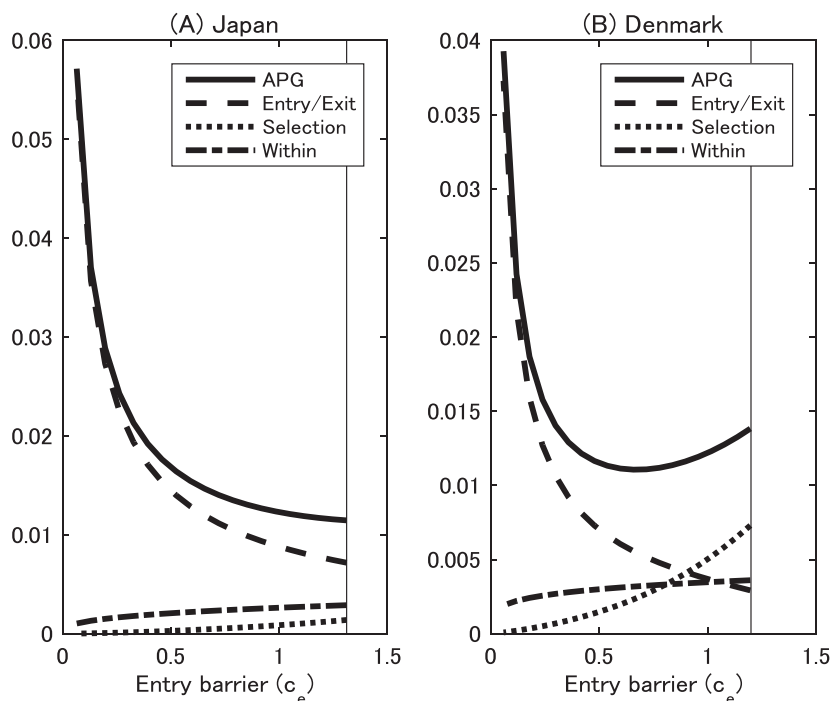
Murao and Nirei (2015) introduce entry cost and free entry conditions into Lentz and Mortensen (2015) and estimate structural parameters using firm panel data of Japan. Then, they set the counterfactual value of  $c_e$  and re-solve the model to quantitatively evaluate each term in the LM decomposition formula. Moreover, Murao and Nirei (2015) estimate  $c_e$  in Denmark by using estimates from Lentz and Mortensen (2008), and then conduct counterfactual simulations of entry deregulation in Denmark.<sup>9</sup>

Figure 1 shows results of the counterfactual simulation that the strength of the entry barrier is reduced from the estimated value.

Results are summarized as follows.

1. Reducing the strength of the entry barrier, two types of resource reallocation, the entry/exit effect and the selection effect, changes in the opposite direction: the entry/exit effect increases and selection effect decreases monotonically.
2. As the strength of the entry barrier decreases, the aggregate productivity growth rate

Figure 1. Results of counterfactual entry deregulation. A vertical line in each panel shows the estimated value of the strength of the entry barrier.



Source: Murao and Nirei (2015).

<sup>9</sup> The strength of the entry barrier in Denmark is estimated from Lentz and Mortensen's original estimates, because it is shown that there is no bias in original LM estimates (except for the level of aggregate labor supply which is not used for the LM decomposition) which is estimated under the assumption that there is no entry cost. See Murao and Nirei (2015) for more detail.

increased monotonically: that is, the increase in the entry/exit effect dominates the decrease in the selection (and within) effect.

3. The relationship between the strength of the entry barrier and aggregate productivity growth rate exhibits a U-shape: that is, if  $c_e$  is constantly reduced, an increase in the entry/exit effect is dominated by a decrease in the selection (and within) effect at some point.

In particular, intuition behind “reduced selection” by entry deregulation requires an explanation among the above. The reduced selection effect indicates that resource reallocation from the lower type to the higher type firms become more sluggish. The intuition is explained as follows. First, there is a well-known “Schumpeterian effect” in the original quality ladder model of Grossman and Helpman (1991) or Aghion and Howitt (1992) that the within effect is reduced by a decrease in  $c_e$  (that is, entry deregulation): this is because, firstly, lowering  $c_e$  promotes new firm entry, and this activates creative destruction by new entrants; thus, R&D incentive by an innovator is lowered because the expected duration that a successful innovator gains monopolistic rents becomes shorter; in other words, lowering  $c_e$  has an effect of reducing “post-innovation rent”, called the *Schumpeterian effect*.

The reason of reduced selection by lowered  $c_e$  is that this Schumpeterian effect varies from firm to firm. Note that, in the LM framework, firms are different in innovation efficiency. Estimates of Murao and Nirei (2015) show that the least innovation-efficient firms do not conduct R&D at all, once they enter the market. In Japanese firm panel data used by Murao and Nirei (2015), these firms amount to 60% of the whole sample of the manufacturing sector. Under these circumstances, the Schumpeterian effect is different among firms. The reason is explained as follows. If the strength of entry barrier  $c_e$  is reduced, while the R&D incentive of firms with higher innovation efficiency is reduced (by the standard Schumpeterian effect), firms with the least innovation efficiency do not change the level of R&D (remains zero). Thus, the difference of R&D incentives between the higher- and lower-innovation efficiency firms decrease, and thus selection pressure to the lower innovation efficient firms decreases.

Then, is (moderate amount of) entry deregulation harmful to economic growth? From figure 1, such a possibility exists in Denmark while not in Japan: the positive effect for growth by deregulation of entry from the status quo (that is, increase in entry/exit effect) dominates the negative effect (that is, decrease in the selection and the within effect).

## VII. Conclusion

In this article, we introduce several recent decomposition methods of aggregate productivity growth utilizing firm/plant level panel datasets. We broadly categorize these methods into (i) reduced form decomposition methods and (ii) the structural APG decomposition method: we discuss pros and cons of the reduced form decomposition method to clarify significance of the structural decomposition method. Next, we discuss the usefulness of the second method to counterfactual policy evaluation by introducing Murao and Nirei

(2015) as an example. There is increasing interest in the research on productivity dynamics and much further effort should be devoted to it.

## References

- Aghion, P. and P. Howitt (1992), "A Model of Growth through Creative Destruction," *Econometrica*, vol. 60(2), 323-51.
- Asplund, M. and V. Nocke (2006), "Firm Turnover in Imperfectly Competitive Markets," *Review of Economic Studies*, vol. 73(2), 295-327.
- Bailey, M., G. Hulten, and D. Campbell (1992) "Productivity dynamics in manufacturing plants." In *Brookings Papers on Economic Activity: Microeconomics*, vol. 4, 187-267.
- Barseghyan (2004), "Entry costs and cross-country differences in productivity and output," *Journal of Economic Growth*, 13:145-167.
- Bartelsman, E., J. Haltiwanger and S. Scarpetta (2013) "Cross-Country Differences in Productivity: The Role of Allocation and Selection," *American Economic Review*, 103(1), 305-334.
- Blanchard, O. and Giavazzi (2003), "Macroeconomic Effects of Regulation and Deregulation in Goods and Labour Markets," *Quarterly Journal of Economics*, 118(3), 879-907.
- Cohen, W. M. and Klepper, S. (1992), "The Anatomy of Industry R&D Intensity Distributions," *American Economic Review*, vol. 82(4), 773-99.
- Collard-Wexler, A. and J. De Loecker (2015) "Reallocation and Technology: Evidence from the US Steel Industry," *American Economic Review*, 105(1), 131-171.
- Dasgupta, P. and Stiglitz, J., (1980), "Industrial structure and the nature of innovative activity," *Economic Journal*, vol. 90(358), 266-93.
- Djankov, R. La Porta, and F. Lopez-de-Silanes, A. S. Shleifer (2002), "The Regulation of Entry," *Quarterly Journal of Economics*, CXVII (1), 1-37.
- Foster, L., J. C. Haltiwanger and C. J. Krizan (2001), "Aggregate Productivity Growth. Lessons from Microeconomic Evidence," in *New Developments in Productivity Analysis*, 303-372.
- Fukao, K. and H. U. Kwon (2006) "Why Did Japan's TFP Growth Slow Down in the Lost Decade? An Empirical Analysis Based on Firm-Level Data of Manufacturing Firms," *Japanese Economic Review*, 57 (2), 195-228.
- Grossman, G. and E. Helpman (1991), *Innovation and growth in the global economy*, MIT Press, Boston.
- Hopenhayn, H. A (1992), "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, vol. 60(5), 1127-50.
- Hopenhayn, H. and R. Rogerson (1993), "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, vol. 101(5), 915-38.
- Hsieh, C-T. and P., Klenow (2009), "Misallocation and manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403-1448.
- Kaufmann, D., A. Kraay, and M. Massimo (2010), "The worldwide governance indicators:

- methodology and analytical issues,” *Policy Research Working Paper Series* 5430, The World Bank.
- Klette, T. J. and S. Kortum (2004), “Innovating Firms and Aggregate Innovation,” *Journal of Political Economy*, vol. 112(5), 986-1018.
- Kwon, H.U., F. Narita and M. Narita (2015), “Resource Reallocation and Zombie Lending in Japan in the 1990s,” *Review of Economic Dynamics*, 18(4), 709-732.
- Levinsohn, J. and A. Petrin (2003), “Estimating Production Functions Using Inputs to Control for Unobservables,” *Review of Economic Studies*, 70 (2), 317-341.
- Lentz, R. and D. Mortensen (2008), “An Empirical Model of Growth through Product Innovation,” *Econometrica*, 76(6), 1317-1373.
- Loayza, N., A.M. Oviedo., and L. Servén (2005), “Regulation and macroeconomic performance,” *World Bank Research Paper*, 3469.
- Melitz, M. J. (2003), “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, vol. 71(6), 1695-1725.
- Melitz, M. J., and S. J Redding (2014), “Heterogeneous firms and trade,” in *Handbook of International Economics*, 4th ed., 4: 1-54.
- Melitz, M.J. and S. Polanec (2015), “Dynamic Olley-Pakes productivity decomposition with entry and exit,” *Rand Journal of Economics*, 46(2), 362-375.
- Murao, T. and M. Nirei (2015), “Non-monotonic Relationship between Competition and Growth: The Role of Resource Reallocation,” mimeograph, Kyushu University.
- Nicoletti, G. and S. Scarpetta (2003), “Regulation, productivity, and growth: OECD evidence,” *Economic Policy*, 18(36), 9-72.
- Nicoletti, G., S. Scarpetta, and O. Boylaud (2000), “Summary Indicators of Product Market Regulation with an Extension to Employment Protection Legislation,” *OECD Economics Department Working Papers*.
- Nishimura, K. G., T. Nakajima, and K. Kiyota, (2005) “Does the Natural Selection Mechanism Still Work in Severe Recessions? Examination of the Japanese Economy in the 1990s,” *Journal of Economic Behavior and Organization*, 58(1), 53-78.
- Nishiwaki, M. and H.U. Kwon (2013) “Are Losers Picked? An Empirical Analysis of Capacity Divestment and Production Reallocation in the Japanese Cement Industry,” *Journal of Industrial Economics*, 61(2), 430–467.
- Olley, S and A. Pakes (1996), “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 64, 1263-1298.
- Pavcnik, N. (2002), “Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants,” *Review of Economic Studies*, 69(1), 245-276.
- Petrin, A. and J. Levinsohn (2012), “Measuring aggregate productivity growth using plant-level data,” *Rand Journal of Economics*, 43(4), 705-712.
- Petrin, A. and J. Sivadasan (2013) “Estimating Lost Output from Allocative Inefficiency, with an Application to Chile and Firing Costs,” *Review of Economics and Statistics*, 95(1), 286-301.
- Pryor, F. L. (2002), “Quantitative Notes on the Extent of Governmental Regulations in



Various OECD Nations,” *International Journal of Industrial Organization*, 20(5), 693-714.

Restuccia, D. and R. Rogerson (2004), “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,” *Review of Economic Dynamics*, 11(4), 707-720.

